CHAPTER EIGHT
SOLVING SIMULTANEOUS EQUATIONS

In this chapter the system of linear equations will be solved, and the matrix inversion is found by using Gauss- Jordan's method.

. Simultaneous algebraic equations arise in many engineering applications.

. Systems of simultaneous equations can be either linear or nonlinear.

. In a system of linear equations, none of the unknown quantities (x1, x2 , x3 , …) are raised to a power $(e.g., x_1^3 ; x_2^2)$ or appear as arguments in functions [*e.g.*, sin (x2) ; lin (x1) ; log (x3)].

. Systems of linear simultaneous equations can be easily represented in matrix notation. Thus, solutions of such systems of equations can be developed by simple matrix operations, as will be seen in this chapter.

8.1. LINEAR SIMULTANEOUS EQUATIONS IN MATRIX NOTATION

Using traditional algebraic notation, linear simultaneous equations would be typically written in the form:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \ldots + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \ldots + a_{2n} x_n = b_2$$

$$\ldots\ldots\ldots$$

$$a_{n1} x_1 + a_{n2} x_2 + a_{n3} x_3 + \ldots + a_{nn} x_n = b_n$$

Equation (8.1) can be written in matrix form as:

$$A\,X = B$$

Where the matrix A and the vectors X and B are written as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

The advantage of writing the system of Equations in the matrix form given in the equation is that the solution of equation can be easily obtained by simple matrix operations as given below:

In the Equation,

premultiply by A^{-1}:

$$A^{-1} A X = A^{-1} B$$

But since $A^{-1} A = I$ (the identity matrix)

Then, the Equations can be written as,

$$IX = A^{-1}B$$

or $\quad X = A^{-1}B$

Therefore, we conclude that a system of simultaneous linear equations $AX = B$ can be solved by determining the inverse A^{-1}, of the coefficient matrix A and then forming the product $A^{-1}B$. The resulting product will be a vector, X, whose components are the desired unknown quantities.

Example (1)

Solve the system of Equations:

$$2x_1 + 3x_2 = 8$$
$$4x_1 - 3x_2 = -2$$

Solution:

In matrix form,

$$AX = B$$

where,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 8 \\ -2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/6 & 1/6 \\ 2/9 & -1/9 \end{bmatrix}$$

$$X = A^{-1} B = \begin{bmatrix} 1/6 & 1/6 \\ 2/9 & -1/9 \end{bmatrix} \begin{bmatrix} 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Thus, the required solution is

$x_1 = 1$

$x_2 = 2$

MATRIX OPERATIONS IN EXCEL

• Matrix operations can easily be carried out in Excel. A matrix is represented as an array in Excel.

• Any array is a block of cells that is referenced collaboratively in the same manner as a single cell.

• Any operation that is carried out on any array will produce the same outcome for all of the cells within the array.

• In addition, an array can be specified as a single argument to a function. This is equivalent to specify the individual cells as multiple arguments.

• Any array is represented as a block of cells enclosed in braces ({}). For example {A1:C3} refers to an array that is composed of the block of cells extending from cell A1 to cell C3.

• The braces are added automatically by Excel as a part of the array specification. Thus, you should not attempt to include the braces when writing an array.

• To specify an array operation (i.e. a matrix operation) within an excel worksheet, proceed as follows;

1. Select the block of cells that makes up the array. Then move the cell marker to the upper left cell within the array.

2. Enter an array formula, as required. You may include cell ranges within the formula. However, after you have entered the formula, do not include the formula in braces.

3. Press Ctrl-Shift-Enter simultaneously. This will cause the formula to appear within a pair of braces, thus identifying the block of cells as an array.

The indicated operation will then be carried out.

__Example__

For solving a linear system of equations, you can use Matrix operations. For instance, the system of equations:

$$\begin{cases} 3X + 6Y = 9 \\ 2X + 0.54Y = 4 \end{cases}$$
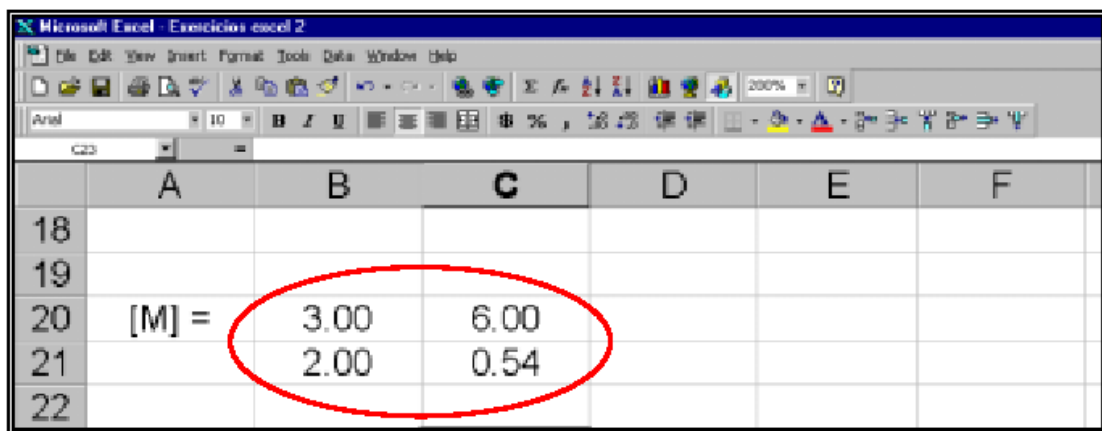
can be written in matrix format as shown below:

$$\underbrace{\begin{pmatrix} 3 & 6 \\ 2 & 0.54 \end{pmatrix}}_{[M]} \text{ X } \underbrace{\begin{pmatrix} X \\ Y \end{pmatrix}}_{[X]} = \underbrace{\begin{pmatrix} 9 \\ 4 \end{pmatrix}}_{[R]}$$

The solution for this system is given by

$$[X] = [M]^{-1} \text{ x } [R]$$

where $[M]^{-1}$ is the inverse of **[M]** matrix.

　　　Please return to your worksheet and type the matrix in cells (B20:C21) as shown in the Figure:

[M] = Coefficients of the Linear Equations

The next step is to calculate the inverse of **[M]** = $[M]^{-1}$. Highlight cells (B23:C24) and select **"MINVERSE"** function by clicking on **"Inser**t" → **"Function"** → **"Math & Trig"**. Press **"OK"** button

and the "**MINVERSE**" box becomes visible. In the "**Array**" box, type the range of cells referred to the matrix **[M]**, i.e., (B20:C21). Then press both "**Ctrl**" and "**Shift**" keys and click on "**OK**" button.
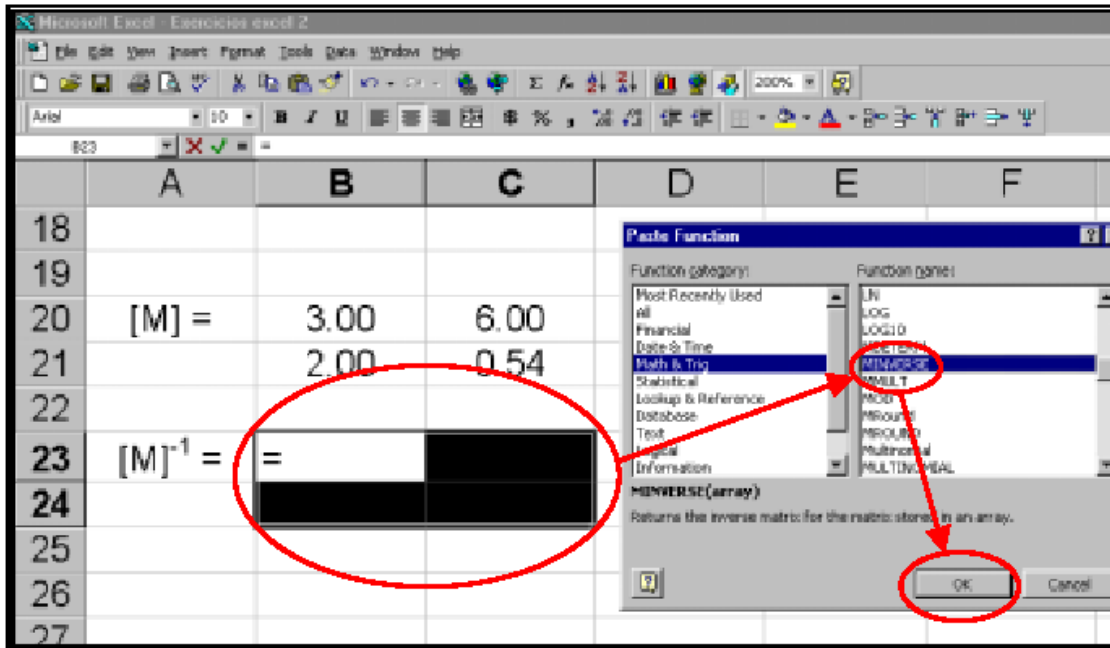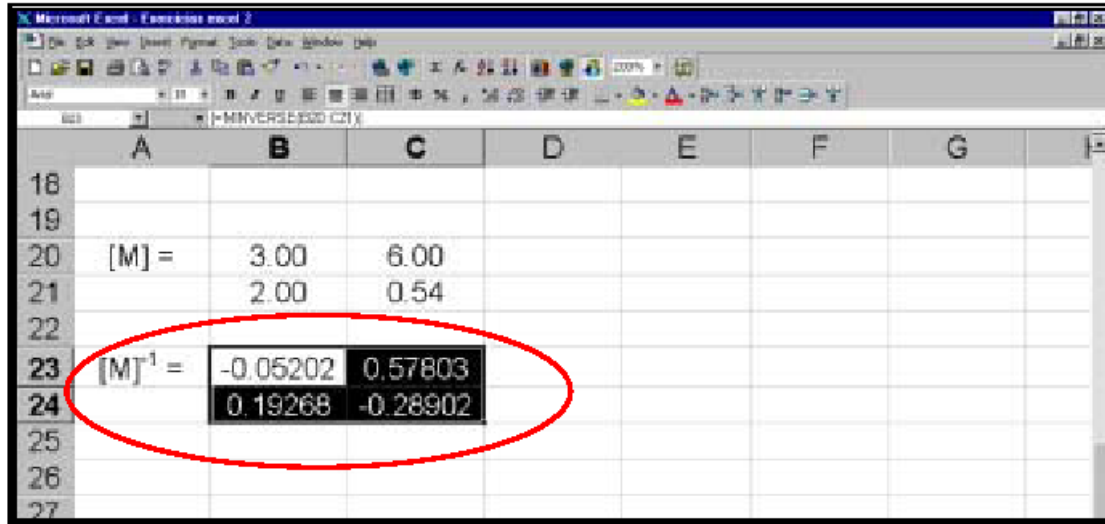

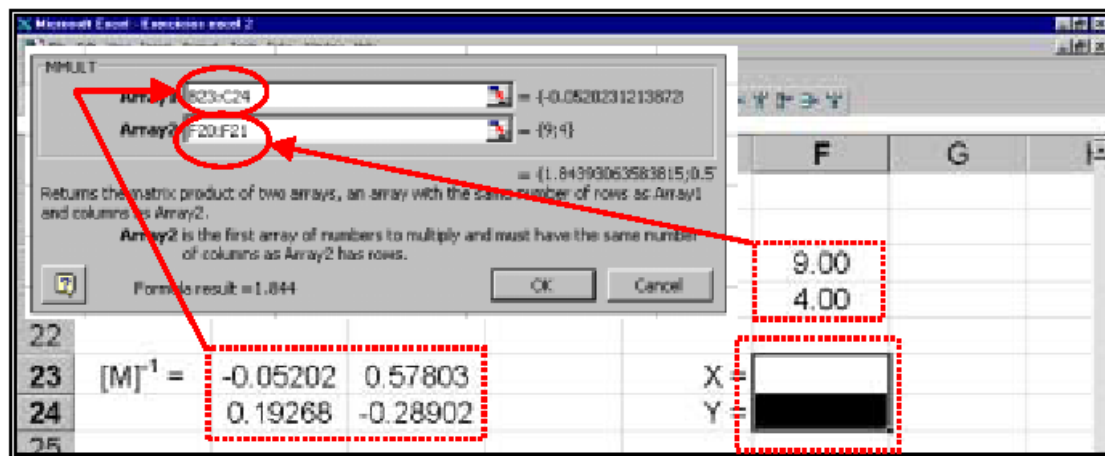**Selecting "MINVERSE" Function**
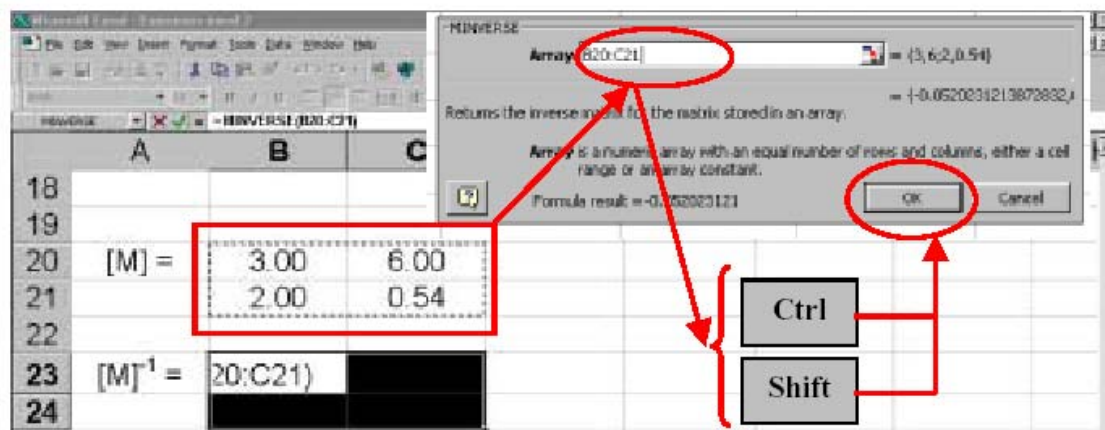
Inserting "MINVERSE" Command

The results for $[M]^{-1}$ will be shown in your worksheet the following Figure.

Results for [M]-1

Now type the **R** vector in cells F20:F21. Select cells F23:F24. To multiply $[M]^{-1}$ by **[R]** use the function "**MMULT**" from "**Math & Trig**" category:

Function "MMULT" for Multiplying Matrices

After typing the range of cells referred to $[M]^{-1}$ and to **[R]** in boxes **"Array 1"** and **"Array 2"**, respectively, press both **"Ctrl"** and **"Shift"** keys and click on **"ENTER"** button.

The results will be displayed in cells F23 and F24 as shown;

Results for the Equation System

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