Electrical Circuits (2)

Lecture 3
Parallel Resonance

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Parallel Resonance Circuit

It is usually called tank circuit

Ideal Circuits

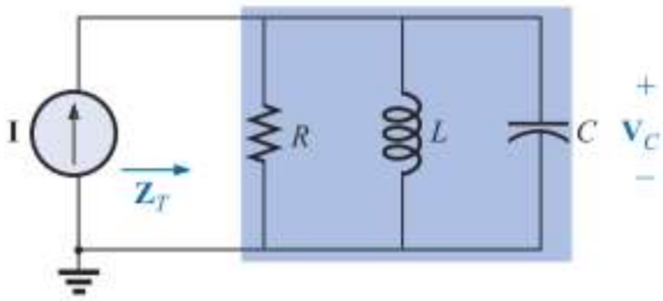


FIG. 20.21
Ideal parallel resonant network.

Practical Circuits

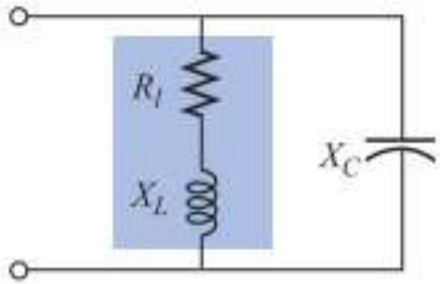
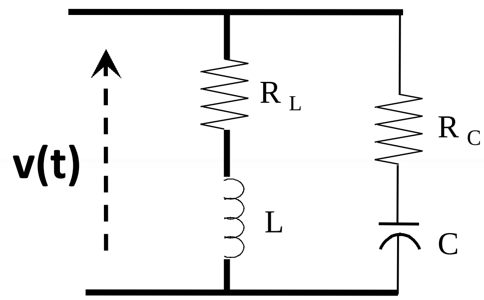


FIG. 20.22
Practical parallel L-C network.

Complex Configuration



Ideal Parallel Resonance Circuit

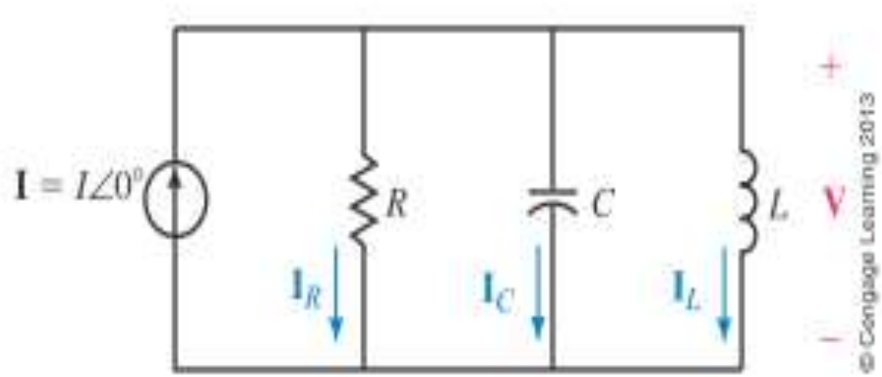
The total admittance

$$Y = Y_1 + Y_2 + Y_3$$

$$Y = \frac{1}{R} + \frac{1}{(j\omega.L)} + \frac{1}{(-j/\omega.C)}$$

$$Y = \frac{1}{R} + \frac{-j}{\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j(\omega C - 1/\omega L)$$



Condition for Ideal Parallel Resonance

Resonance occurs when the imaginary part of **Y** is zero

$$\omega C - \frac{1}{\omega L} = 0$$

$$X_C = X_L.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

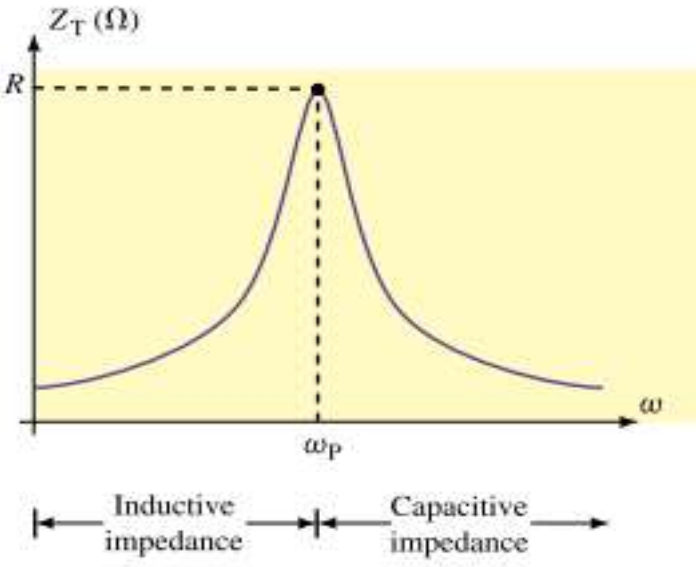
Ideal Parallel Resonance Circuit

At parallel resonance:

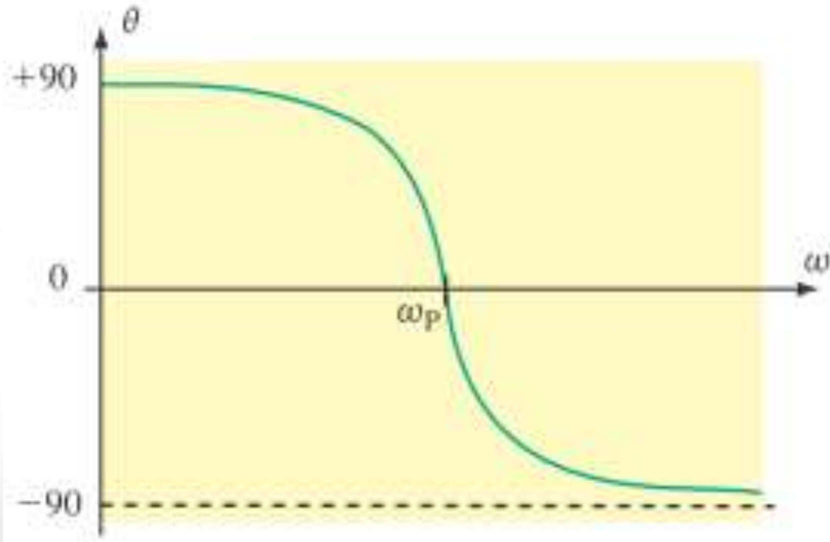
- ✓ At resonance, the admittance consists only conductance G = 1/R.
- ✓ The value of current will be minimum since the total admittance is minimum.
- ✓ The voltage and current are in phase (Power factor is unity).
- ✓ The inductor and capacitor reactances cancel, resulting in a circuit voltage simply determined by Ohm's law as:

$$\mathbf{V} = \mathbf{I}R = IR\angle 0^\circ$$

- ✓ The frequency response of the impedance of the parallel circuit is shown



exactly opposite to that in series resonant circuits,



Ideal Parallel Resonance Circuit

The Q of the parallel circuit is determined from the definition as

$$Q_P = \frac{\text{reactive power}}{\text{average power}} = \frac{V^2/X_L}{V^2/R}$$

$$Q_P = \frac{R}{X_{LP}} = \frac{R}{X_C}$$

Reciprocal of series case

The current

$$\mathbf{I}_R = \frac{\mathbf{V}}{\mathbf{R}} = \mathbf{I}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{X_L\angle 90^\circ} = \frac{V}{R/Q_P}\angle -90^\circ = Q_P I\angle -90^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{X_C\angle -90^\circ} = \frac{V}{R/Q_P}\angle 90^\circ = Q_P I\angle 90^\circ$$

- ✓ The currents through the inductor and the capacitor have the same magnitudes but are 180 out of phase.
- ✓ Notice that the magnitude of current in the reactive elements at resonance is Q times greater than the applied source current.

Ideal Parallel Resonance Circuit

- **Parallel resonant circuit has same parameters as the series resonant circuit.**

Resonance frequency:

$$\omega_p = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Half-power frequencies:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

Bandwidth and Q-factor:

$$\text{BW} = \omega_2 - \omega_1 = \frac{1}{RC} \quad \text{(rad/s)}$$

$$\text{BW} = \frac{\omega_P}{R(\omega_P C)} = \frac{X_C}{R}\omega_P$$

$$\text{BW} = \frac{\omega_P}{Q_P} \quad \text{(rad/s)}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q},$$

$$\omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

Practical Parallel Resonance Circuit

Effect of Winding Resistance on the Parallel Resonant Frequency

- The internal resistance of the coil must be taken into consideration because it is no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes.
- Even though RL is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition,

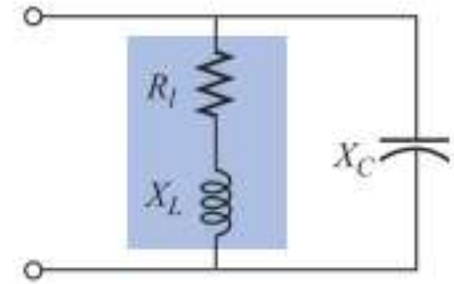


FIG. 20.22
Practical parallel L-C network.

1. Find a parallel network equivalent to the series R-L branch

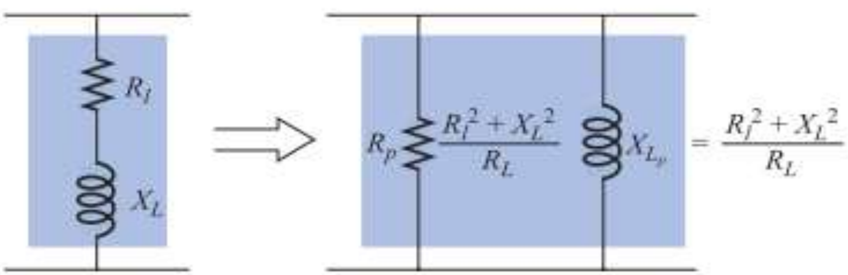


FIG. 20.23
Equivalent parallel network for a series R-L combination.

$$\mathbf{Z}_{R\text{-}L} = R_l + j\,X_L$$

$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\mathbf{Z}_{R\text{-}L}} = \frac{1}{R_l + j\,X_L} = \frac{R_l}{R_l^2 + X_L^2} - j\,\frac{X_L}{R_l^2 + X_L^2}$$

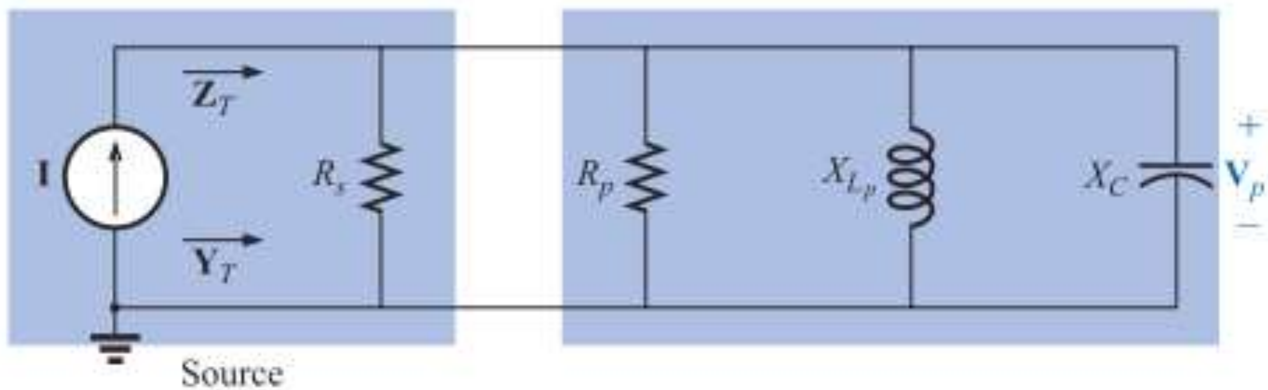
Practical Parallel Resonance Circuit

$$R_p = \frac{R_l^2 + X_L^2}{R_l}$$

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}$$

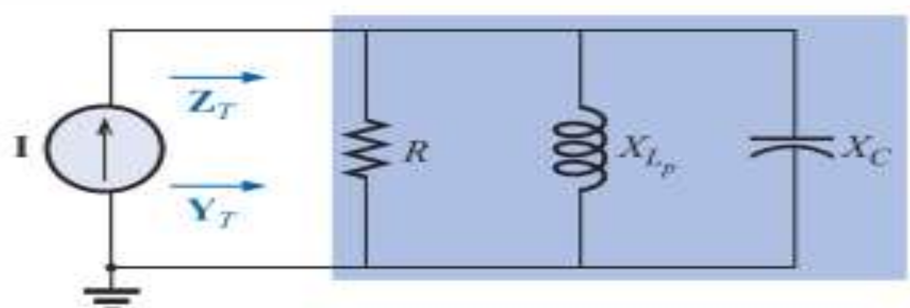
$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{j X_{Lp}}$$

Redrawing the network



If we define the parallel combination of R_s and R_p by the notation

$$R = R_s \| R_p$$



Practical Parallel Resonance Circuit

$$\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)$$

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

$$X_{L_p} = X_C$$

$$\frac{R_l^2 + X_L^2}{X_L} = X_C$$

The resonant frequency, fp , can now be determined as follows:

$$R_l^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R_l^2$$

$$2\pi f_p L = \sqrt{\frac{L}{C} - R_l^2}$$

$$f_p = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_l^2}$$

Multiplying within the square-root sign by C/L and rearranging produces :

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}$$

Practical Parallel Resonance Circuit

1. Maximum impedance

- At f = fp the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of Rp .

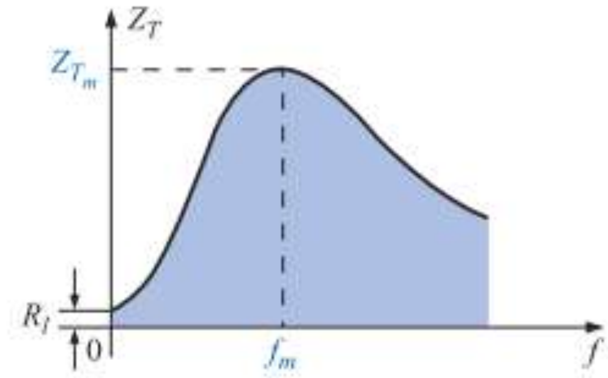


FIG. 20.26
Z_T versus frequency for the parallel resonant circuit.

- The frequency at which maximum impedance will occur is:

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

fm is determined by differentiating the general equation for ZT with respect to frequency

2. Minimum impedance

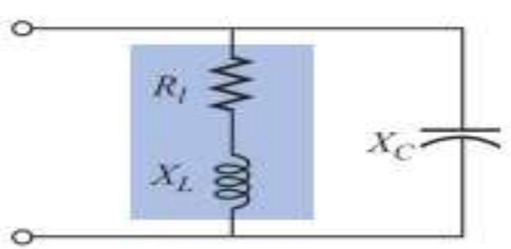


FIG. 20.22
Practical parallel L-C network.

At f = 0 Hz,

Xc is O.C, XL = zero

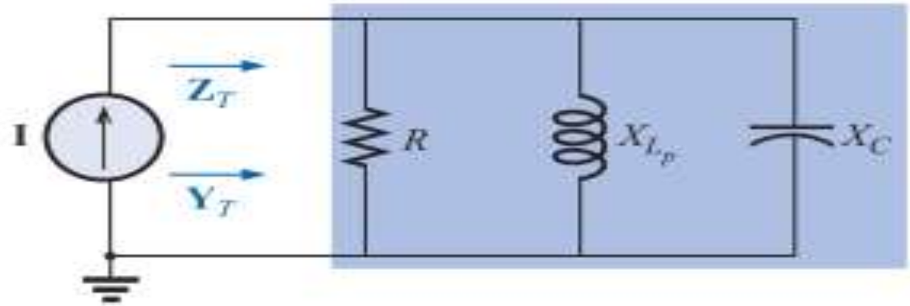
$$Z_T = R_s \parallel R_l \cong R_l.$$

As Rs is sufficiently large for the current source (ideally infinity)

Practical Parallel Resonance Circuit

➢ The quality factor of the practical parallel resonant circuit

determined by the ratio of the reactive power to the real power at resonance



$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

$$R = R_s \| R_p$$

V_p is the voltage across the parallel branches.

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s \| R_p}{X_{L_p}}$$

$$Q_p = \frac{R_s \| R_p}{X_C}$$

For the ideal current source ($R_s = \infty\ \Omega$)

$$R = R_s \| R_p \cong R_p$$

$$Q_p = \frac{R_s \| R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

$$Q_p = \frac{X_L}{R_l} = Q_l$$

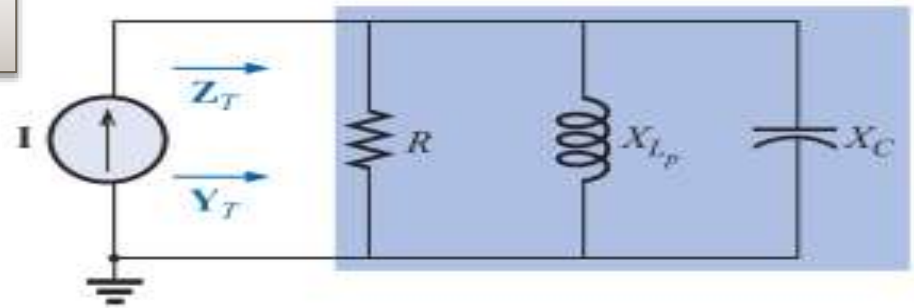
$R_s \gg R_p$

which is simply the quality factor Q_l of the coil.

Practical Parallel Resonance Circuit

- Bandwidth and Half-Power point

$$BW = f_2 - f_1 = \frac{f_r}{Q_p}$$



- The cutoff frequencies f1 and f2 can be determined using the equivalent network shown in the figure:

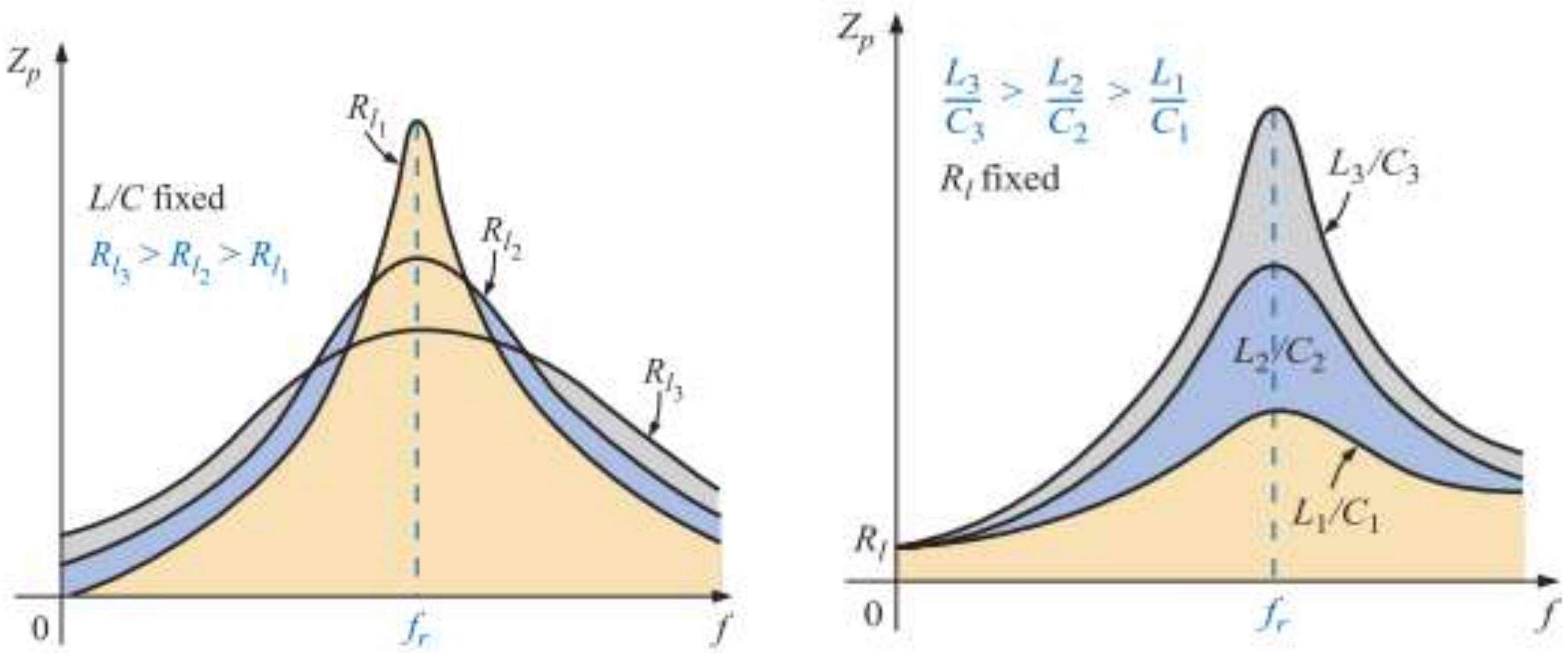
$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = 0.707R$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

Practical Parallel Resonance Circuit

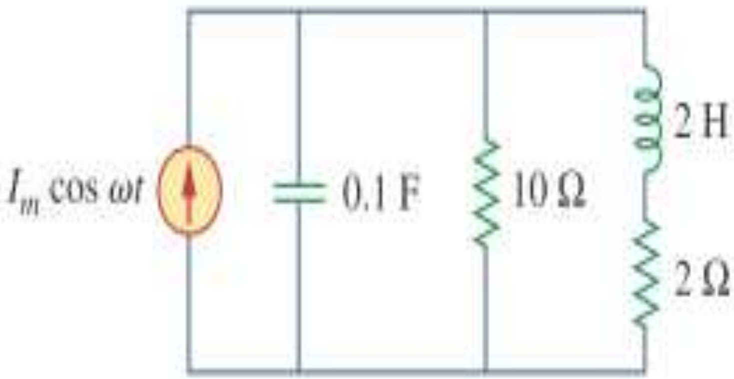
The effect of R_l, L, and C on the shape

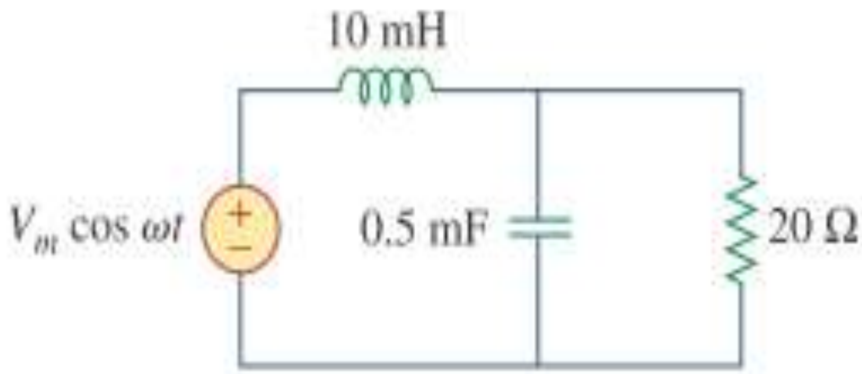


- The same effect as in series resonance

Assignment No(1)

Determine the resonant frequency of the circuit in Fig.





- **Validate your analysis using simulation (Proteus or Multisim)**

- **Hint (1): review properties of resonant case to know how to validate the analysis using simulation**
- **Hint (2): you may use “current probe” + Oscilloscope in Multisim**
- **Hint (3): you may use “current probe” + Mixed Graph in Proteus**

1. **Group solution is not permitted**
2. **Cheating or copying other students work will not be tolerated)**

Thank you