Introduction To Analog Filters

Filters

Background:

- Filters may be classified as either digital or analog.

- Analog filters are used to filter out unwanted bands of frequency.
- It may be classified as either passive or active and are usually implemented with R, L, and C components and operational amplifiers.

- Digital filters
  - are implemented using a digital computer or special purpose digital hardware.
  - A digital filter, in general, is a computational process, or algorithm that converts one sequence of numbers representing the input signal into another sequence representing the output signal.
  - Accordingly, a digital filter can perform functions as differentiation, integration, estimation, and, of course, like an analog filter, it can filter out unwanted bands of frequency.

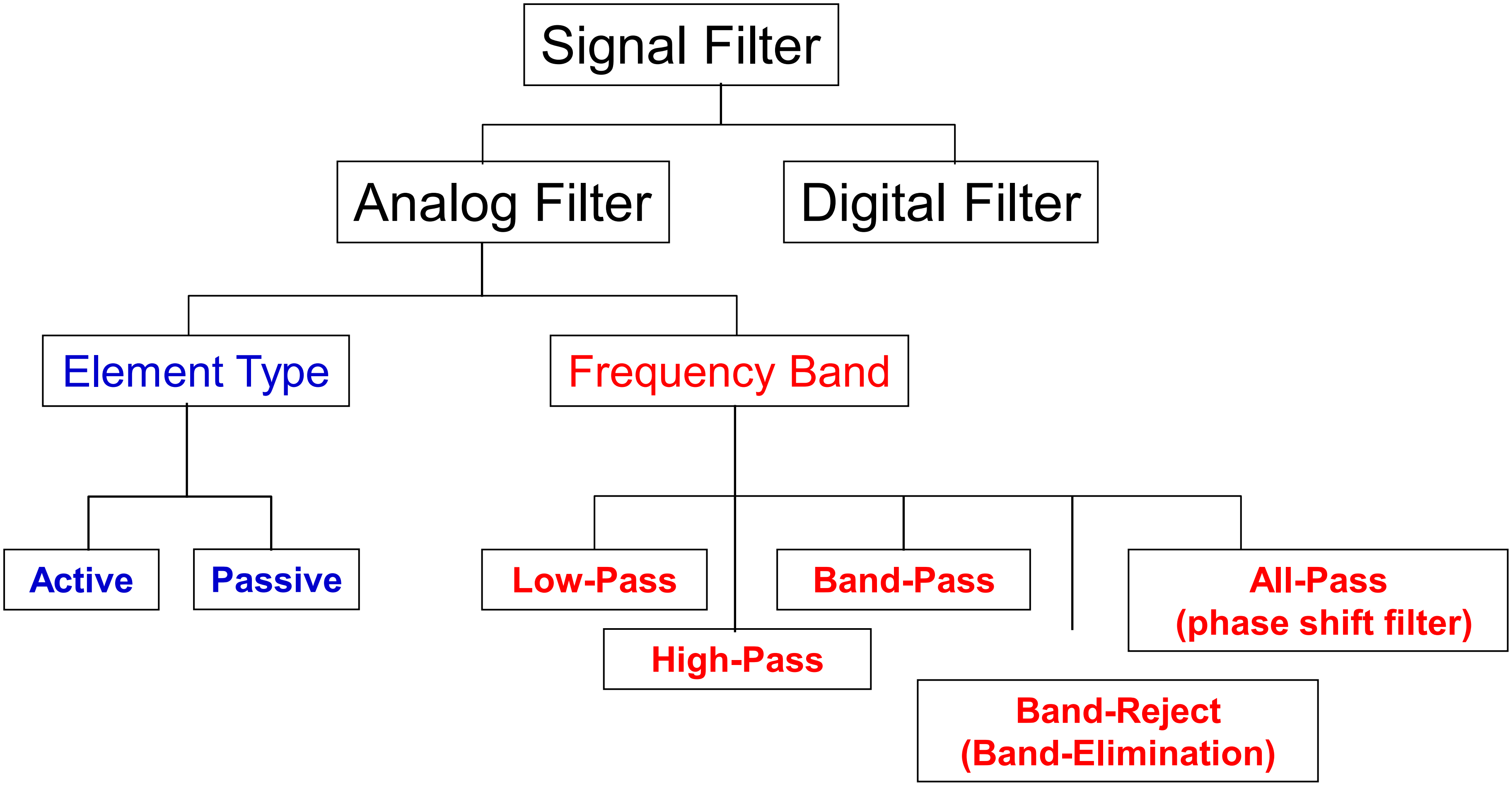
Filters

Background

- An active filter is one that, along with R, L, and C components, also contains an energy source, such as that derived from an operational amplifier.

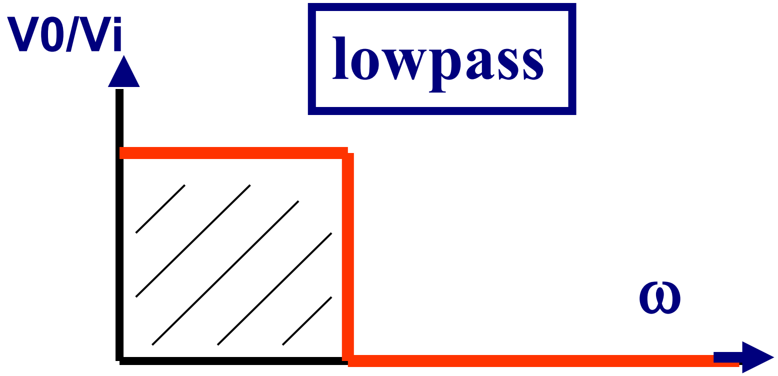
- A passive filter is one that contains only R, L, and C components. It is not necessary that all three be present. L is often omitted (on purpose) from passive filter design because of the size and cost of inductors – and they also carry along an R that must be included in the design.

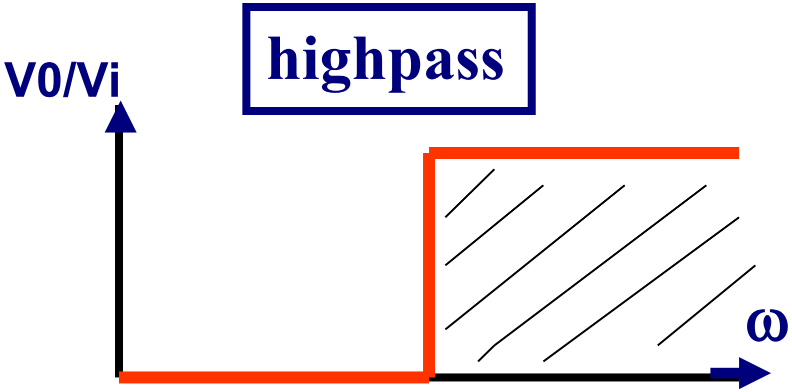
Classification of Filters

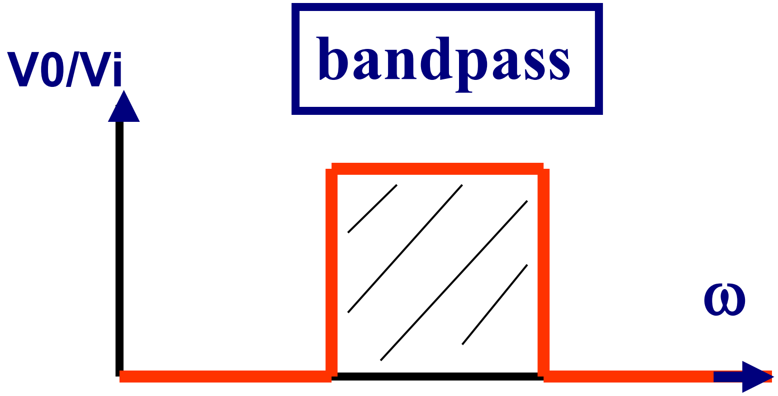


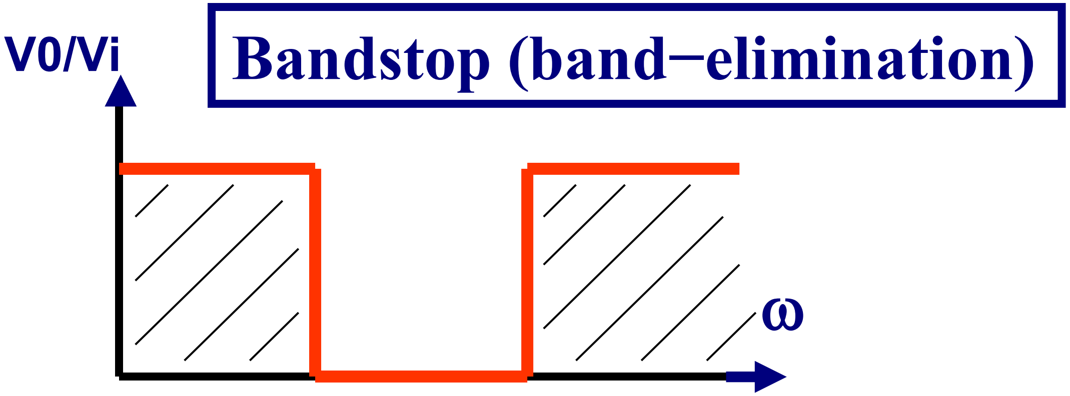
Passive Analog Filters

Four types of filters - “Ideal”

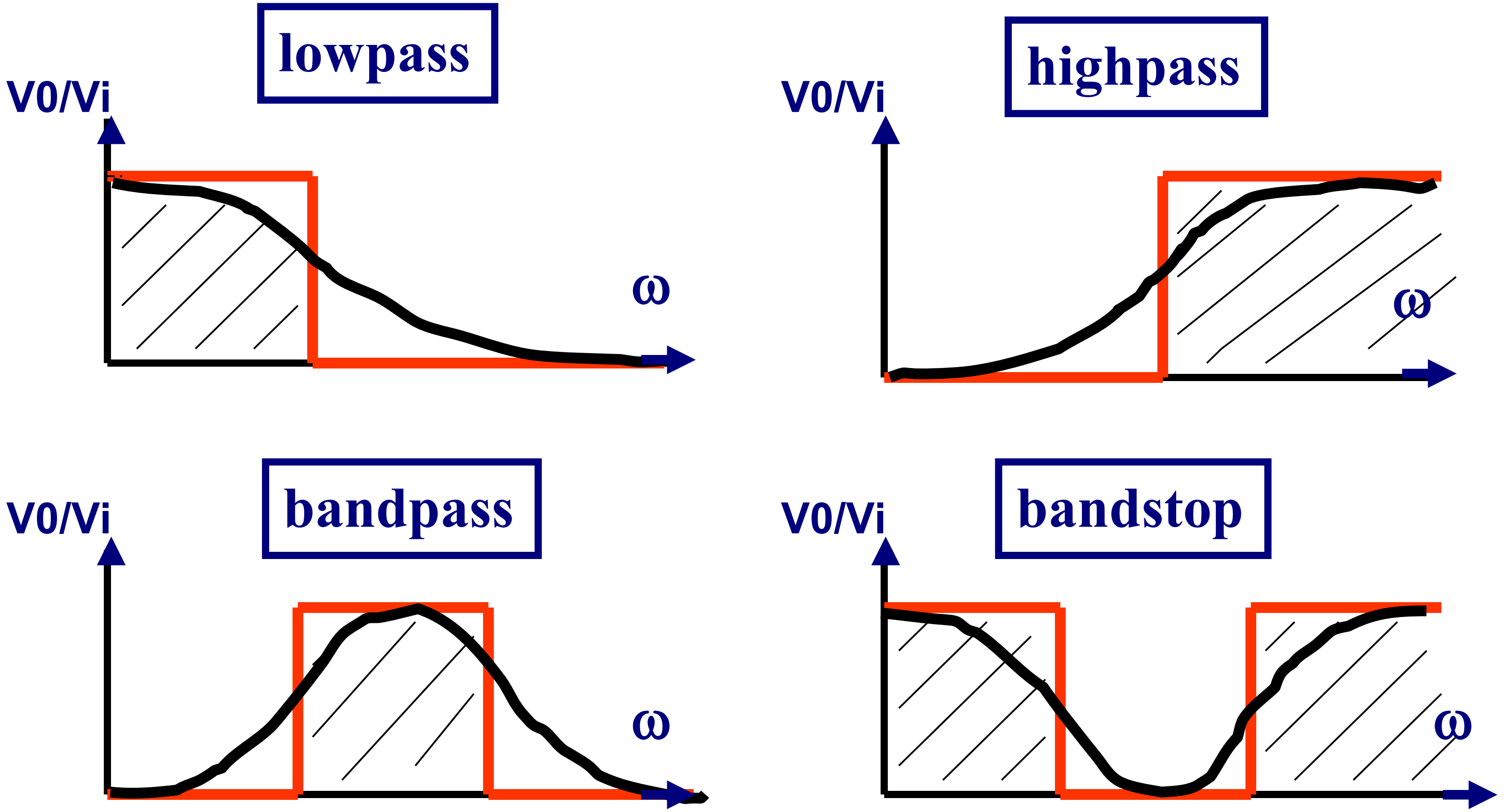








Realistic Filters:



Passive Analog Filters

It will be shown later that the ideal filter, sometimes called a “brickwall” filter, can be approached by making the order of the filter higher and higher.

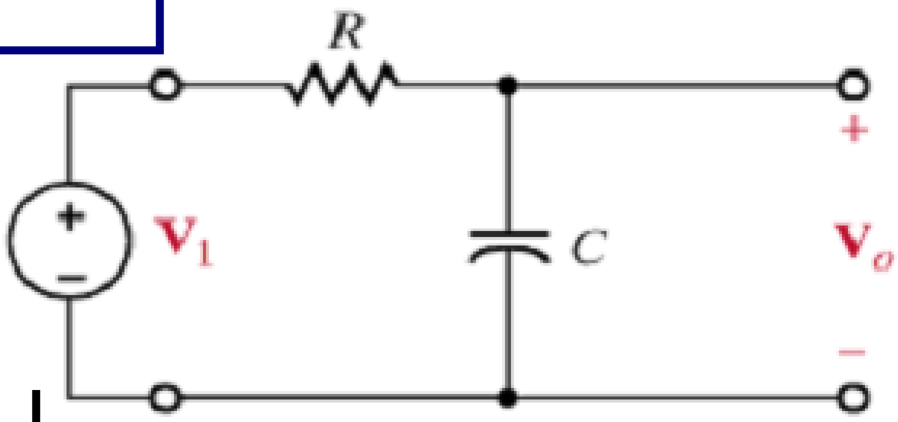
The order here refers to the order of the polynomial(s) that are used to define the filter. Matlab examples will be given later to illustrate this.

Filter Terms

- Passband - range of signal frequencies that pass through filter relatively unimpeded.
- Stopband - range of frequencies that pass through filter and undergo a relatively strong attenuation.
- Cut-off frequency - the end of the passband region. For the band-pass and band-reject filters there are 2 cut-off frequencies.
- Cut-off (or half power) frequency - the frequency where the magnitude to of the transfer function is 0.707 down from its maximum value.
- Passive filter – filter circuit without amplifier elements (no external power). The gain for passive filters is always less than or equal to 1 for all frequencies.
- Active filter – filter circuit requires power external to the input signal . Thes filters use an amplifier element (i.e. an op amp).

Simple low-pass filter RC L.P.F

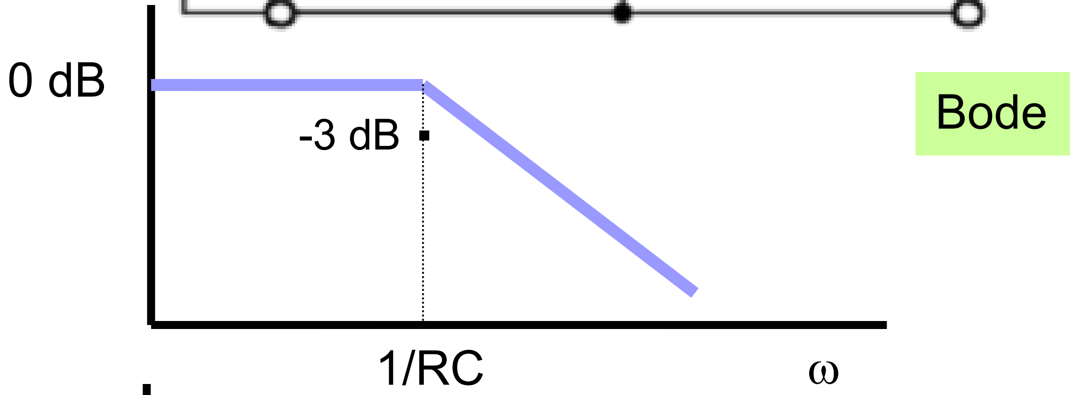
$$G_v = \frac{V_0}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$



$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = RC$$

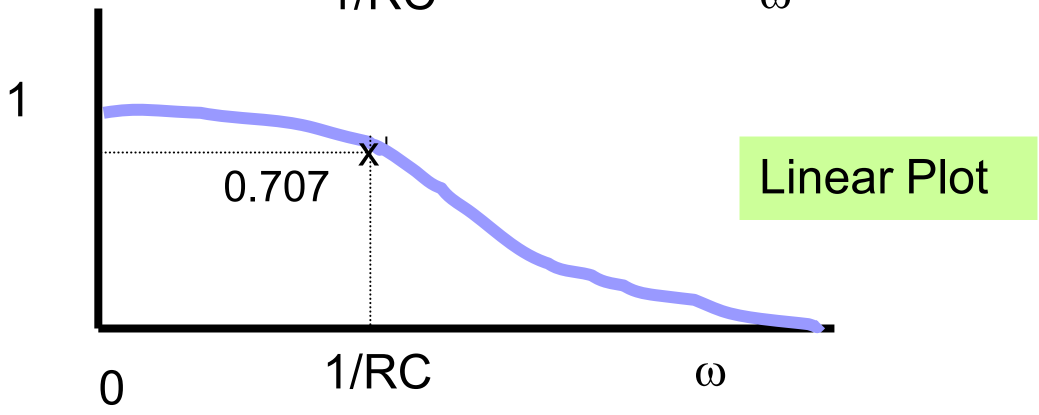
$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = 0 - \tan^{-1}\omega\tau$$



$$M_{max} = 1, \quad M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

At w = 0



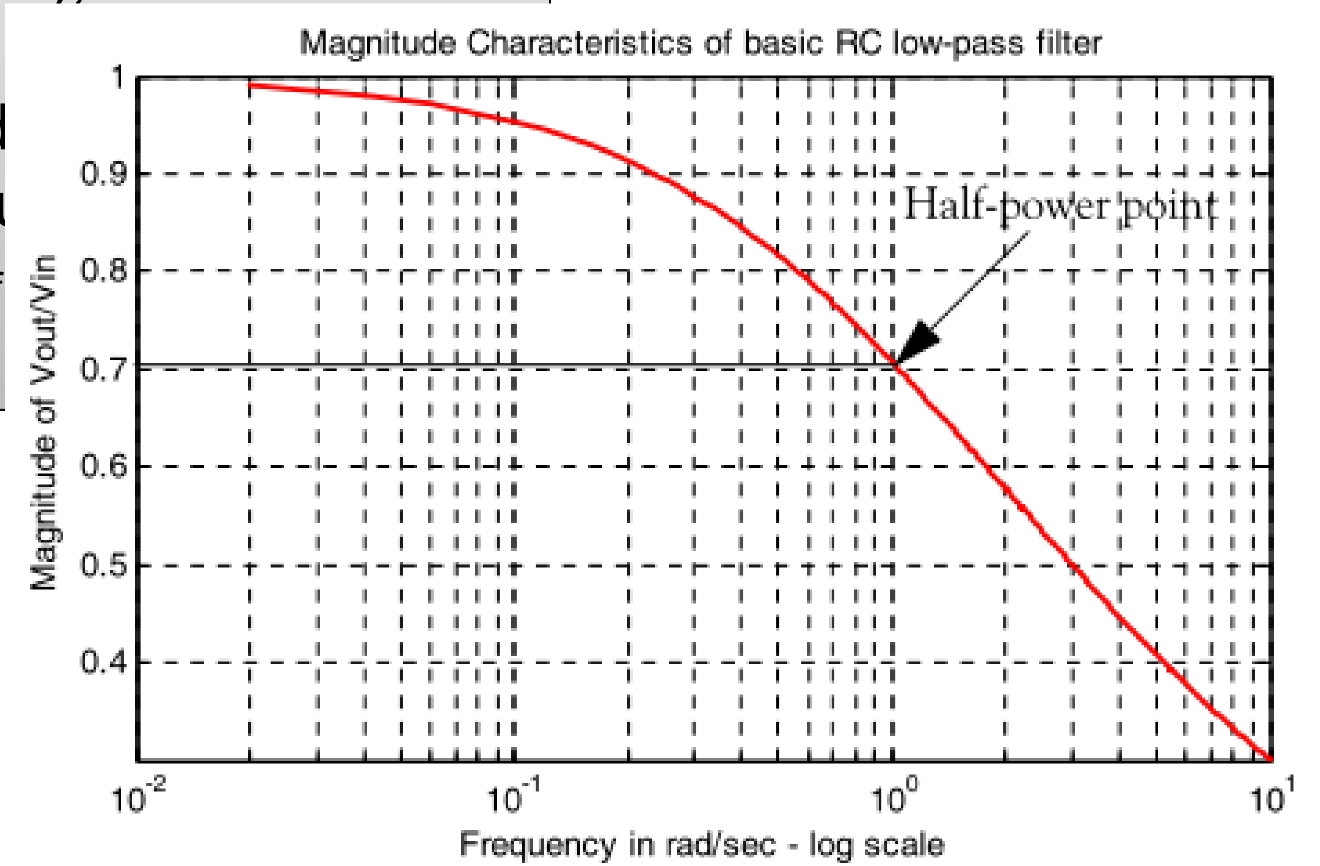
$\omega = \frac{1}{\tau}$ = half power frequency $\qquad BW = \frac{1}{\tau}$

Simple low-pass filter (RC L.P.F)

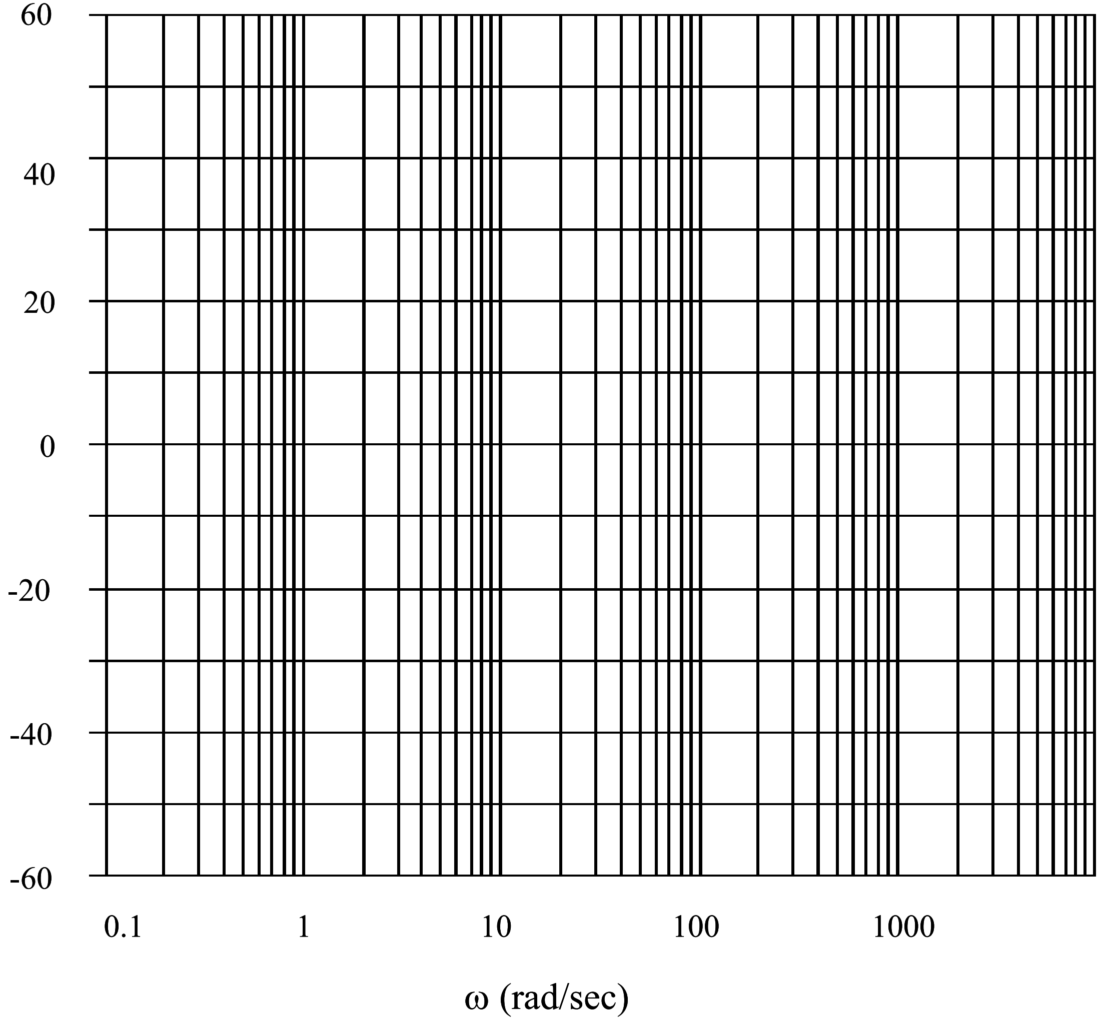
We will use the MATLAB script below to plot $|G(j\omega)|$ versus radian frequency ω

let $RC = 1$

```
w=0:0.02:10;
RC=1;
magGjw=1./sqrt(1+w.*RC);
semilogx(w,magGjw);
xlabel('Frequency in rad
ylabel('Magnitude of Vou
title('Magnitude Characteristics of
grid
```



Semi log Curves



Simple low-pass filter (RC L.P.F)

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let $RC = 1$

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w=0:0.02:10;
RC=1;
phaseGjw=−atan(w.*
semilogx(w,phaseGjv
xlabel('Frequency in
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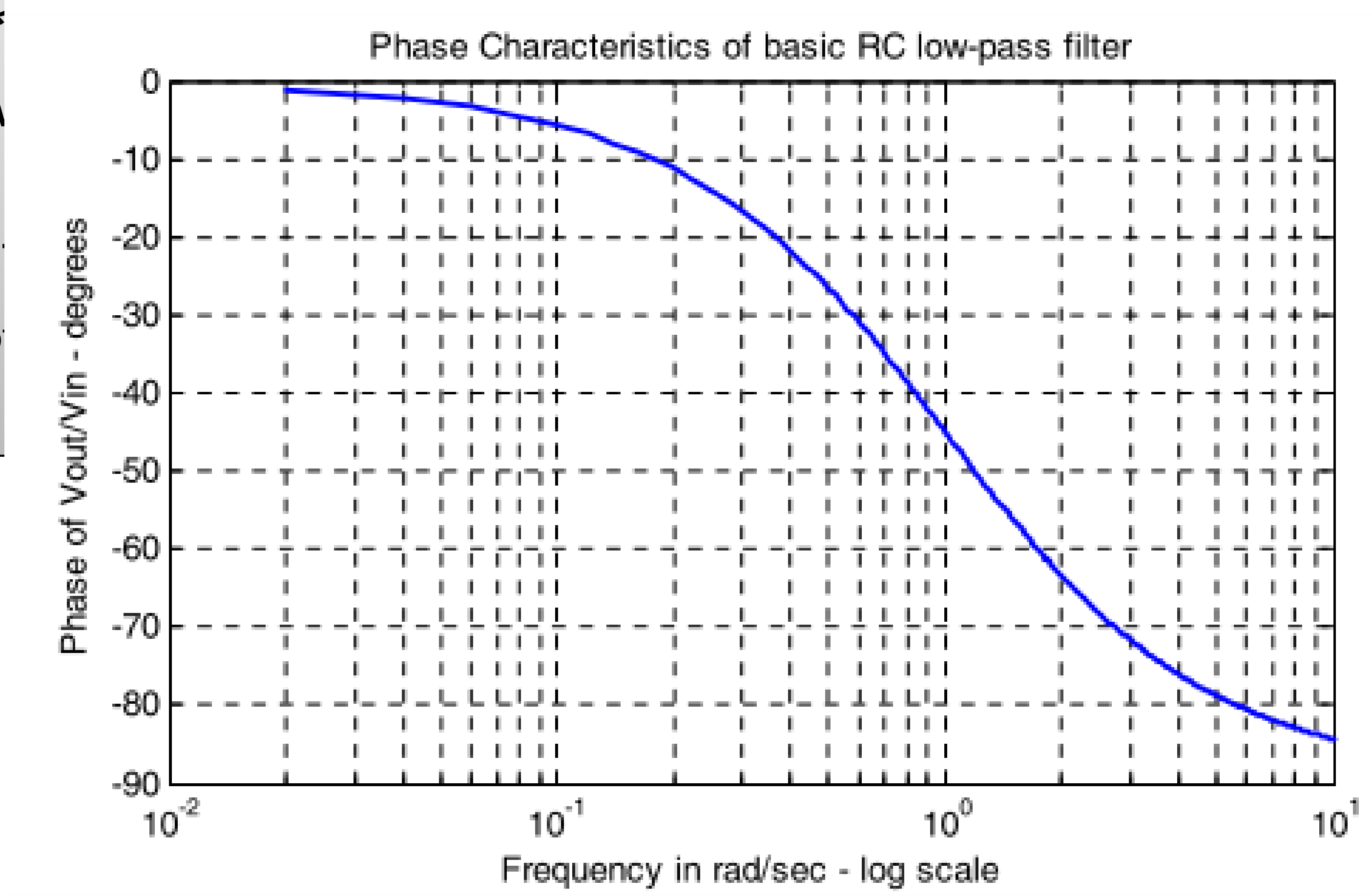
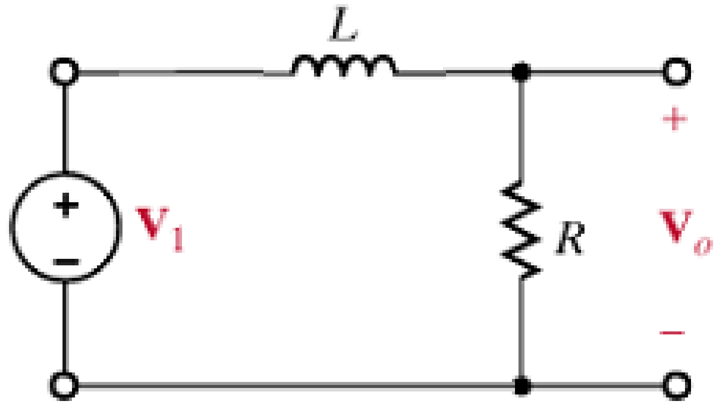
title('Phase Characteristics o
grid
```



Simple low-pass filter
RL L.P.F



$$G_v = \frac{V_0}{V_1} = \frac{R}{R + jwl} = \frac{1}{1 + j\omega \frac{L}{R}}$$

$$G_v = \frac{1}{1 + j\omega\tau}; \quad \tau = \frac{L}{R}$$

$$M(\omega) = |G_v| = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

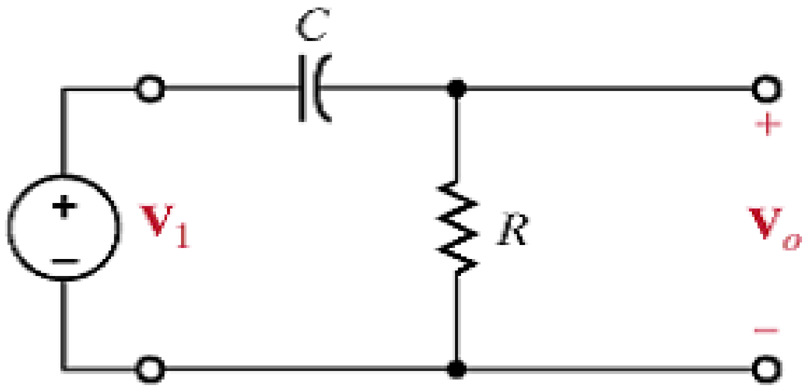
$$\angle G_v = \phi(\omega) = -\tan^{-1}\omega\tau$$

$$M_{\max} = 1, \quad M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$$\omega = \frac{1}{\tau} = \text{half power frequency}$$

$$BW = \frac{1}{\tau}$$

Simple high-pass filter
RC H.P.F



$$G_v = \frac{V_0}{V_1} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

$$G_v = \frac{j\omega\tau}{1 + j\omega\tau}; \ \tau = RC$$

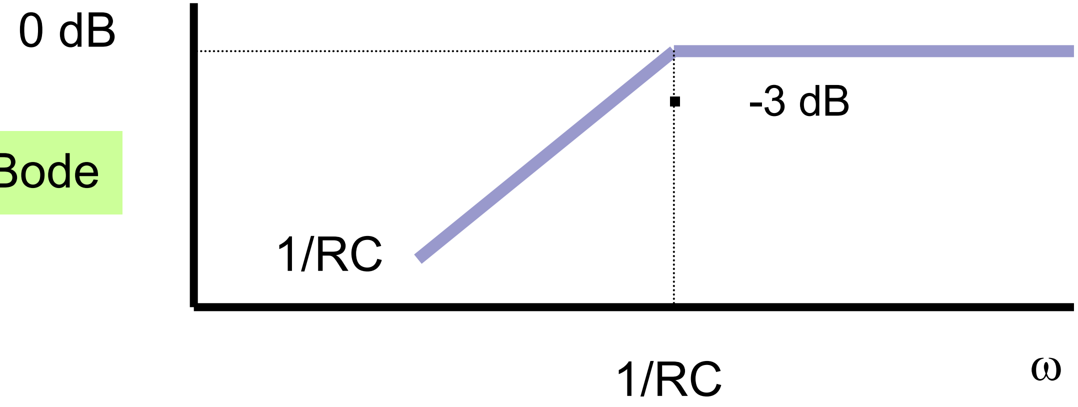
$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau = \tan^{-1}\frac{1}{\omega\tau}$$

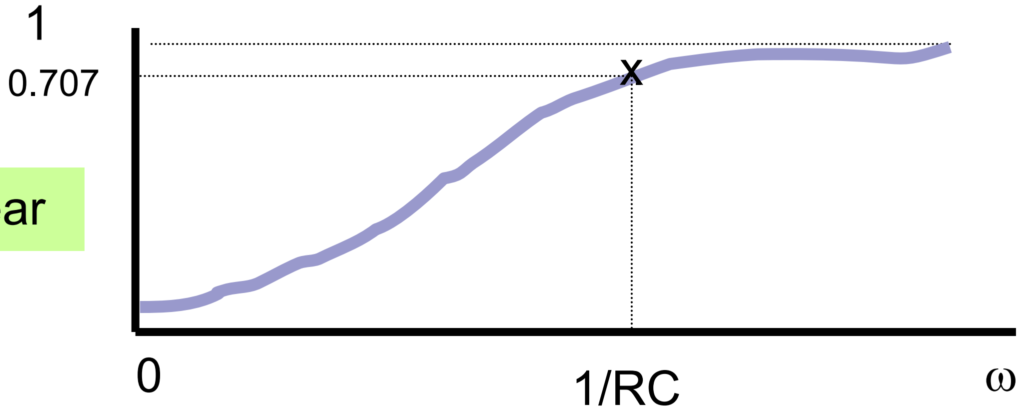
$$M_{\max} = 1, \ M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

$$\omega = \frac{1}{\tau} = \text{half power frequency}$$

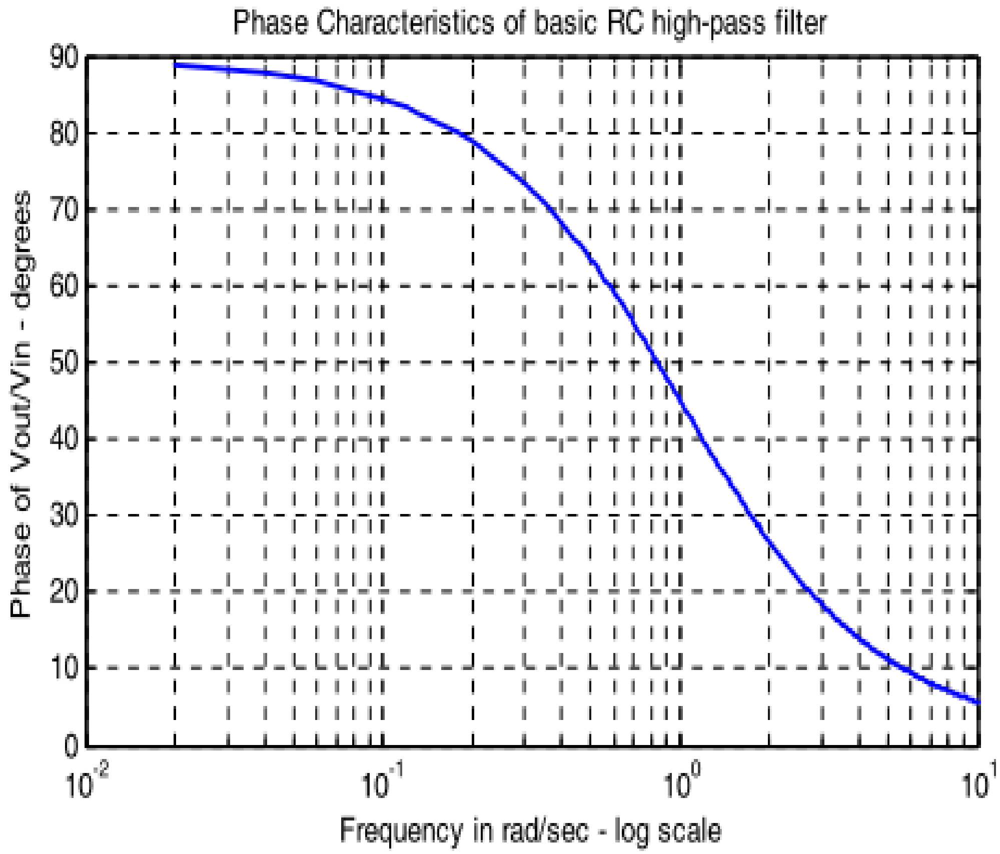
Bode

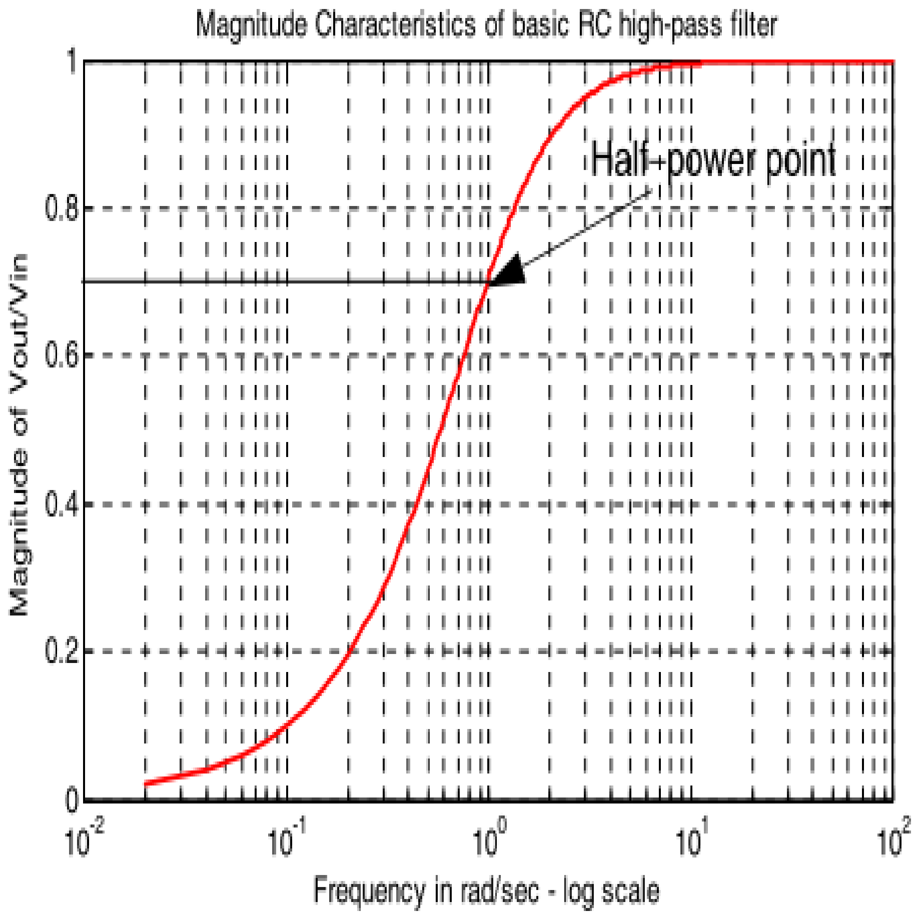


Linear



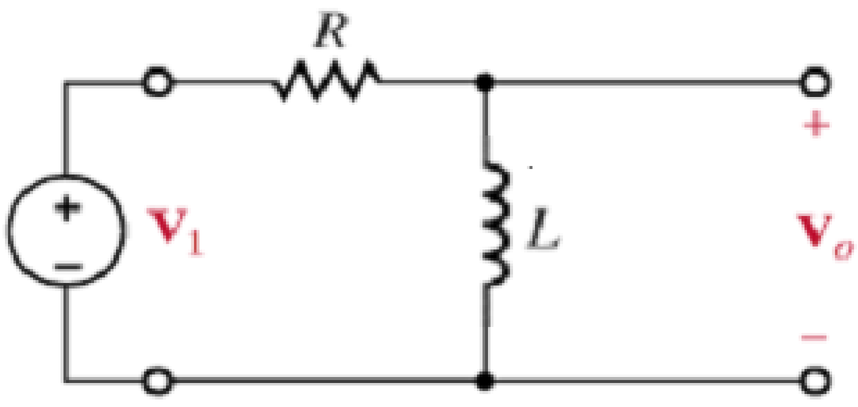
Simple high-pass filter
RC H.P.F





Simple high-pass filter
RL H.P.F

$$G_v = \frac{V_0}{V_1} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L / R}{1 + j\omega L / R}$$



$$G_v = \frac{j\omega\tau}{1 + j\omega\tau};\ \tau = L / R$$

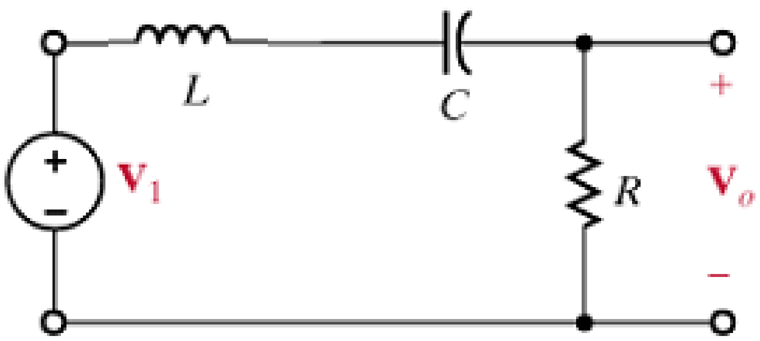
$$M_{\max} = 1,\ M\left(\omega = \frac{1}{\tau}\right) = \frac{1}{\sqrt{2}}$$

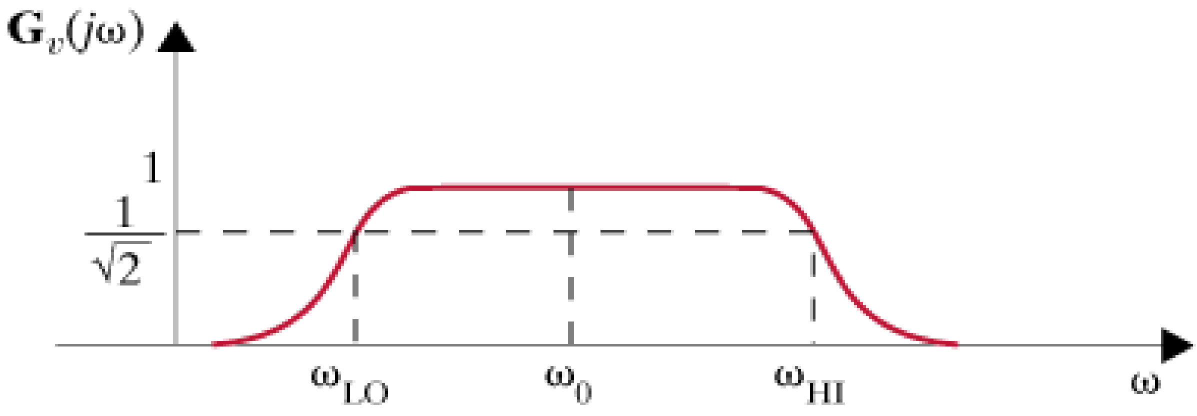
$$M(\omega) = |G_v| = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}$$

$$\omega = \frac{1}{\tau} = \text{half power frequency}$$

$$\angle G_v = \phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$$

Simple band-pass filter





(e)

$$G_v = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + \left(\omega^2 LC - 1\right)^2}}$$

$$M\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1 \qquad M(\omega = 0) = M(\omega = \infty) = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$M(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

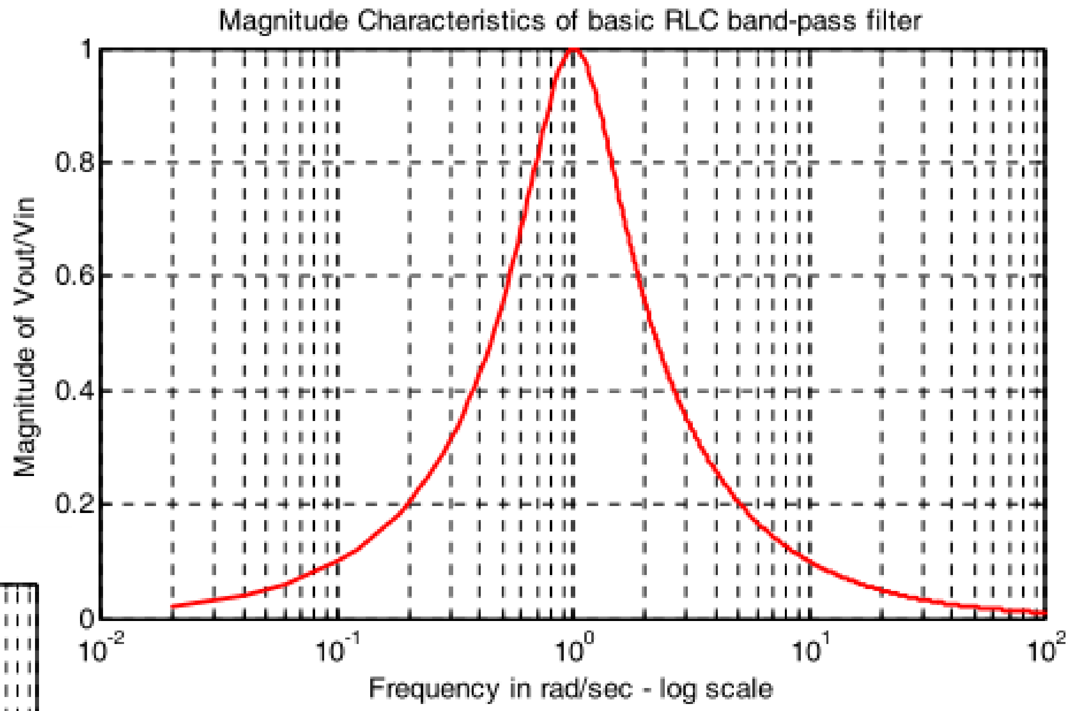
$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

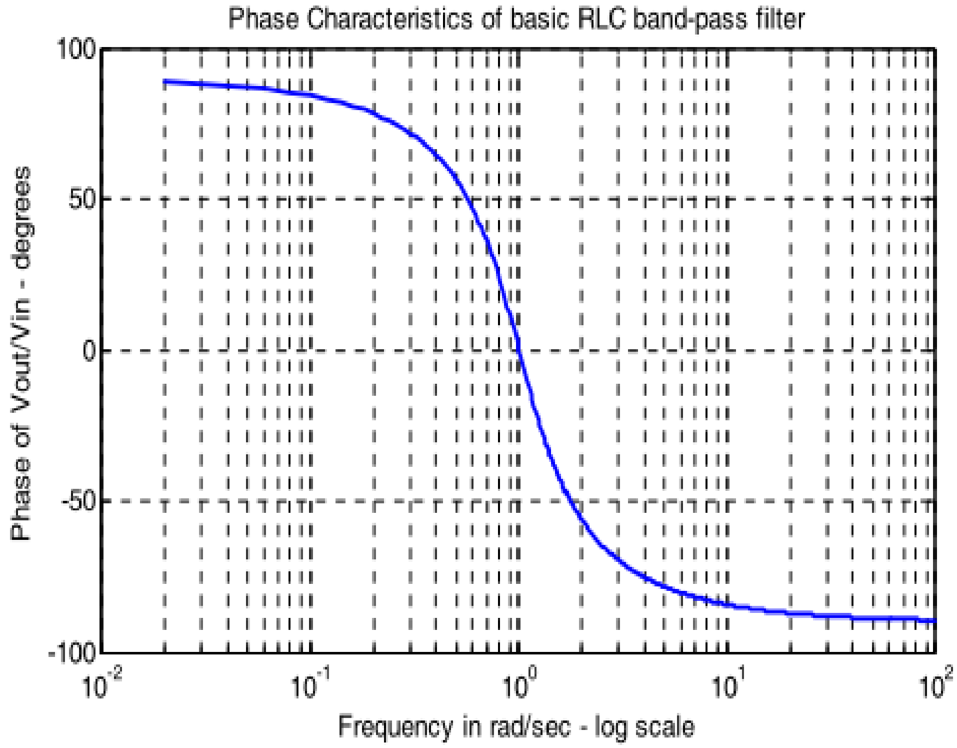
$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

Simple band-pass filter

RLC band–pass filter with R = L = C = 1

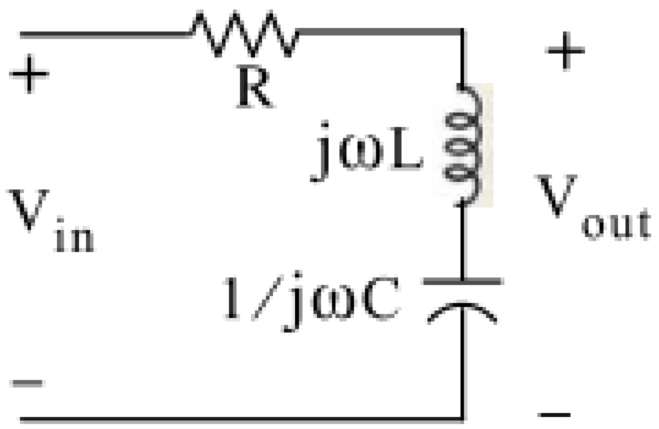




RLC Band−Elimination Filter

$$G(j\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega L + 1/j\omega C}{j\omega L + 1/j\omega C + R} =$$

$$= \frac{-\omega^2 LC + 1}{-\omega^2 LC + 1 + j\omega RC} = \frac{\omega^2 - 1}{\omega^2 - j\omega(R/L) - 1/LC}$$

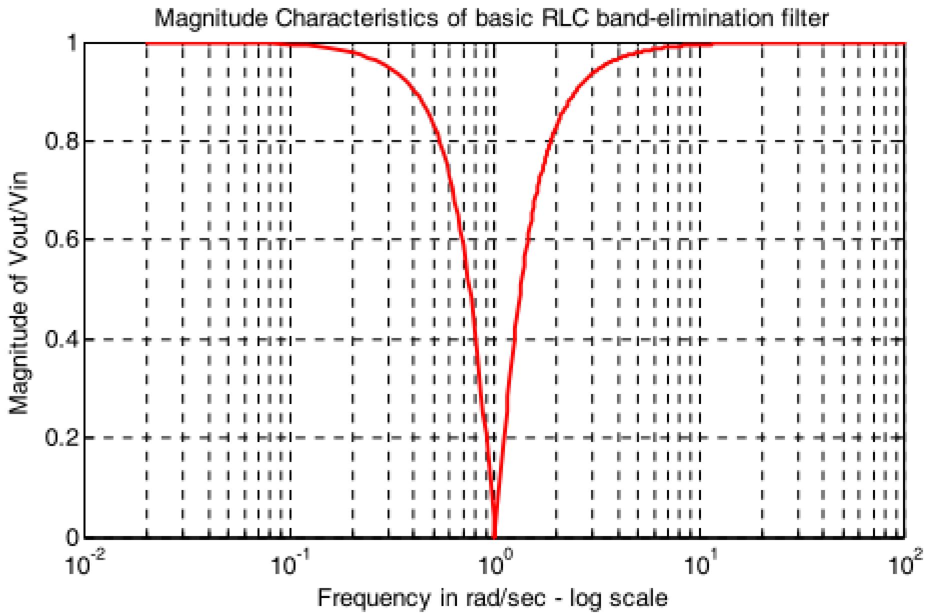


- we can make use of the abs(x) and angle(x) MATLAB functions for the magnitude and phase plots.

For Example if : $R = L = C = 1$

$$G(j\omega) = \frac{\omega^2 - 1}{\omega^2 - j\omega - 1}$$

```
w=0:0.02:100;
magG=abs((w.^2−1)./(w.^2−j.*w−1);
semilogx(w,magG);
grid
```



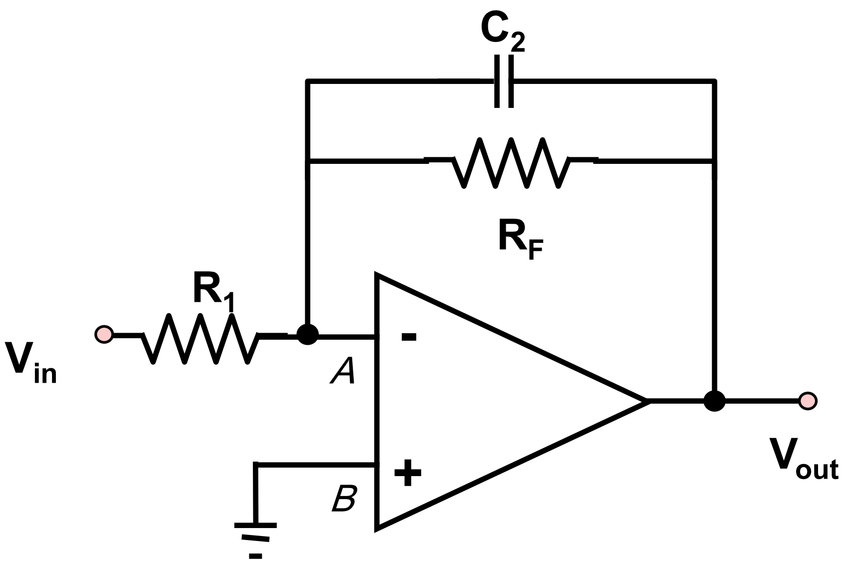
Active Filters

- Active filters employ Op-Amps to attenuate select frequencies and amplify signal during filtering process.

- Q factor of a filter is defined as the ratio of the center frequency f_c to the bandwidth $f_H - f_L$:

$$Q = f_C \big/ \left(f_H - f_L\right)$$

Design of Low Pass Active Filters



$$T.F. = \frac{V_0}{V_{in}} = -\frac{R_F // (-jX_c)}{R_1}$$

$$T.F. = \frac{V_0}{V_{in}} = -\frac{R_F / R1}{1 + j\omega R_F C}$$

The cut-off frequency:

$$\omega_c = 1 / (R_F C_2)$$

The DC gain:

$$K_{LP} = \frac{R_F}{R_1}$$

T.F. is the Transfer Function

Designing a Butterworth Filter using Matlab

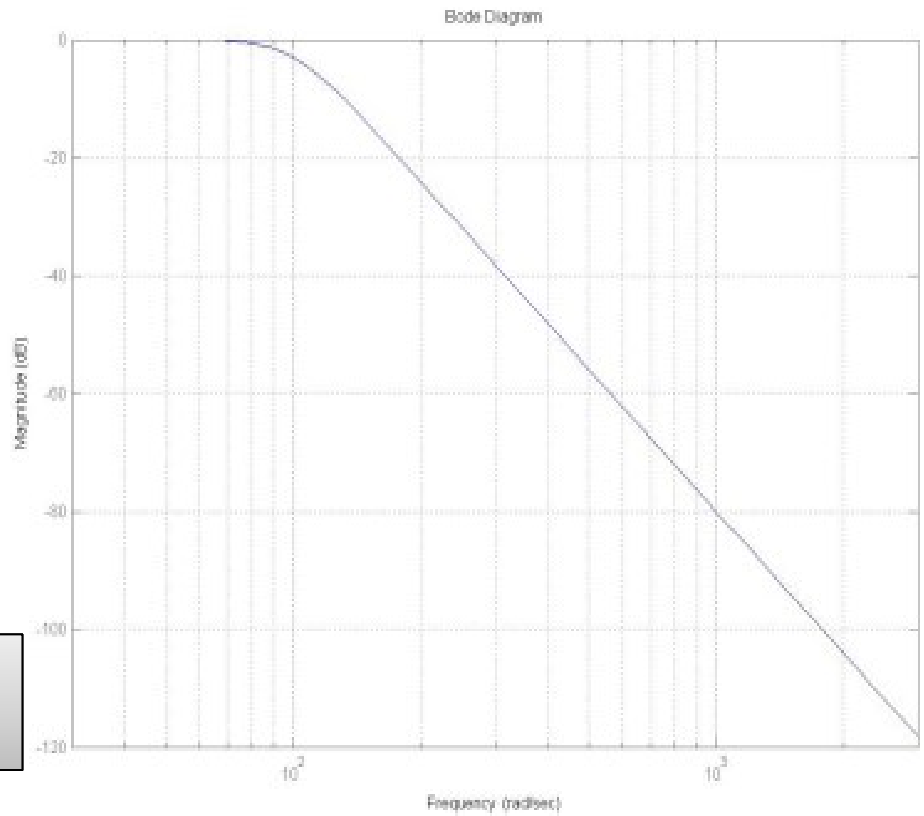
➢ To design an analog low-pass Butterworth filter using MATLAB:

```
[b a] = butter(order, cutoff_freq, 's')
```

- The 's' tells MATLAB to design an analog filter.
- The vectors a and b hold the coefficients of the denominator and the numerator (respectively) of the filter's transfer function.

```
[b a] = butter(4, 100, 's');
G = tf(b,a)
bodemag(G,{30,3000})
```

```
Transfer function:
                          1e008
-----------------------------------------------------
s^4 + 261.3 s^3 + 3.414e004 s^2 + 2.613e006 s + 1e008
```



bodemag used to plot the magnitude response from 30 rad/s out to 3,000 rad/s.

- ✓ At 100 rad/s the response seems to have decreased by about 3 dB

- ✓ From 100 rad/s to 1,000 rad/s the response seems to drop by about 80 dB. As this is a fourth order filter its rolloff should be 4 × 20 dB/decade.