Evaluation of Statistical Quality Control of Concrete

1- Variation in strength of concrete

The strength of concrete produced in sites is varied between mixes and even in in the same mix due to the following reasons:

Table 2.1—Principal sources of strength variation

Variations due to the properties of concrete	Variations due to testing methods
•Changes in *w/cm* caused by: -Poor control of water -Excessive variation of moisture in aggregate or variable aggregate moisture measurements -Retempering	•Improper sampling procedures
•Variations in water requirement caused by: -Changes in aggregate grading, absorption, particle shape -Changes in cementitious and admixture properties -Changes in air content -Delivery time and temperature changes	•Variations due to fabrication techniques: -Handling, storing, and curing of newly made cylinders -Poor quality, damaged, or distorted molds
•Variations in characteristics and proportions of ingredients: -Aggregates -Cementitious materials, including pozzolans -Admixtures	•Changes in curing: -Temperature variation -Variable moisture control -Delays in bringing cylinders to the laboratory -Delays in beginning standard curing
•Variations in mixing, transporting, placing, and consolidation	•Poor testing procedures: -Specimen preparation -Test procedure -Uncalibrated testing equipment
•Variations in concrete temperature and curing	

2- Analysis of Strength Data

2.1 Definitions

- **Concrete sample**—a portion of concrete, taken at one time, from a single batch or single truckload of concrete.
- **Single cylinder (cube) strength or individual strength**—thestrength of a single cylinder; a single cylinder strength doesnot constitute a test result.
- **Companion cylinders**—cylinders made from the same sample of concrete.
- **Strength test or strength test result**—the average of two or more single-cylinder strengths of specimens made from the same concrete sample (companion cylinders) and tested at the same age.
- **Range or within-test range**—the difference between the maximum and minimum strengths of individual concrete specimens comprising one strength test result.
- **Test record—**a collection of strength test results of a single concrete mixture.

2.2 Statistical Functions

A sufficient number of tests are needed to indicate accurately the variation in the concrete produced and to permit appropriate statistical procedures for interpreting the test results. Statistical procedures provide a sound basis for determining from such results the potential quality and strength of the concrete and for expressing results in the most useful form.

A strength test result is defined as the average strength of all specimens of the same age, fabricated from a sample taken from a single batch of concrete. A strength test cannot be based on only one cylinder; a minimum of two cylinders is required for each test.

Concrete tests for strength are typically treated as if they fall into a distribution pattern similar to the normal frequency distribution curve illustrated in Fig. 3.1.

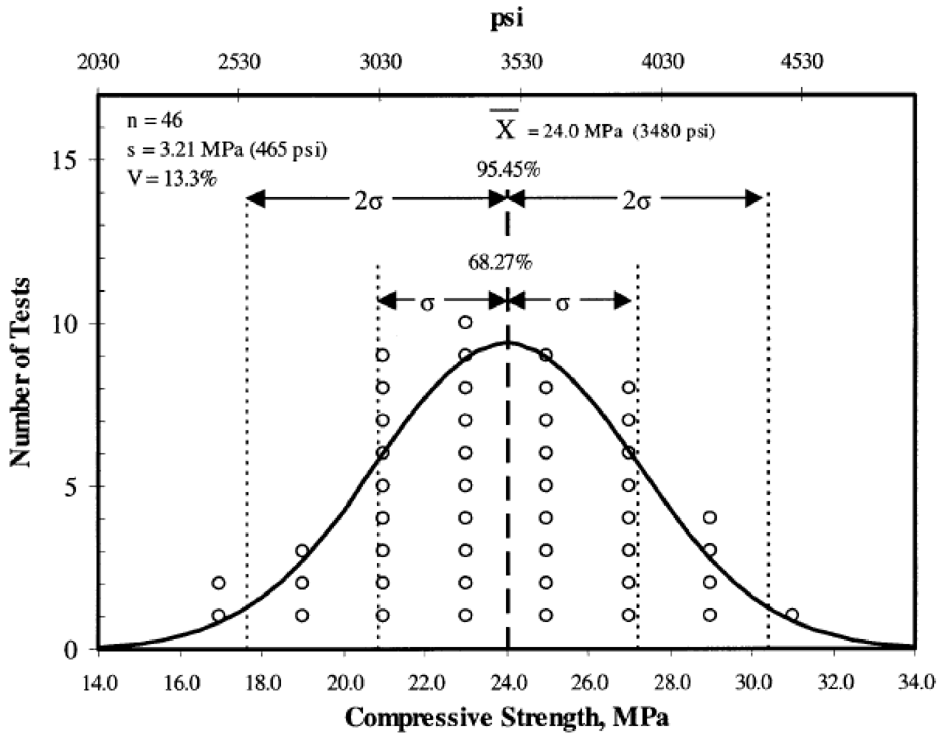

Fig. 3.1—Frequency distribution of strength data and corresponding assumed normal distribution.

When there is good control, the strength test values will tend to cluster near to the average value, that is, the histogram of test results is tall and narrow. As variation instrength results increases, the spread in the data increases and the normal distribution curve becomes lower and wider (Fig. 3.2).

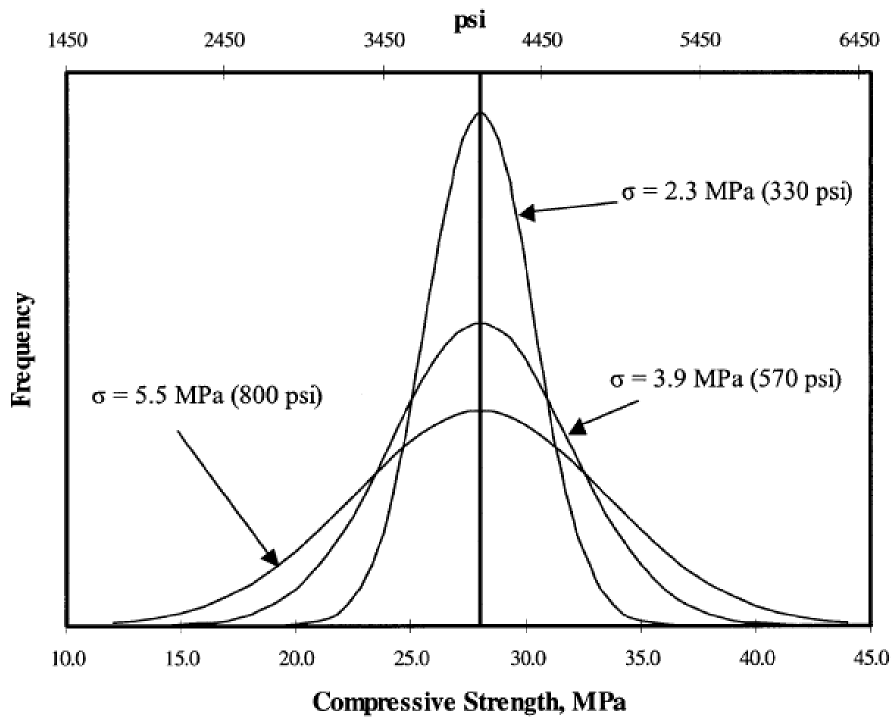

Fig. 3.2—Normal frequency curves for three different distributions with the same mean but different variability.

The normal distribution can be fully defined mathematically by two statistical parameters: the mean and standard deviation. These statistical parameters of the strength can be calculated as shown below:

- **Mean $\bar{X}$** , The average strength tests result $\bar{X}$ is calculated using the following equation

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{1}{n}\sum X_i = \frac{1}{n}(X_1 + X_2 + X_3 + \ldots + X_n) \quad (3\text{-}1)$$

where X_i is the i-th strength test result, the average of at least two cylinder strength tests. X_2 is the second strength test result in the record, ΣX_i is the sum of all strength test results and n is the number of tests in the record.

- **Standard deviations,** the standard deviation is the most generally recognized measure of dispersion of the individual test data from their average.

$$s = \sqrt{\frac{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}} \quad (3\text{-}2b)$$

where s is the sample standard deviation, n is the number of strength test results in the record, $\bar{X}$ is the mean, or average, strength test result, and ΣX is the sum of the strength test results.

- **Coefficient of variation V** - the sample standard deviation expressed as a percentage of the average strength is called the coefficient of variation

$$V = \frac{s}{\bar{X}} \times 100 \quad (3\text{-}4)$$

where V is the coefficient of variation, s is the sample standard deviation, and $\bar{X}$ is the average strength test result.

The coefficient of variation is less affected by the magnitude of the strength level, and is therefore more useful than the standard deviation in comparing the degree of control for a wide range of compressive strengths. The coefficient of variation is typically used when comparing the dispersion of strength test results of records with average compressive strengths more than about 7 MPa different.

- **Range R** - Range is the statistic found by subtracting the lowest value in a data set from the highest value in that data set.

In evaluation of concrete test results, **the within-test range R** of a strength test result is found by subtracting the lowest single cylinder strength from the highest single cylinder strength of the two or more cylinders used to comprise a strength test result. The average within-test range is used for estimating the within-test standard deviation.

2.3 Strength variations

Variations in strength test results can be traced to two different sources:

1. Variations in testing methods; and
2. Variations in the properties or proportions of the constituent materials in the concrete mixture, variations in the production, delivery or handling procedures, and variations in climatic conditions.

It is possible to compute the variations attributable to each source using analysis of variance (ANOVA) techniques or with simpler techniques.

1- Within-test variation—**Variability due to testing** is estimated by the within-test variation based on differences in strengths of companion (replicate) cylinders comprising a strength test result. The within-test variation is affected by variations in sampling, molding, consolidating, transporting, curing, capping, and testing specimens. A single strength test result of a concrete mixture, however, does not provide sufficient data for statistical analysis. As with any statistical estimator, the confidence in the estimate is a function of the number of test results.

The ***within-test standard deviation*** is estimated from the average range $\bar{R}$of at least 10, and preferably more, strength test results of a concrete mixture, tested at the same age, and the appropriate values of d_2 in Table 3.1 using Eq. (3-5). In Eq. (3-6), the within test coefficient of variation, in percent, is determined from the within-test standard deviation and the average strength.

$$s_1 = \frac{1}{d_2}\bar{R} \qquad (3\text{-}5)$$

$$V_1 = \frac{s_1}{\bar{\bar{X}}} \times 100 \qquad (3\text{-}6)$$

Where s_1 is the sample within-test standard deviation, $\bar{R}$ is the average within-test range of at least 10 tests, d_2 is the factor for computing within-test standard deviation from the average range, V_1 is the sample within-test coefficient of variation, and $\bar{X}$is the mean, or average, strength test result.

Table 3.1—Factors for computing within-test standard deviation from range

No. of specimens	d_2
2	1.128
3	1.693
4	2.059

Note: From Table 49, *ASTM Manual on Presentation of Data and Control Chart Analysis*, MNL 7.

2-Batch-to-Batch variation - These variations reflect differences in strength from batch to batch, which can be attributed to variations in:

(a) Characteristics and properties of the ingredients; and

(b) Batching, mixing, and sampling.

Batch-to-batch variation can be estimated from strength test results of a concrete mixture if each test result represents a separate batch of concrete.

The ***overall variation s*** has two component variations, the within-test s_1 , and batch-to-batch $\boldsymbol{s_2}$ variations. The sample variance—the squareof the sample standard deviation—is the sum of the sample within-test and sample batch-to-batch variances

$$s^2 = s_1^2 + s_2^2 \qquad (3\text{-}7)$$

From which the batch – to – batch standard deviation can be computed as

$$s_2 = \sqrt{s^2 - s_1^2} \qquad (3\text{-}8)$$

The within-test sample standard deviation estimates the variation attributable to sampling, specimen preparation, curing and testing, assuming proper testing methods are used. The batch-to-batch sample standard deviation estimates the variations attributable to constituent material suppliers, and the concrete producer.

2.4 Interpretation of statistical parameters

Once the statistical parameters have been computed, and with the assumption or verification that the results follow a normal frequency distribution curve, additional analysis of the test results is possible. Figure 3.3 indicates an approximate division of the area under the normal frequency distribution curve. For example, approximately 68% of the area (equivalent to 68% of the results) lies within ±1σof the average, and 95% lies within ±2σ.

This permits an estimate of the portion of the test results expected to fall within given multiples z of σ of the average or of any other specific value.

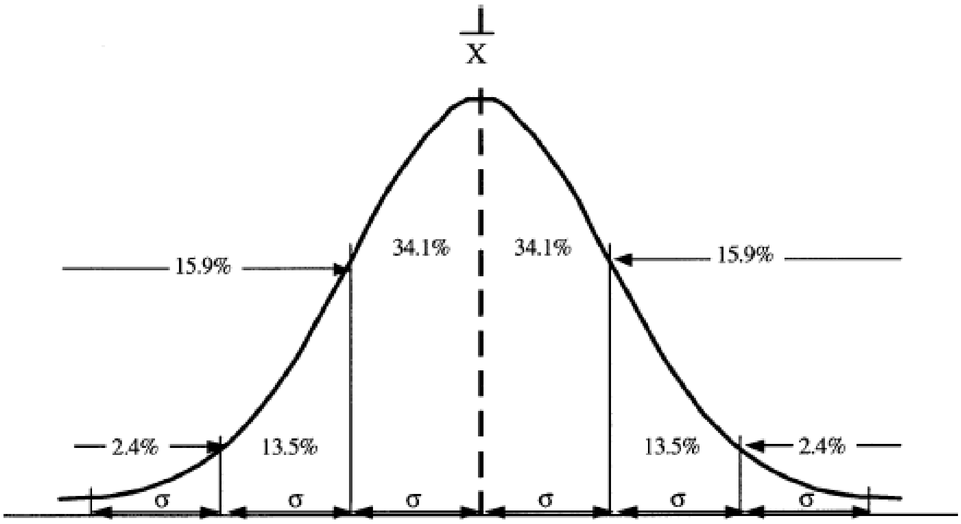

Fig. 3.3—Approximate distribution of area under normal frequency distribution curve.

Table 3.4 was adapted from the normal cumulative distribution (the normal probability integral) and shows the probability of a fraction of tests falling below fc′in terms of the average strength of the population of test results when the population average strength μ equals fc′ + zσ.

Table 3.4—Expected percentages of individual tests lower than f'_c *

Average strength μ	Expected percentage of low tests	Average strength μ	Expected percentage of low tests
$f'_c + 0.10\sigma$	46.0	$f'_c + 1.6\sigma$	5.5
$f'_c + 0.20\sigma$	42.1	$f'_c + 1.7\sigma$	4.5
$f'_c + 0.30\sigma$	38.2	$f'_c + 1.8\sigma$	3.6
$f'_c + 0.40\sigma$	34.5	$f'_c + 1.9\sigma$	2.9
$f'_c + 0.50\sigma$	30.9	$f'_c + 2.0\sigma$	2.3
$f'_c + 0.60\sigma$	27.4	$f'_c + 2.1\sigma$	1.8
$f'_c + 0.70\sigma$	24.2	$f'_c + 2.2\sigma$	1.4
$f'_c + 0.80\sigma$	21.2	$f'_c + 2.3\sigma$	1.1
$f'_c + 0.90\sigma$	18.4	$f'_c + 2.4\sigma$	0.8
$f'_c + 1.00\sigma$	15.9	$f'_c + 2.5\sigma$	0.6
$f'_c + 1.10\sigma$	13.6	$f'_c + 2.6\sigma$	0.45
$f'_c + 1.20\sigma$	11.5	$f'_c + 2.7\sigma$	0.35
$f'_c + 1.30\sigma$	9.7	$f'_c + 2.8\sigma$	0.25
$f'_c + 1.40\sigma$	8.1	$f'_c + 2.9\sigma$	0.19
$f'_c + 1.50\sigma$	6.7	$f'_c + 3.0\sigma$	0.13

*where μ exceeds f'_c by amount shown.

2.4 Standards of Control

One of the primary purposes of statistical evaluation of concrete data is to identify sources of variability. This knowledge can then be used to help determine appropriate steps to maintain the desired level of control. Several different techniques can be used to detect variations in concrete production, materials processing and handling, and contractor and testing agency operations. One simple approach is to compare **overall variability** and **within-test variability**, using eitherstandard deviation or coefficient of variation, as appropriate, with previous performance.

Table 3.2 gives the standards of control whichare appropriate for concrete having specified strengths up to 35 MPa (5000 psi), whereas Table 3.3 gives the appropriate standards of control for specified strengths over 35 MPa (5000 psi). These standards of control were adopted based on examination and analysis of compressive strength data by ACI Committee 214 and ACI Committee 363. The strength tests were conducted using 150 x 300 mm (6 x12 in.) cylinders.

Table 3.2—Standards of concrete control*

Overall variation					
Class of operation	Standard deviation for different control standards, MPa (psi)				
	Excellent	Very good	Good	Fair	Poor
General construction testing	Below 2.8 (below 400)	2.8 to 3.4 (400 to 500)	3.4 to 4.1 (500 to 600)	4.1 to 4.8 (600 to 700)	Above 4.8 (above 700)
Laboratory trial batches	Below 1.4 (below 200)	1.4 to 1.7 (200 to 250)	1.7 to 2.1 (250 to 300)	2.1 to 2.4 (300 to 350)	Above 2.4 (above 350)
Within-test variation					
Class of operation	Coefficient of variation for different control standards, %				
	Excellent	Very good	Good	Fair	Poor
Field control testing	Below 3.0	3.0 to 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory trial batches	Below 2.0	2.0 to 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

*$f_c' \leq 34.5$ MPa (5000 psi).

Table 3.3—Standards of concrete control*

Overall variation					
Class of operation	Coefficient of variation for different control standards,%				
	Excellent	Very good	Good	Fair	Poor
General construction testing	Below 7.0	7.0 to 9.0	9.0 to 11.0	11.0 to 14.0	Above 14.0
Laboratory trial batches	Below 3.5	3.5 to 4.5	4.5 to 5.5	5.5 to 7.0	Above 7.0
Within-test variation					
Class of operation	Coefficient of variation for different control standards, %				
	Excellent	Very good	Good	Fair	Poor
Field control testing	Below 3.0	3.0 to 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory trial batches	Below 2.0	2.0 to 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

*$f_c' > 34.5$ MPa (5000 psi).

3- Criteria

The strength of concrete in a structure and the strength of test cylinders cast from a sample of that concrete are not necessarily the same. The strength of the cylinders obtained from that sample of concrete and used for contractual acceptance are to be cured and tested under tightly controlled conditions. The strengths of these cylinders are generally the primary evidence of the quality of concrete used in the structure. The engineer specifies the desired strength, the testing frequency, and the permitted tolerance in compressive strength.

Any specified quantity, including strength, should alsohave a tolerance. It is impractical to specify an absolute minimum strength, because there is always the possibility of even lower strengths simply due to random variation, even when control is good.There will always be a certain probability of tests fallingbelow fc'. ACI 318 and most other building codes and specifications establish tolerances for meeting the specified compressive strength acceptance criteria, analogous to the tolerances for other building materials.

To satisfy statistically based strength-performance requirements, the average strength of the concrete should be in excess of the specified compressive strength fc'. **The required average strength fcr' which is the strength used in mixture proportioning**, depends on the expected variability of test results as measured by the coefficient of variation or standard deviation, and on the allowable proportion of tests below the appropriate, specified acceptance criteria.

3.1 Data used to establish the minimum required average strength

To establish the required average strength fcr' (target strength), an estimate of the variability of the concrete to be supplied for construction is needed. The strength test record used to estimate the standard deviation or coefficient of variation should represent a group of at least 30 consecutive tests.

The requirement for 30 consecutive strength tests can be satisfied by using a test record of 30 consecutive batches of the same class of concrete or the statistical average of two test records totaling 30 or more tests. If the number of test results available is less than 30, a more conservative approach is needed. Test records with as few as 15 tests can be used to estimate the standard deviation; however, the calculated standard deviation should be increased by as much as 15% to account for the uncertainty in the estimate of the standard deviation. In the absence of sufficient information, a very conservative approach is required and the concrete is proportioned to produce relatively high average strengths.

If only a small number of test results are available, the estimates of the standard deviation and coefficient of variation become less reliable. When the number of strength test results is between 15 and 30, the calculated standard deviation, multiplied by the appropriate modification factors obtained from Table 4.1, which was taken from ACI 318, provides a sufficiently conservative estimate to account for the uncertainty in the calculated standard deviation.

Table 4.1—Modification factors for standard deviation

Number of tests	Modification factors
Less than 15	See Table 4.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

Table 4.2—Minimum required average strength without sufficient historical data

$f'_{cr} = f'_c + 6.9$ MPa (1000 psi)	when $f'_c < 20.7$ MPa (3000 psi)
$f'_{cr} = f'_c + 8.3$ MPa (1200 psi)	when $f'_c \geq 20.7$ MPa (3000 psi) and $f'_c \leq 34.5$ MPa (5000 psi)
$f'_{cr} = 1.10f'_c + 4.8$ MPa (700 psi)	when $f'_c > 34.5$ MPa (5000 psi)

3.2 Criteria for strength requirements

The minimum required average strength fcr'can be computed using Eq. (4-1a), (4-1b), Table 4.2, depending on whether the coefficient of variation or standard deviation is used. The value of fcr'will be the same for a given set of strength test results regardless of whether the coefficient of variation or standard deviation is used.

$$f'_{cr} = f'_c /(1 - zV) \tag{4-1a}$$

$$f'_{cr} = f'_c + zs \tag{4-1b}$$

where z is selected to provide a sufficiently high probability of meeting the specified strength, assuming a normal distribution of strength test results. In most cases, fc' is replaced by a specified acceptance criterion, such as fc' – 3.5 MPa or 0.90fc'.

Figure 4.3 shows that as the variability increases, fcr' increases and thereby illustrates the economic value of good control.

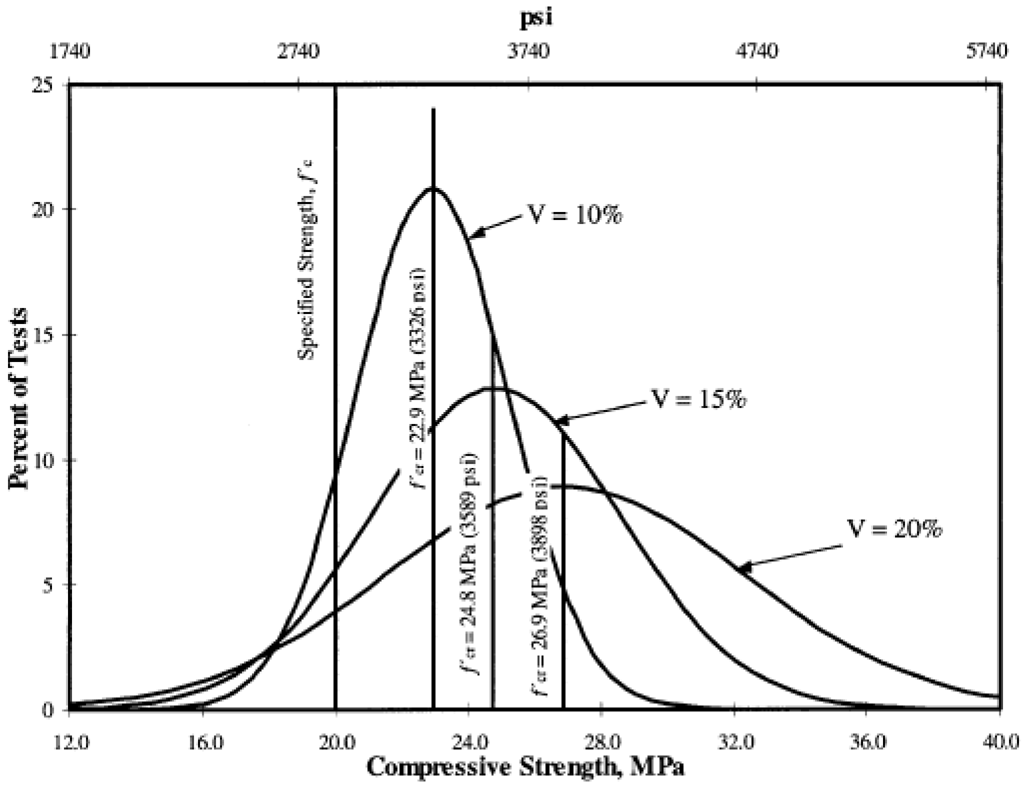

Fig. 4.3—Normal frequency curves for coefficients of variation of 10, 15, and 20%.

Table 4.3 provides values of z for various percentages of testsfalling between the mean + zσ and the mean –zσ.

Table 4.3—Probabilities associated with values of *z*

Percentages of tests within ± $z\sigma$	Chances of falling below lower limit	z
40	3 in 10 (30%)	0.52
50	2.5 in 10 (25%)	0.67
60	2 in 10 (20%)	0.84
68.27	1 in 6.3 (15.9%)	1.00
70	1.5 in 10 (15%)	1.04
80	***1 in 10 (10%)***	***1.28***
90	1 in 20 (5%)	1.65
95	1 in 40 (2.5%)	1.96
95.45	1 in 44 (2.3%)	2.00
98	***1 in 100 (1%)***	***2.33***
99	1 in 200 (0.5%)	2.58
99.73	1 in 741 (0.13%)	3.00

Note: Commonly used values in bold italic.

The amount by which the required average strength fcr′ should exceed the specified compressive strength fc′depends on the acceptance criteria specified for a particular project. **The following are criteria examples used to determine the required average strength for various specifications or elements of specifications**. The numerical examples are presented in both SI and inch-pound units in a parallel format that have been hard converted and so are not exactly equivalent numerically.

Criterion no. 1—The engineer may specify a statedmaximum percentage of individual, random strength testsresults that will be permitted to fall below the specified compressivestrength. This criterion is no longer used in theACI 318 Building Code, but does occur from time to time inspecifications based on allowable strength methods or in situationswhere the average strength is a fundamental part of thedesign methodology, such as in some pavement specifications.A typical requirement is to permit no more than 10%of the strength tests to fall below fc′. The specified strengthin these situations will generally be between 21 and 35 MPa.

Standard deviation method—Assume sufficientdata exist for which a standard deviation of 3.58 MPa has been calculated for a concrete mixture with a specifiedstrength of 28 MPa. FromTable 4.3, 10% of the normal probability distribution liesmore than 1.28 standard deviations below the mean. Using Eq. (4-1b)

$$f'_{cr} = f_c' + zs$$

$$f'_{cr} = 28 \text{ MPa} + 1.28 \times (3.58) \text{ MPa} = 32.6 \text{ MPa}$$

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an averagestrength of not less than 32.6 MPa so that, on average, no morethan 10% of the results will fall below fc′.

Coefficient of variation method—Assume sufficientdata exist for which a coefficient of variation of 10.5% hasbeen calculated for a concrete mixture with a specified strength of 28 MPa. From Table 4.3, 10% of the normal probability distribution lies more than 1.28 standard deviations below the mean. Using Eq. (4-1a)

$$f'_{cr} = f'_c / (1 - zV)$$

$$f'_{cr} = 28 \text{ MPa} / [1 - (1.28 \times 10.5/100)] = 32.3 \text{ MPa}$$

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an averagestrength of not less than 32.3 MPa so that, on average, no morethan 10% of the results will fall below fc′.

4- EVALUATION OF DATA

Evaluation of strength data is required in many situations.Three commonly required applications are:

- Evaluation for mixture submittal purposes;
- Evaluation of level of control (typically called quality control); and
- Evaluation to determine compliance with specifications.

A major purpose of these evaluations is to identify departuresfrom desired target values and, where possible, to assist with theformulation of an appropriate response. In all cases, the usefulnessof the evaluation will be a function of the amount of testdata and the statistical rigor of the analysis.

Numbers of tests - For a particular project, a sufficient number of tests should be made to ensure accurate representation of the concrete. A test is defined as the average strength of at least two specimens of the same age fabricated from a sample taken from a single batch of concrete. The frequency of concrete tests can be established on the basis of time elapsed or volume placed. The engineer should establish the number of tests needed based on job conditions.

Rejection of doubtful specimens - The practice of arbitrary rejection of strength test results that appear too far out of line is not recommended because the normal distribution anticipates the possibility of such results. Discarding test results indiscriminately can seriously distort the strength distribution, making analysis of results less reliable. Occasionally, the strength of one cylinder from a group made from a sample deviates so far from the others as to be highly improbable. If questionable variations have been observed during fabrication, curing, or testing of a specimen, the specimen should be rejected on that basis alone.

ASTM E 178 provides criteria for rejecting the test result for one specimen in a set of specimens. In general, the result from a single specimen in a set of three or more specimens can be discarded if its deviation from a test mean is greater than three times the previously established within-test standard deviation, and should be accepted with suspicion if its deviation is greater than two times the within test standard deviation. The test average should be computed from the remaining specimens. A test, that is, the average of all specimens of a single sample tested at the same age, should not be rejected unless it is very likely that the specimens are faulty. The test represents the best available estimate for the sample.

Applications

1- Calculate the required average strength (fcr') for a mix design if the specified compressive strength (characteristic strength) is 25 MPa. Assume sufficient data exist for which a standard deviation of 5.61 MPa has been calculated. Assume 10% of the normal probability distribution lies below the average compressive strength (i.e. 10 % of the results below the average strength, 90% confidence)

$$f'_{cr} = f_c' + zs$$

From Table (4.3) 10 % probability corresponds to z=1.28 , therefore,

Fcr' = 25 + 1.28*5.61 = 32.2 MPa

2- It is required to comment on the quality of a concrete for a raft foundation, the measurement of compressive strength of 17 test results. What is the actual compressive strength if the degree of confidence is 95% and do approve or disapprove this concrete, knowing that the specified (characteristic strength is 20 MPa).

Test Result No	Fc', MPa	Deviation	Squared deviation
1	21.8	2.8	7.84
2	18.4	-0.6	0.36
3	17.7	- 1.3	1.69
4	21.5	2.5	6.25
5	18.6	- 0.4	1.6
6	17.3	-1.7	2.89
7	20.9	1.9	3.61
8	14.2	-4.8	23.04
9	15.3	-3.7	13.69
10	18.7	- 0.3	0.09
11	18.1	- 0.9	0.81
12	19.3	0.3	0.09
13	14.7	- 4.3	18.49
14	21.3	2.3	5.29
15	23.1	4.1	16.81
16	20	1.0	1.0
17	22.1	3.1	9.61
	Average=19		Sum = 111.72

$$\sigma = \sqrt[2]{\frac{111.72}{17}} = \ 2.56\ kg/cm^2$$

V = 2.56 (100)/19 = 13.5 %

$$f'_{cr} = f'_c/(1 - zV)$$

Fc' = fcr' (1-zv)

Fc' = 19.0*(1 – 1.64*0.135) = 14.7 MPa, much less than 20MPa

Therefore this concrete is disapproved. Redesign should be carried out based on fc' = 14.7MPa.

3- Evaluate the results of compressive strength of two groups of concrete samples consider 95% degree of confidence. Then calculate the specified compressive strength of both groups of concrete based on only 90% degree of confidence.

	Concrete samples, Group 1	Concrete samples, Group 2
Measured compressive strength	395, 410, 412, 415	305, 385, 402, 540
Average, kg/cm^2	408	408
Range, kg/cm^2	20 (4.9 % from average)	235 (57.6 from average)
Deviations, kg/cm^2	-13, 2, 4, 7	-103, -23, -6, 132
Standard deviations, kg/cm^2	7.7 (0.77 MPa)	84.6 (8.46 MPa)
Coefficient of deviation, %	1.9	20.7
Level of quality control	Excellent (Table 3.2)	Poor (Table 3.3)
Specified strength (95% confidence)	395	269
Specified strength (90% confidence)	398	300

4- Comment on the following test results of samples of a concrete cover for raft foundation

	7-day test result, kg/cm^2	28-day results, kg/cm^2	Additional 28-day results, kg/cm^2
1	209.5	230	192.6
2	179.7	269.1	115.3
3	147.6	121.4	212.5
4	106.4	306	157.1
5	76.7	158	189.6
6	187.9	202.2	173.1
7	167.3	145.5	129.4
8	176	146	200.6
9	135	130.6	102.5
10	135.4	198.4	178
11	130	150.5	170.3
12	57.9	217.2	248.2
13	129.3	178.8	237
14	123.4	182.3	228.6
15	118.9	172.4	89.6
16	112.6	131.5	157.3
17	123.1	247.5	161.2
18	123.9	186.8	144.1
19	104.6	142.7	167.4
20	93.6	163.1	141.3
21	105.3	82	
22	181.6	147.5	
23	182.2	150.3	
24	150.8	183.1	
25	204.3	243.6	

<u>Statistical Functions</u>

Average of the 45 test results at 28 day = 175.16 kg/cm^2

Standard deviation = 47.37 kg/cm^2 (4.7 MPa)

Coefficient of variation = 27.05 %

Max value = 306 kg/cm^2

Min value = 82 kg/cm^2

Range = 224 kg/cm^2

Analysis of Results

- Based on Table 3.2 for standard deviation 4.7 MPa, the concrete is at the upper limit of "**Fair**"
- Based on Table 3.3, for coefficient of variation of 27.05 %, the concrete is "**Poor**"

Specified strength = 175.16 (1-1.28*.2705) = 114.5 kg/cm2 (90 % confidence)
Specified strength = 175.16 (1-1.64*.2705) = 97.5 kg/cm2 (95 % confidence)

% of discount = (target strength – actual average specified strength)/ Target strength
Assume target strength = 225 kg/cm^2
% discount = (200-114.5)/200 = 0.4275

Total discount inUSD = cost per 1m^3*amount of concrete* % of discount

= 204 USD * 155 * 0.4275 = 13517.6 USD