

Lecture notes .in

CHAPTER 12 FREQUENCY RESPONSE ANALYSIS (BODE PLOTS)

After completing this chapter, the students will be able to:

- Plot asymptotic approximations to the frequency response of an open-loop control system,
- Use the Bode plot to determine the stability of open-loop systems
- Find the bandwidth, peak magnitude, and peak frequency of a closed-loop frequency response.

1. Introduction

Frequency response methods, developed by Nyquist (1930) and Bode (1945), are older than the root locus method, which was discovered by Evans in 1948.

Obtaining the frequency response from the transfer function by substituting the value of (ω) directly in the system transfer function is a tedious task. The frequency range required in frequency response is often so wide that it is inconvenient to use a linear scale for the frequency axis. Also, there is a more systematic way of locating the important features of the magnitude and phase plots of the transfer function. For these reasons, it has become standard practice to use a logarithmic scale for the frequency



axis and a linear scale in each of the separate plots of magnitude and phase. Such semi-logarithmic plots of the transfer function—known as *Bode plots*—have become the industry standard. Bode plots contain the same information as the non-logarithmic plots, but they are much easier to construct.

The transfer function $GH(s)$ can be expressed as:

$$GH(s) = |GH| \angle \varphi$$

Since Bode plots are based on logarithms, it is important that we keep the following properties of logarithms in mind:

$$\log X_1 X_2 = \log X_1 + \log X_2$$

$$\log X_1 / X_2 = \log X_1 - \log X_2$$

$$\log X_1^2 = 2 \log X_1$$

$$\log 1 = 0$$

2. The Decibel Scale

In communications systems, gain is measured in *Bels*. The bel is used to measure the ratio of two levels of power or power gain G ; that is,

$$G = \log \frac{P_1}{P_2} \quad \text{Bels}$$

Deci is a suffix express 10 times of the quantity.

$$G = 10 \times \log \frac{P_1}{P_2} \quad \text{decibelBels}$$

decibelBels or (dB) provides less magnitude. Decibels is 1/10 of bels.

Consider the electric network shown in Fig. 1.

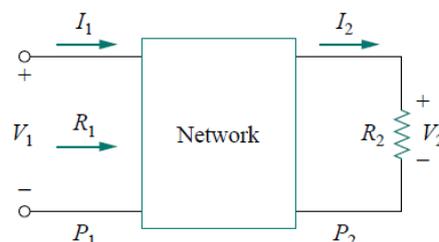


Fig. 1, Simple electric circuit

If P_1 is the input power, P_2 is the output (load) power, R_1 is the input resistance, and R_2 is the load resistance, then:



$$P_1 = \frac{V_1^2}{R_1} = I_1^2 R_1$$
$$P_2 = \frac{V_2^2}{R_2} = I_2^2 R_2$$

Assuming that $R_1 = R_2$ a condition that is often assumed when comparing voltage levels, then:

$$G_{dB} = 10 \times \log \frac{P_1}{P_2} = 10 \times \log \left(\frac{V_1}{V_2} \right)^2 = 20 \times \log \left(\frac{V_1}{V_2} \right)$$

By the same way, assuming that $R_1 = R_2$ a condition that is assumed for comparing current levels, then:

$$G_{dB} = 10 \times \log \frac{P_1}{P_2} = 10 \times \log \left(\frac{I_1}{I_2} \right)^2 = 20 \times \log \left(\frac{I_1}{I_2} \right)$$

To conclude the above information:

- 10 log is used for power, while 20 log is used for voltage or current, because of the square relationship.
- The dB value is a logarithmic measurement of the ratio of one variable to another of the same type. Therefore, it applies in expressing the transfer function.

In Bode plots, the magnitude is plotted in Decibels (dB) versus frequency. The dB quantity can be obtained as:

$$GH_{dB} = 20 \log GH$$

Moreover, the phase angle (φ) is plotted versus frequency. Both magnitude and phase plots are made on semi-log graph paper.

3. Asymptotic Bode Plots (Open-Loop Frequency Response)

The log-magnitude and phase frequency response curves as functions of $\log \omega$ are called Bode plots or Bode diagrams. Sketching Bode plots can be simplified because they can be approximated as a sequence of straight lines. Straight-line approximations simplify the evaluation of the magnitude and phase frequency response.

Consider the following transfer function that may be written in terms of factors that have real and imaginary parts such as:



$$G(j\omega) = \frac{K \left(1 + \frac{j\omega}{z_1}\right) \left(1 + \frac{j\omega}{z_2}\right) \left\{1 + j2\xi_1 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n}\right)^2\right\} \dots}{(j\omega)^{\pm 1} \left(1 + \frac{j\omega}{p_1}\right) \left(1 + \frac{j\omega}{p_2}\right) \left\{1 + j2\xi_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n}\right)^2\right\} \dots}$$

this is called the Bode (Standard) form of the system transfer function that may contain seven different factors:

- Bode gain K
- Pole at origin $(j\omega)^{-1}$ or zero at origin $(j\omega)^{+1}$
- Real pole $\left(1 + \frac{j\omega}{p_1}\right)^{-1}$ and/or real zero $\left(1 + \frac{j\omega}{z_1}\right)$
- Quadratic pole $\left\{1 + j2\xi_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n}\right)^2\right\}^{-1}$ or quadratic zero $\left\{1 + j2\xi_1 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n}\right)^2\right\}$

In constructing a Bode plot, we plot each factor separately and then combine them graphically. The factors can be considered one at a time and then combined additively because of the logarithms involved. For this mathematical convenience of the logarithm, Bode plots is considered as a powerful engineering tool.

In the following subsections, we will make straight-line plots of the factors listed above. These straight-line plots known as asymptotic (approximate) Bode plots.

3.1 Bode Gain

For the gain K , there are two cases:

K is +ve and less than one: the magnitude $20 \log K$ is negative and the phase is 0° ;

K is +ve and greater than one: the magnitude $20 \log K$ is positive and the phase is 0° ;

K is -ve and less than one: the magnitude $20 \log K$ is negative and the phase is -180° ;

K is -ve and greater than one: the magnitude $20 \log K$ is +ve and the phase is -180° ;

Both of the magnitude and phase are constant with frequency. Thus the magnitude and phase plots of the gain are shown in Fig.2.

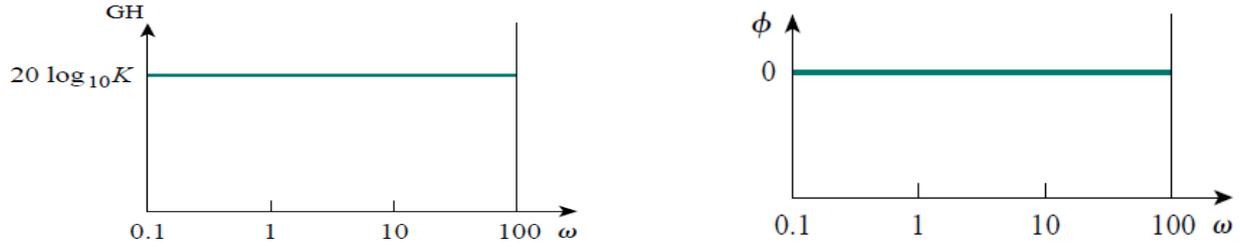


Fig. 2, Magnitude and phase plots of Bode gain

3.2 Zero at origin

For the zero ($j\omega$) at the origin, the magnitude is $20 \log_{10} \omega$ and the phase is 90° . These are plotted in Fig. 3, where we notice that the magnitude is represented by a straight line with slope of 20 dB/decade and intersect the 0dB line at $\omega=1$ and extended to intersect the vertical axis. But the phase is represented by straight line parallel to horizontal axis with constant value at 90° .

القيمة تمثل بخط مستقيم ميله 20 ديسبل لكل ديكاد ويمر بخط الصفر ديسيبل عند $\omega = 1$ ونمده
أما الزاوية فقيمتها ثابتة عند 90 درجة وتمثل بخط مستقيم موازي لمحور ω

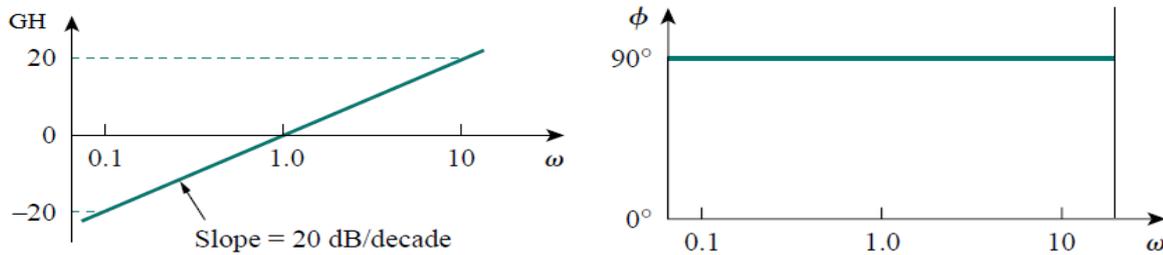


Fig. 3, Magnitude and phase plots of zero at origin

In general, for multiple zeros at origin $(j\omega)^N$, where N is an integer, the magnitude plot will have a slope of $(20 \times N)$ dB/decade. But the phase is $(90 \times N)$ degrees.

3.3 Pole at origin

For the pole $(j\omega)^{-1}$ at the origin, the magnitude is $-20 \log_{10} \omega$ and the phase is -90° . These are plotted in Fig. 4, where we notice that the magnitude is represented by a straight line with slope of -20 dB/decade and intersect the 0dB line at $\omega=1$ and extended to intersect the vertical axis. But the phase is represented by straight line parallel to horizontal axis with constant value at -90° .

القيمة تمثل بخط مستقيم ميله -20 ديسبل لكل ديكاد ويمر بخط الصفر ديسيبل عند $\omega = 1$ ونمده
أما الزاوية فقيمتها ثابتة عند -90 درجة وتمثل بخط مستقيم موازي لمحور ω

In general, for multiple poles at origin $(j\omega)^{-N}$, where N is an integer, the magnitude plot will have a slope of $-(20 \times N)$ dB/decade. But the phase is $-(90 \times N)$ degrees.

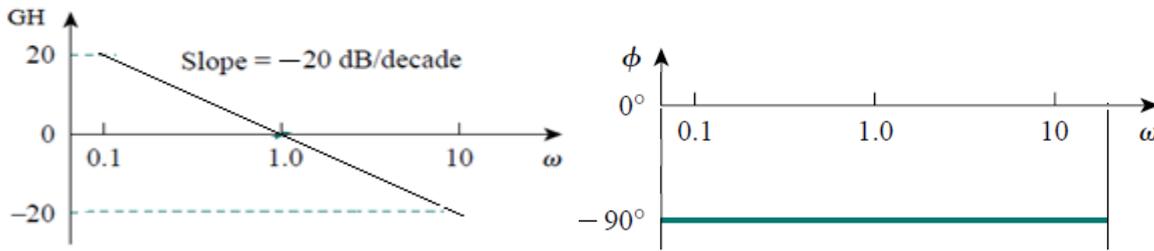


Fig. 4, Magnitude and phase plots of pole at origin

3.4 Real Zero

The magnitude of a real zero $(1 + \frac{j\omega}{z_1})$ is obtained from $20 \log |1 + \frac{j\omega}{z_1}|$, and the phase is obtained from $\tan^{-1}(\frac{\omega}{z_1})$. We notice that:

- For small values of ω , the magnitude is $20 \log |1 + \frac{j\omega}{z_1}| \cong 20 \log 1 = 0$
- For large values of ω , the magnitude is $20 \log |1 + \frac{j\omega}{z_1}| \cong 20 \log |\frac{\omega}{z_1}|$

From the above two points, we can approximate the magnitude of real zero by two straight lines (at $\omega \rightarrow 0$: a straight line with zero slope with zero magnitude) and (at $\omega \rightarrow \infty$: a straight line with slope 20 dB/decade). At the frequency $\omega = z_1$ where the two asymptotic lines meet is called the *corner frequency*. Thus, the approximate magnitude plot is shown in Fig. 5. The actual plot for real zero is also shown in that figure. Notice that the approximate plot is close to the actual plot except at the corner frequency, where $\omega = z_1$ and the deviation is $20 \log |1 + j1| \cong 20 \log \sqrt{2} = 3 \text{ dB}$.

القيمة تمثل بخط مستقيم ميله +20 ديسيبل لكل ديكيد ويمر بخط الصفر ديسيبل عند $Z_1 = \omega$ ولائمه
الزاوية: ديكيد قبل $(Z_1/10)$ الزاوية = صفر، ثم ديكيد بعد $(10Z_1)$ الزاوية = 90 ونصل بينهما بخط مستقيم
ليكون ميل الخط 45 درجة لكل ديكيد

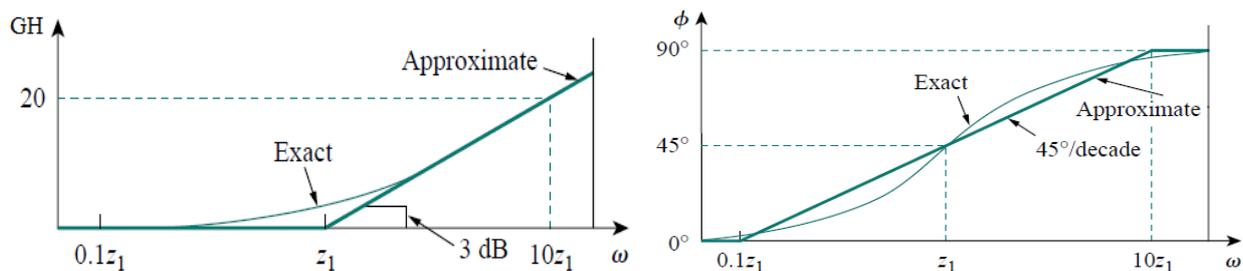


Fig. 5, Magnitude and phase plots of real zero



The phase angle of real zero that given as $\tan^{-1}\left(\frac{\omega}{z_1}\right)$ is represented as a straight-line approximation, $\phi = 0$ for $\omega \leq z_1/10$, $\phi = 45^\circ$ for $\omega = z_1$, and $\phi = 90^\circ$ for $\omega \geq 10z_1$ as shown in Fig. 4. The straight line has a slope of 45° per decade.

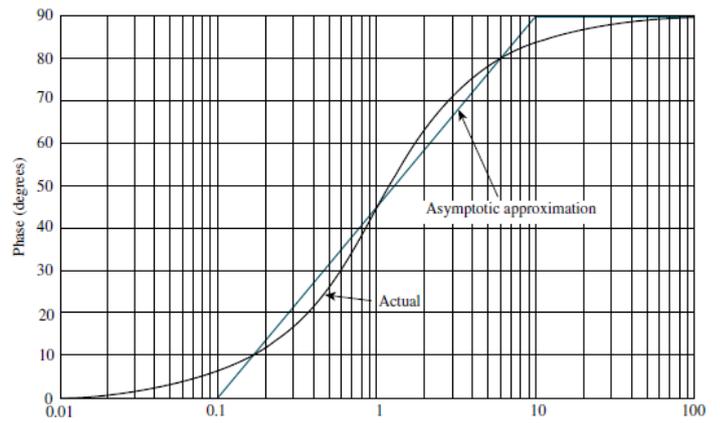
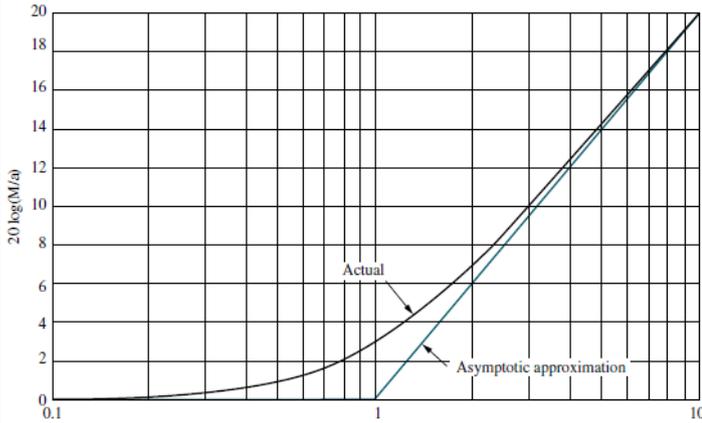
$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \rightarrow \infty \end{cases}$$

For example, consider the real zero (S+1), it will be $(1+j\omega)$ in the Bode form. Then:

$$\text{magnitude} = 20 \log(\sqrt{1 + \omega^2}), \quad \text{phase} = \tan^{-1} \frac{\omega}{1}$$

The following table shows the actual and asymptotic values of the magnitude and phase of that real zero.

Frequency (rad/s)	Magnitude (dB)		Phase (degrees)	
	Asymptotic	Actual	Asymptotic	Actual
0.01	0	0.00	0.00	0.57
0.02	0	0.00	0.00	1.15
0.04	0	0.01	0.00	2.29
0.06	0	0.02	0.00	3.43
0.08	0	0.03	0.00	4.57
0.1	0	0.04	0.00	5.71
0.2	0	0.17	13.55	11.31
0.4	0	0.64	27.09	21.80
0.6	0	1.34	35.02	30.96
0.8	0	2.15	40.64	38.66
1	0	3.00	45.00	45.00
2	6	6.99	58.55	63.43
4	12	12.30	72.09	75.96
6	15.56	15.68	80.02	80.54
8	18	18.13	85.64	82.87
10	20	20.04	90.00	84.29
20	26.02	26.03	90.00	87.14
40	32.04	32.04	90.00	88.57
60	35.56	35.56	90.00	89.05
80	38.06	38.06	90.00	89.28
100	40	40.00	90.00	89.43



3.5 Real Pole

The magnitude of a real pole $\left(1 + \frac{j\omega}{p_1}\right)^{-1}$ is obtained from $-20 \log \left|1 + \frac{j\omega}{p_1}\right|$, and the phase is obtained from $-\tan^{-1} \left(\frac{\omega}{p_1}\right)$. We notice that:

- For small values of ω , the magnitude is $-20 \log \left|1 + \frac{j\omega}{p_1}\right| \cong 20 \log 1 = 0$
- For large values of ω , the magnitude is $-20 \log \left|1 + \frac{j\omega}{p_1}\right| \cong -20 \log \left|\frac{\omega}{p_1}\right|$

From the above two points, we can approximate the magnitude of real pole by two straight lines (at $\omega \rightarrow 0$: a straight line is with zero slope and zero magnitude) and (at $\omega \rightarrow \infty$: the straight line is with slope -20 dB/decade). At the frequency $\omega = p_1$ where the two asymptotic lines meet is called the *corner frequency*. Thus, the approximate magnitude plot is shown in Fig. 6. The actual plot for real pole is also shown in that figure. Notice that the approximate plot is close to the actual plot except at $\omega = p_1$, the deviation is $-20 \log |1 + j1| \cong -20 \log \sqrt{2} = -3 \text{ dB}$.

القيمة تمثل بخط مستقيم ميله -20- ديسبل لكل ديكيد ويمر بخط الصفر ديسيبل عند $\omega = p_1$ ولا نمده

The phase angle of real pole that given as $-\tan^{-1} \left(\frac{\omega}{p_1}\right)$ is represented as a straight-line approximation, $\varphi = 0$ for $\omega \leq p_1/10$, $\varphi = -45^\circ$ for $\omega = p_1$, and $\varphi = -90^\circ$ for $\omega \geq 10p_1$ as shown in Fig. 6. The straight line has a slope of -45° per decade.

الزاوية: ديكيد قبل ($p_1/10$) الزاوية = صفر، ثم ديكيد بعد ($10p_1$) الزاوية = -90 ونصل بينهما بخط مستقيم
ليكون ميل الخط -45 درجة لكل ديكيد

$$\phi = \tan^{-1} \left(\frac{\omega}{p_1} \right) = \begin{cases} 0, & \omega = 0 \\ -45^\circ, & \omega = p_1 \\ -90^\circ, & \omega \rightarrow \infty \end{cases}$$

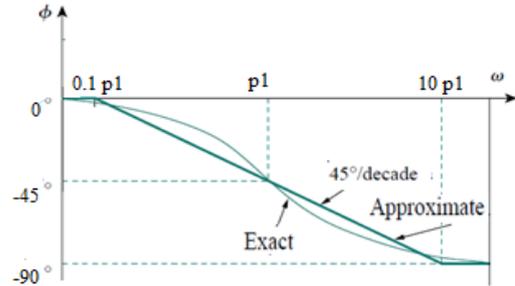
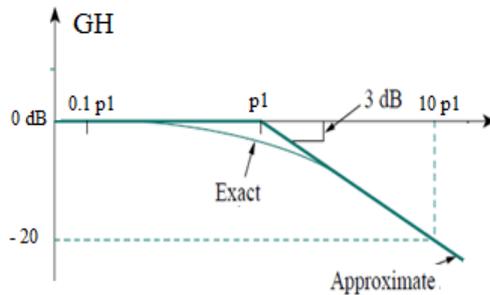


Fig. 6, Magnitude and phase plots of real pole

3.6 Quadratic Zero

The magnitude of a quadratic zero $\left\{ 1 + j2\xi_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n} \right)^2 \right\}$ is obtained as

$$20 \log \left| 1 + j2\xi_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n} \right)^2 \right| = 0 \text{ for } \omega \rightarrow 0$$

$$20 \log \left| 1 + j2\xi_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n} \right)^2 \right| = 20 \log \left| \left(\frac{\omega}{\omega_n} \right)^2 \right| = 40 \log \left| \left(\frac{\omega}{\omega_n} \right) \right| \text{ for } \omega \rightarrow \infty$$

Thus, the amplitude plot consists of two straight asymptotic lines: one with zero slope for $\omega < \omega_n$ and the other one with slope -40 dB/decade for $\omega > \omega_n$, with ω_n as the corner frequency. Figure 7 shows the approximate and actual amplitude plots. Note that the actual plot depends on the damping ratio ξ_2 as well as the corner frequency ω_n . The significant peaking in the neighborhood of the corner frequency should be added to the straight-line approximation if a high level of accuracy is desired. However, we will use the straight-line approximation for the sake of simplicity.

The phase plot is a straight line with a slope of 90° per decade starting at $\omega_n/10$ and ending at $10\omega_n$, as shown in Fig. 7. We see again that the difference between the actual plot and the straight-line plot is due to the damping factor.

$$\phi = \tan^{-1} \frac{2\xi_2 \omega / \omega_n}{1 - \omega^2 / \omega_n^2} = \begin{cases} 0, & \omega = 0 \\ 90^\circ, & \omega = \omega_n \\ 180^\circ, & \omega \rightarrow \infty \end{cases}$$

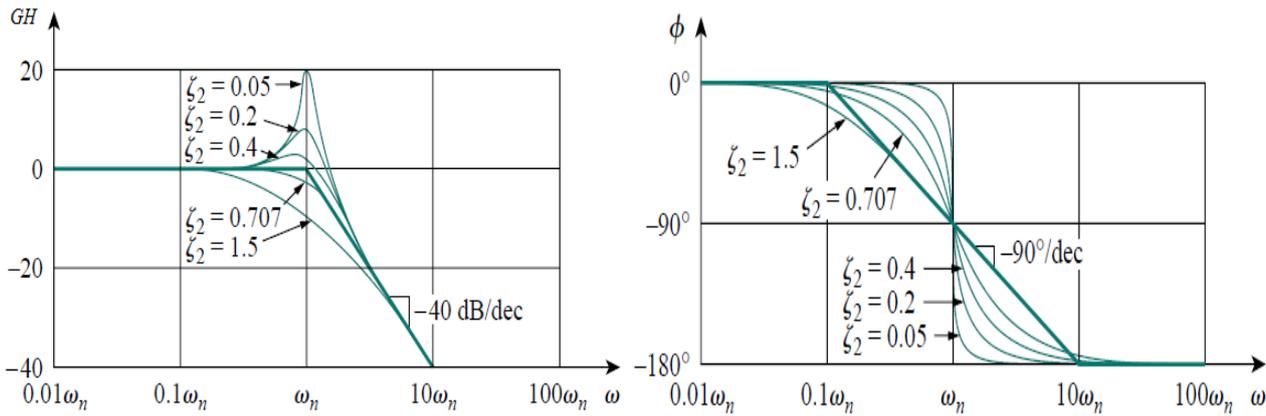


Fig. 7, Magnitude and phase plots of quadratic zero

For the quadratic pole $\left\{1 + j2\zeta_2 \frac{\omega}{\omega_n} + \left(\frac{\omega}{\omega_n}\right)^2\right\}^{-1}$ the plots shown in Fig. 7 are inverted because the magnitude plot has a slope of -40 dB/decade while the phase plot has a slope of -90° per decade.

$$\phi = -\tan^{-1} \frac{2\zeta_2 \omega / \omega_n}{1 - \omega^2 / \omega_n^2} = \begin{cases} 0, & \omega = 0 \\ -90^\circ, & \omega = \omega_n \\ -180^\circ, & \omega \rightarrow \infty \end{cases}$$

Since the asymptotes are quite easy to draw and are sufficiently close to the exact curve, the use of such approximations in drawing Bode diagrams is convenient in establishing the general nature of the frequency-response characteristics quickly with a minimum amount of calculation and may be used for most preliminary design work.

4. Closed-Loop Stability Analysis Using Bode Plots

The gain crossover frequency ω_g is defined as the frequency at which the total magnitude equals 0 dB. Therefore, its value can be determined from the intersection of the total magnitude line with the 0 dB line as shown in Fig. 8. On the other hand, the phase crossover frequency ω_p is defined as the frequency at which the total phase equals -180° . Therefore, its value can be determined from the intersection of the total phase line with the -180° line as shown in Fig. 8.

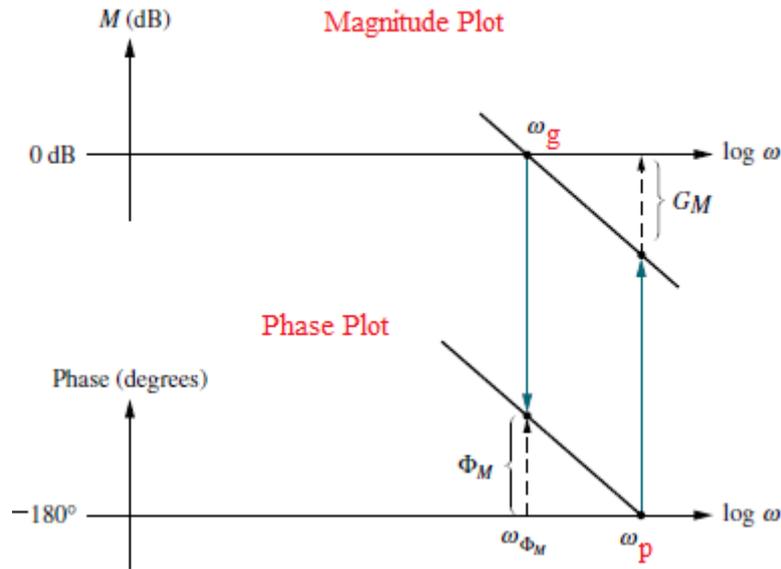


Fig. 8, Gain and Phase crossover frequencies

The system Gain Margin (GM) in dB is the additional gain that makes the system on the edge of instability. GM can be determined by calculating the total magnitude at $\omega = \omega_p$. Also, the system Phase Margin ϕ_{PM} in degrees is the additional phase that makes the system on the edge of instability. ϕ_{PM} can be determined by calculating the total phase at $\omega = \omega_g$ as shown in Fig. 9.

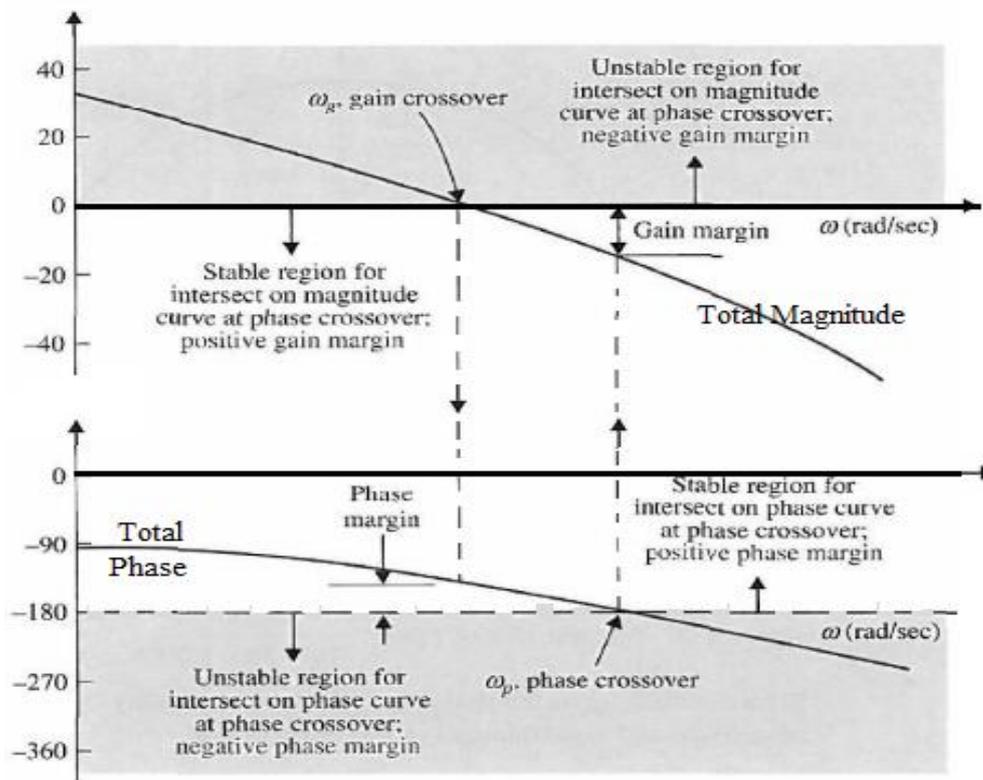


Fig. 9, Gain and Phase Margins

5. Plotting Bode Plots Using Matlab

To specify the frequency range for Bode plots, use the command:

```
>> logspace (d1, d2)
```

This generate 50 points logarithmically equally spaced between 10^{d1} and 10^{d2} .

For example, if we need Bode plot starts at 0.1 rad/sec and finish at 100 rad/sec, enter the command:

```
>> logspace (-1, 2)
```

If we need to change the number of points between d1 and d2 rather than 50, use the command:

```
>> logspace (d1, d2, n)
```

where n is the number of points to be generated.

For example, to generate 100 points between 1 rad/sec and 1000 rad/sec, use:

```
>> W=logspace (0, 3, 100)
```

To draw the Bode plot, we use the command

```
>> sys=tf (num, den)
```

```
>> bode (sys, W)
```

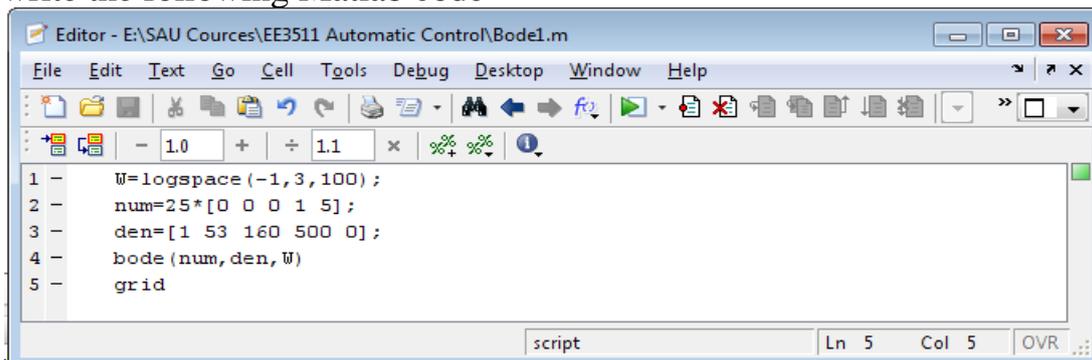
To display the gain and phase margins

```
>> margin (sys)
```

Suppose we need to draw the Bode plot for the control system:

$$GH(s) = \frac{25(S + 5)}{S(S^2 + 3S + 10)(S + 50)}$$

So we write the following Matlab code



```

Editor - E:\SAU Courses\EE3511 Automatic Control\Bode1.m
File Edit Text Go Cell Tools Debug Desktop Window Help
1 - W=logspace (-1, 3, 100);
2 - num=25*[0 0 0 1 5];
3 - den=[1 53 160 500 0];
4 - bode (num, den, W)
5 - grid
script Ln 5 Col 5 OVR
    
```

The magnitude and phase plots are as shown below in Fig. 10, and it is clear that this is an unstable system.

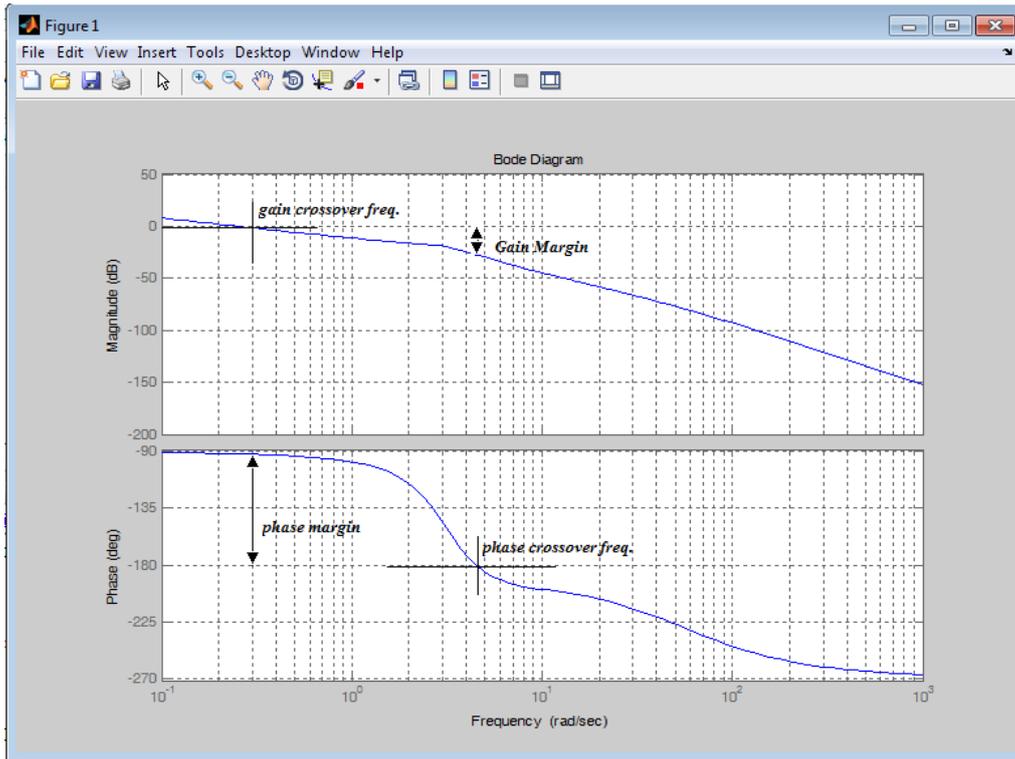
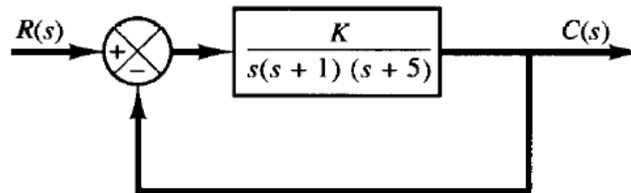


Fig. 10, Magnitude and phase plots

Example (1):

Obtain the phase and gain margins of the system shown below, for the two cases $K=10$ and $K=100$.



First, we draw the Bode plot at $K=10$ as shown in Fig. 11, and calculate the Gain and Phase margins as:

G.M. = 8 dB (+ve) and Phase Margin = 21° (+ve). This gives **Stable System**

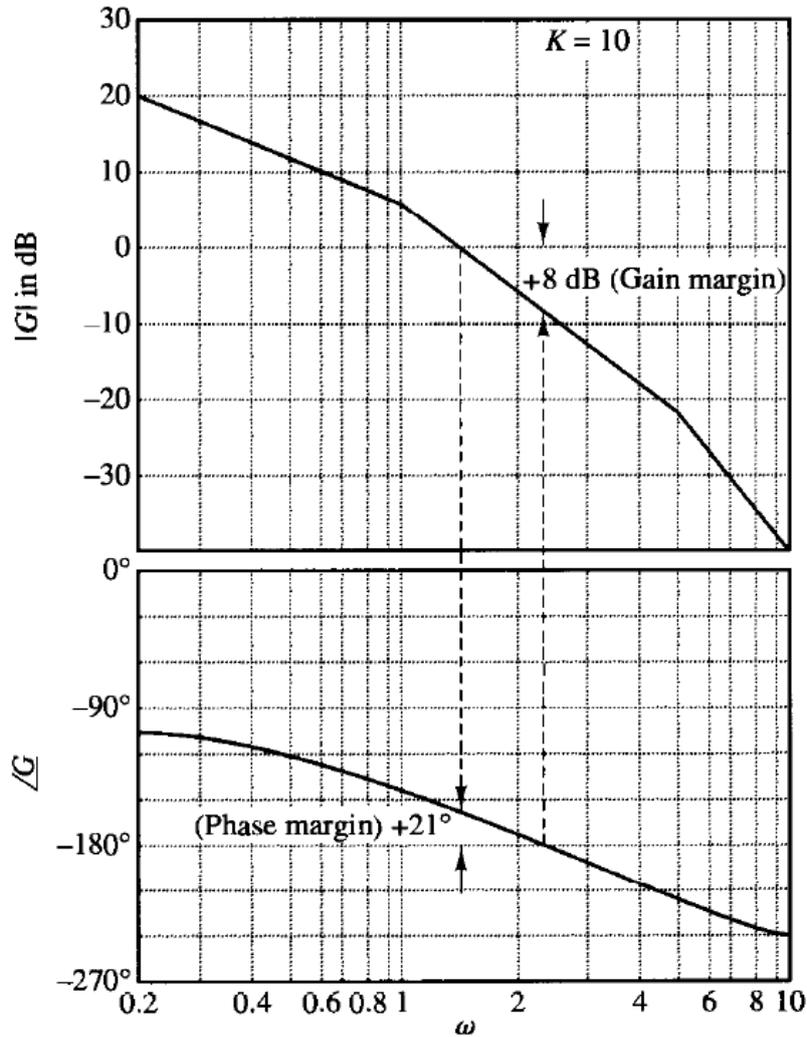


Fig. 11, Bode plot at K=10

Now, the system gain is changed to 100.

Changing the system gain change the magnitude plot up or down depending on K. But the phase plot is not affected at all.

Therefore, no need to re-plot the gain plots at the new gain, but we can modify each point on the magnitude plot up by 20 dB ($20 \log 100 - 20 \log 10$) as shown in Fig. 12.

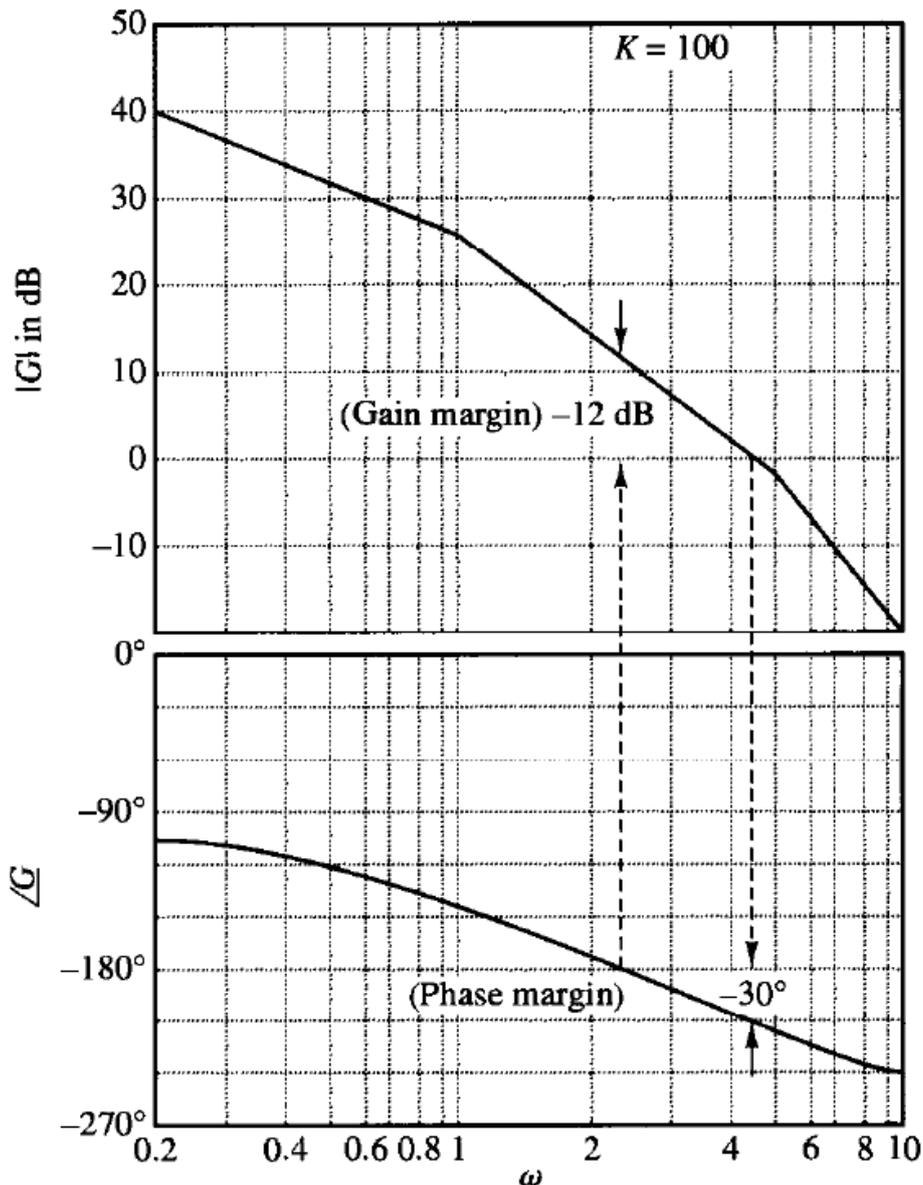


Fig. 12, Bode plot at $K=100$

From Fig. 11, the Gain and Phase margins are:

G.M. = 12 dB (-ve) and Phase Margin = 30° (-ve). This gives **Unstable System**

6. Finding T.F. & Steady-State Error Coefficient from Bode Plot

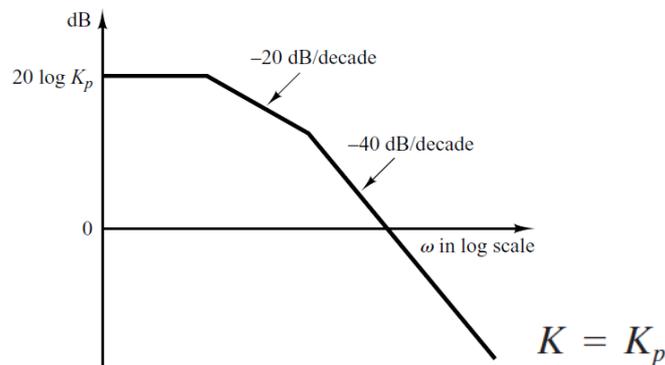
The asymptotic Bode plot must have slopes of multiples of ± 20 dB/decade. If the slope is changed from -20 to -40 dB/decade at $\omega = \omega_1$, this means there is one pole at ω_1 . Also, If the slope is changed from -20 to -60 dB/decade at $\omega = \omega_2$, this means there is quadratic poles at ω_2 .

On the other hand, if the slope is changed from -20 to 0 dB/decade at $\omega=\omega_1$, this means there is one zero at ω_1 . Also, If the slope is changed from -20 to $+20$ dB/decade at $\omega=\omega_2$, this means there is quadratic zeros at ω_2

The type of the system determines the slope of the log-magnitude curve at low frequencies. Now, to calculate how many poles at origin (system type) and the system gain (K), we need to examine the first portion of the plot as follows:

6.1 Type zero systems:

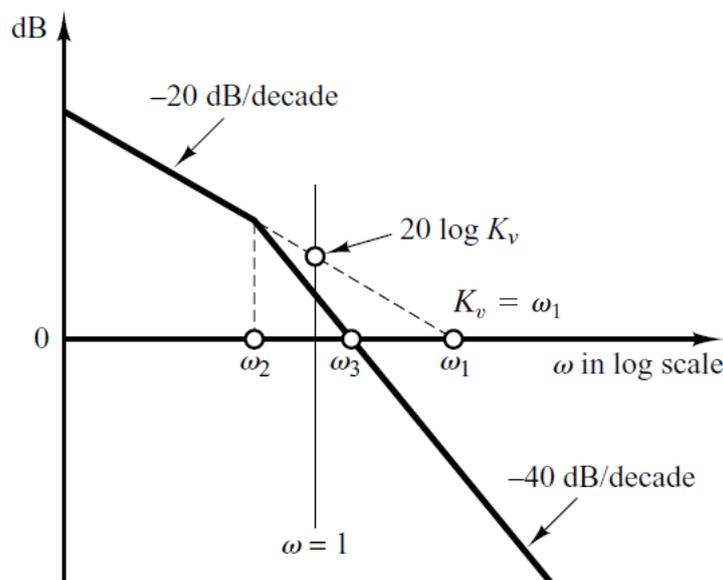
It follows that the low-frequency asymptote is a horizontal line at $20 \log K_p$ dB.



6.2 Type one systems

The intersection of the initial -20 -dB/decade segment (or its extension) with the line $\omega = 1$ has the magnitude $20 \log K_v$

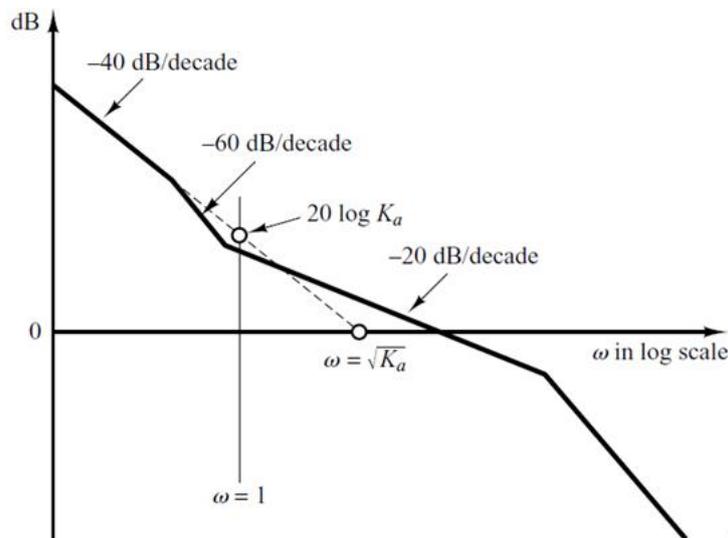
The intersection of the initial -20 -dB/decade segment (or its extension) with the 0 -dB line has a frequency numerically equal to K_v .



6.3 Type two systems

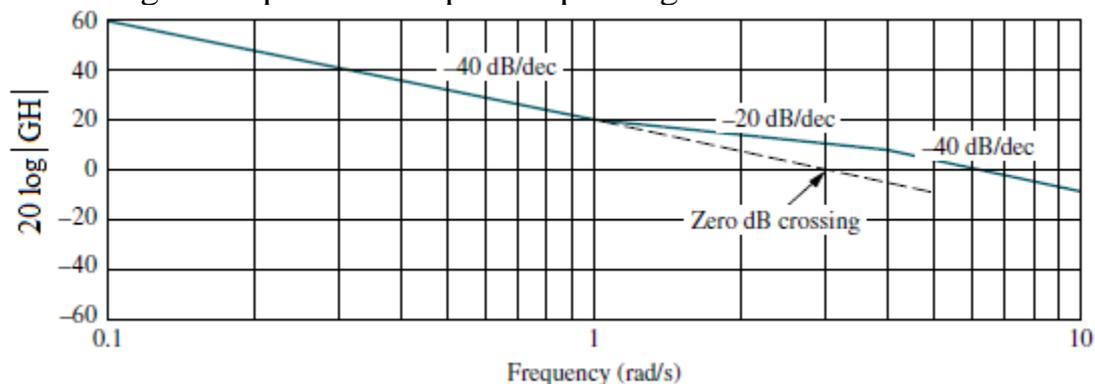
The intersection of the initial -40 -dB/decade segment (or its extension) with the $\omega = 1$ line has the magnitude of $20 \log K_a$.

The frequency ω_a at the intersection of the initial -40 -dB/decade segment (or its extension) with the 0 -dB line gives the square root of K_a numerically.



Example (2)

Consider the magnitude plot of the open-loop T.F. given below:



Obtain:

- The steady-state error coefficient
- The open-loop T.F.

Referring to the first section in the magnitude plot given above, we find the initial slope is -40 dB/dec, this means this system is type 2.

We can calculate the acceleration error coefficient K_a from the point of intersection between the extension of -40 dB/dec line with 0 dB line which is 3.0 rad/s



$$\sqrt{K_a} = 3 \rightarrow K_a = 9$$

Also, we can get K_a by obtaining the magnitude at $\omega = 1$

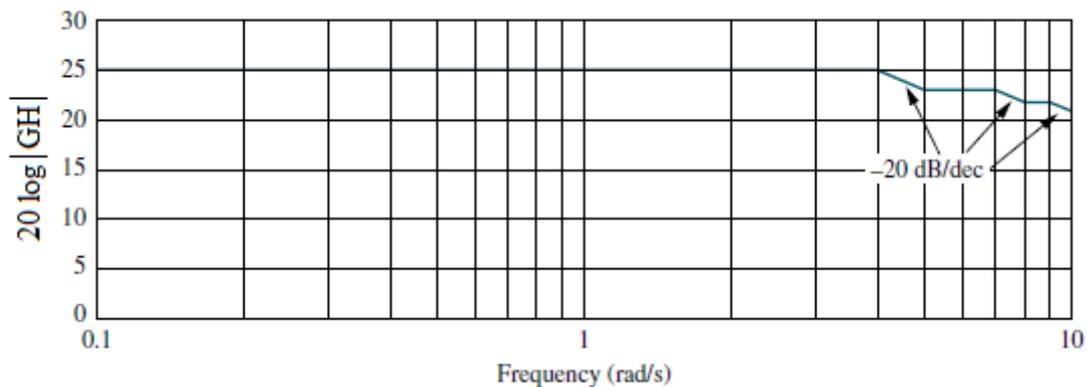
$$20 \log(K_a) = 19.1 \rightarrow K_a = 9$$

Therefore, the system gain $K = K_a = 9$ and the open-loop T.F. is given as:

$$GH(j\omega) = \frac{9 \left(1 + \frac{j\omega}{1}\right)}{j\omega^2 \left(1 + \frac{j\omega}{4}\right)}$$

Example (3)

Consider the magnitude plot of the open-loop T.F. given below:



Obtain:

- The steady-state error coefficient
- The open-loop T.F.

Referring to the first section in the magnitude plot given above, we find the initial slope is 0 dB/dec, this means this system is type 0.

We can calculate the position error coefficient K_p as

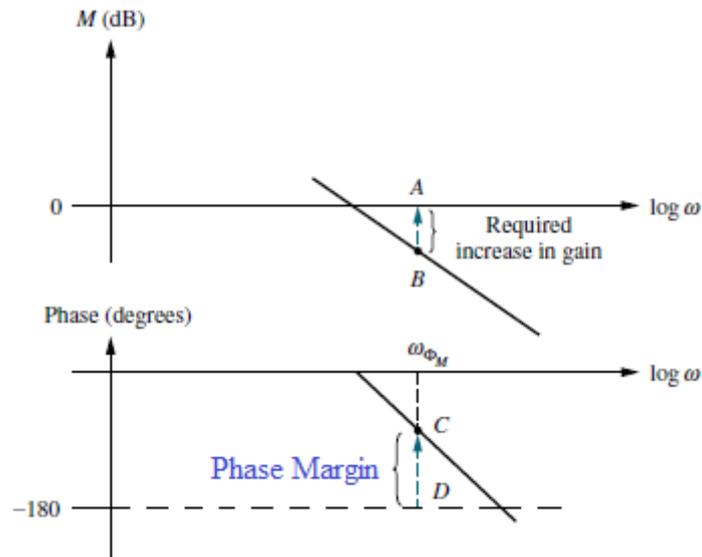
$$20 \log K_p = 25 \rightarrow K_p = 17.7828$$

Therefore, the system gain $K = K_p = 17.7828$ and the open-loop T.F. is given as:

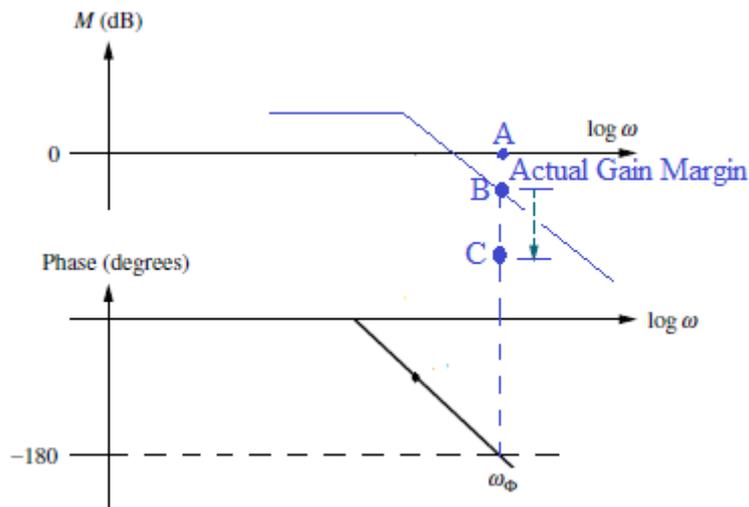
$$GH(j\omega) = \frac{17.7828 \left(1 + \frac{j\omega}{5}\right) \left(1 + \frac{j\omega}{8}\right)}{\left(1 + \frac{j\omega}{4}\right) \left(1 + \frac{j\omega}{7}\right) \left(1 + \frac{j\omega}{9}\right)}$$

7. Design Via Gain Adjustment

Referring to Fig. below, we see that if we desire certain phase margin, represented by CD, we would have to raise the magnitude curve by AB. Thus, a simple gain adjustment can be used to design phase margin.



On the other hand, if we desire certain gain margin, represented by AC, and from the system Bode plot we find the actual gain margin is AB (as shown in Fig. below). Therefore, we would have to decrease the magnitude curve by BC. Thus, a simple gain adjustment can be used to design gain margin also.



Design Procedure:

- Assume the system gain $K = 1$ (At this value, the magnitude plot is not affected as $20 \log(1) = 0$. And the phase plot is not affected as the $\angle 1 = 0^\circ$)
- Draw the system Bode magnitude and phase plots.
- Based on the required Gain Margin or required Phase Margin, determine the amount of increment or decrement in the magnitude plot.



Example (4)

For the open loop transfer function given below:

$$GH(s) = \frac{500K(S + 2)}{S(S + 1)(S^2 + 5S + 100)} \quad (S = j\omega)$$

- Construct the Bode plots,
- The Gain crossover frequency and the Gain Margin,
- The phase crossover frequency and the Phase Margin,
- Design the value of K to change the gain crossover frequency to 1 rad/s, then find the corresponding Gain and Phase Margins,
- Design the value of K to give a phase margin of 54° , then find the gain crossover frequency and the corresponding Gain Margin,
- Design the value of K to give a critical stable system.

For the transfer function given below:

$$H(s) = \frac{500 K(S + 2)}{S(S + 1)(S^2 + 5S + 100)}$$

we put the T.F. in Bode form as:

$$H(s) = \frac{10K \left(1 + \frac{j\omega}{2}\right)}{j\omega \left(1 + \frac{j\omega}{1}\right) \left\{1 + \frac{5j\omega}{100} + \left(\frac{j\omega}{10}\right)^2\right\}}$$

Assuming $K=1$,

The Bode gain = $20 \log(10) = 20 \text{ dB}$

The total magnitude and total phase are plotted as given in semi-log paper.

- Bode plots are sketched as given in semilog paper.

From Bode plots:

- The gain crossover frequency = 4.8 rad/s & Gain Margin = 2dB
- The phase crossover frequency = 6.4 rad/s & Phase Margin = 14°
- To change the gain crossover frequency to 1.0 rad/s, the total magnitude at this point must be zero dB. But from Bode plot, it is found that the total magnitude is +20 dB, so it is recommended to add -20 dB at that point



$$20 \log K = -20$$

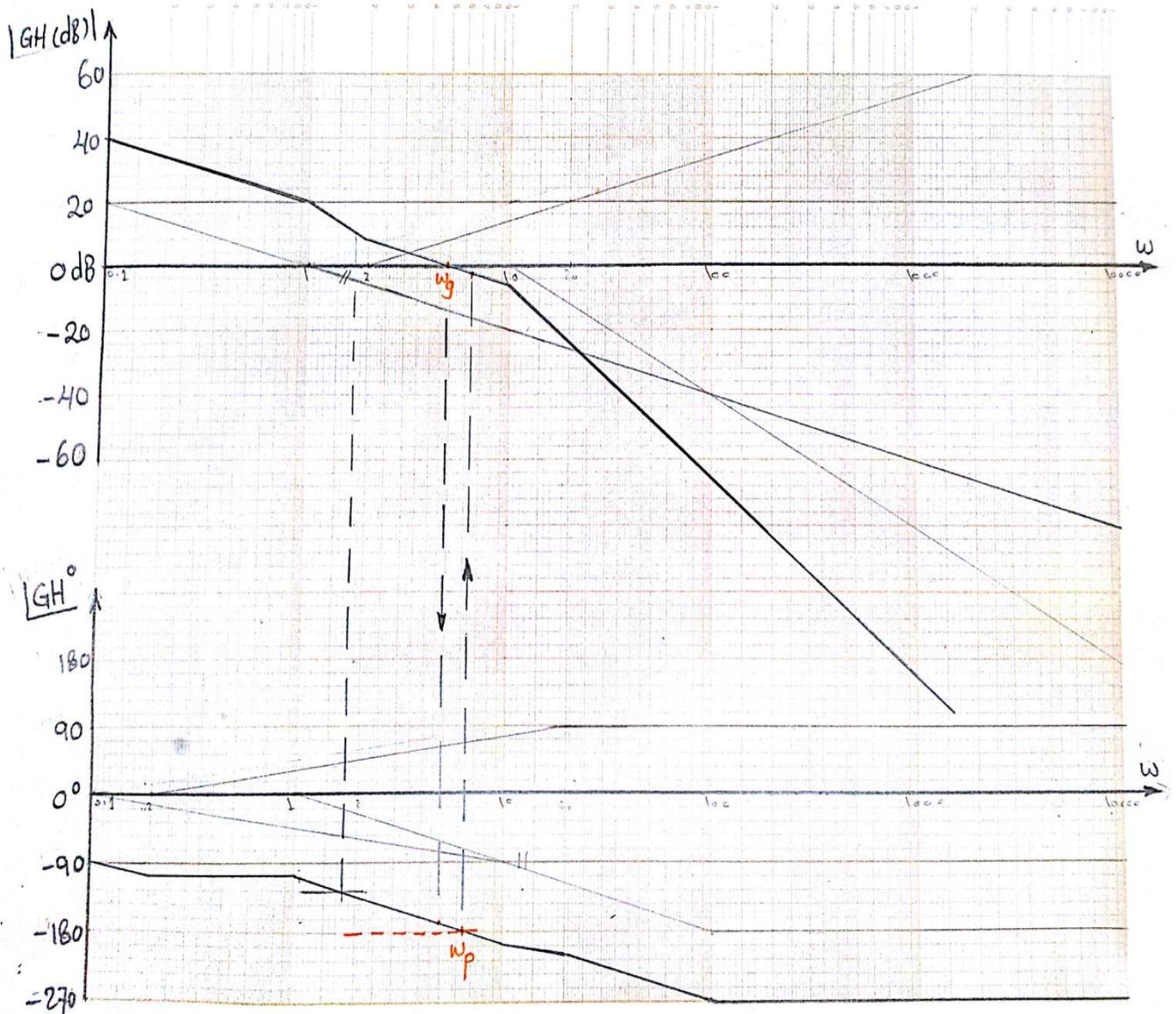
$$K = 0.1$$

At this value of K, Gain Margin= 22 dB and Phase Margin = 72°

e) To change the phase margin to 54°, this mean the gain crossover frequency must be changed to 1.8 rad/s, and the total magnitude at this point must be zero dB. But from Bode plot, it is found that the total magnitude is +10 dB, so it is recommended to add -10 dB at that point

$$20 \log K = -10$$

K = 0.316, At this value of K, Gain Margin= 12 dB





Example (5)

A system for controlling the performance of an aircraft in the lateral mode is equivalent to a single loop with open loop transfer function:

$$G(S)H(S) = \frac{250K(S + 20)}{S(S + 5)(S + 10)^2}$$

- a) Draw Bode plots at gain $K = 1$ and discuss the system stability.
- b) Find the value of K to get phase margin of 45°
- c) Find the value of K to get gain margin of 20 dB

$$G(S)H(S) = \frac{250K(S + 20)}{S(S + 5)(S + 10)^2}$$

$$G(j\omega)H(j\omega) = \frac{250 \times 20K(1 + \frac{j\omega}{20})}{5 \times 100 j\omega (1 + \frac{j\omega}{5}) (1 + \frac{j\omega}{10})^2}$$

$$G(j\omega)H(j\omega) = \frac{10K(1 + \frac{j\omega}{20})}{j\omega (1 + \frac{j\omega}{5}) (1 + \frac{j\omega}{10})^2}$$

At $K = 1$,

For the Bode gain 10, it can be represented as $20 \log (10) = 20$ dB

Pole at origin: the magnitude is represented by a straight line with slope of -20 dB/decade and intersect the 0dB line at $\omega=1$ and extended to intersect the vertical axis. But the phase is represented by straight line parallel to horizontal axis with constant value at -90° .

Real zero (S+20): the magnitude is represented by a straight line with slope of +20 dB/decade and intersect the 0dB line at $\omega=20$. The phase is represented as:

$$\varphi = \begin{cases} 0 & \text{at } \omega < 20 \\ +45 & \text{at } \omega = 20 \\ +90 & \text{at } \omega > 200 \end{cases}$$

Real Pole (S+5): the magnitude is represented by a straight line with slope of -20 dB/decade and intersect the 0dB line at $\omega=5$. The phase is represented as:

$$\varphi = \begin{cases} 0 & \text{at } \omega < 0.5 \\ -45 & \text{at } \omega = 5 \\ -90 & \text{at } \omega > 50 \end{cases}$$

Repeated Real Pole (S+10)²: the magnitude is represented by a straight line with slope of -40 dB/decade and intersect the 0dB line at $\omega=10$. The phase is represented as:



$$\varphi = \begin{cases} 0 & \text{at } \omega < 1 \\ -90 & \text{at } \omega = 10 \\ -180 & \text{at } \omega > 100 \end{cases}$$

From Bode plot shown on semi-log paper, we find that

- a) $\omega_1 = 6.6 \text{ rad/s} \rightarrow$ Phase margin = -13°
 $\omega_c = 4.7 \text{ rad/s} \rightarrow$ Gain margin = -5 dB , this mean the system is unstable ##

- b) At Phase margin = 45° , the system magnitude is $+20 \text{ dB}$
 Then $20 \log(K) = -20 \rightarrow K = 0.1$ ##

- c) To get Gain margin of 20 dB ,
 $20 \log(K) = -(20+5)$ $K = 0.05623$ ##

7. Closed-Loop Frequency Response

Consider the second-order system given in Fig. 13, whose open-loop T.F. is given as:

$$G(s)H(s) = \frac{\omega_n^2}{S(S + 2\zeta\omega_n)}$$

Also, the closed-loop T.F. is given as:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

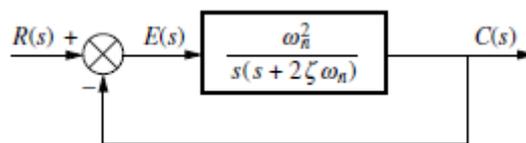


Fig. 13, Second-order system

To obtain the closed-loop frequency response, replace each S in the closed-loop T.F. by $j\omega$ as follows:

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega}$$

The magnitude $[M(j\omega)]$ of the above closed-loop T.F. is:

$$|M(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

The magnitude plot obtained from the representation of $M(j\omega)$ with respect to ω is shown in Fig. 14.

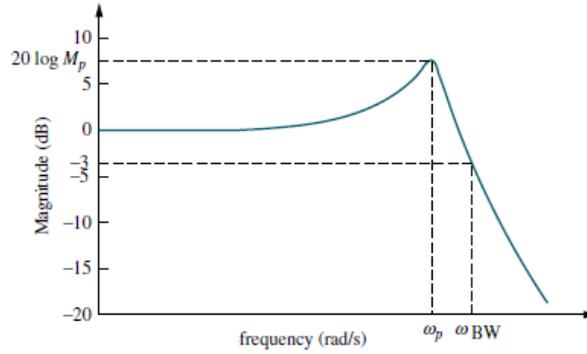


Fig. 14, Magnitude plot of closed-loop T.F.

From the above figure, there is a peak value which is $20 \times \log(M_p)$ in dB or M_p in actual scale. To obtain this peak value (M_p), we get the derivative $dM(j\omega)/d\omega$ and equating it to zero as follows:

$$\frac{dM(j\omega)}{d\omega} = \frac{-0.5\omega_n^2(4\omega^3 - 4\omega_n^2\omega + 8\zeta^2\omega_n^2\omega)}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2} = 0$$

The frequency at which peak value is occurred is called peak frequency (ω_p)

$$4\omega_p^3 - 4\omega_n^2\omega_p + 8\zeta^2\omega_n^2\omega_p = 0$$

$$\omega_p(\omega_p^2 + 2\zeta^2\omega_n^2 - \omega_n^2) = 0$$

$$\omega_p^2 = \omega_n^2(1 - 2\zeta^2) \quad \rightarrow \quad \omega_p = \omega_n\sqrt{1 - 2\zeta^2}$$

To get the peak magnitude M_p , substitute with the value of ω_p in the equation

$$M_p = |M(j\omega)|_{\omega=\omega_p} = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega_n^2(1 - 2\zeta^2))^2 + 4\zeta^2\omega_n^2\omega_n^2(1 - 2\zeta^2)}}$$

$$M_p = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

The effect of changing damping ratio on M_p and ω_p is shown in Fig. 15.

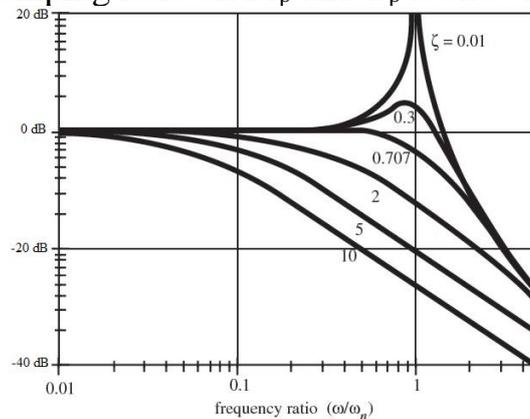


Fig. 15, Peak amplitude at different damping ratios



The Band Width frequency (ω_{BW}), is frequency at which the magnitude response curve is 3 dB down (on log scale) (or $1/\sqrt{2}=0.7071$ on normal scale) from its value at zero frequency.

$$\frac{1}{\sqrt{2}} \times |M(j\omega)|_{\omega=0} = \frac{1}{\sqrt{2}} \times 1 = 0.7071 = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega_{BW}^2)^2 + 4\zeta^2 \omega_n^2 \omega_{BW}^2}}$$

$$\omega_{BW} = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

Example (6)

Find the closed-loop bandwidth required for 20% overshoot and 2-seconds settling time.

The maximum overshoot is 0.2, and as we know it is given by:

$$0.2 = e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \rightarrow \zeta = 0.45595$$

The settling time (based on $\pm 2\%$ tolerance) is given as:

$$T_s = 2 = \frac{4}{\zeta \omega_n} \rightarrow \omega_n = 4.38645 \text{ rad/s}$$

The BW frequency is calculated from:

$$\omega_{BW} = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\omega_{BW} = 4.38645 \sqrt{1 - 2 \times 0.20789 + \sqrt{4 \times 0.04322 - 4 \times 0.20789 + 2}} = 5.79 \text{ rad/s}$$

Example (7)

For the closed-loop T.F given below,

$$\frac{C(s)}{R(s)} = \frac{5}{s^2 + 2s + 5}$$

Calculate the peak amplitude (M_p), frequency (ω_p) at which M_p is occurred and the band width (BW) of the closed-loop frequency response. Then give a free-hand sketch for the closed-loop frequency response.

Since the system is 2nd order system, we can use the equations given above. But, first we need to calculate ζ and ω_n . By comparing the characteristic eqn. of the given system with the standard one:



$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$

We obtain that:

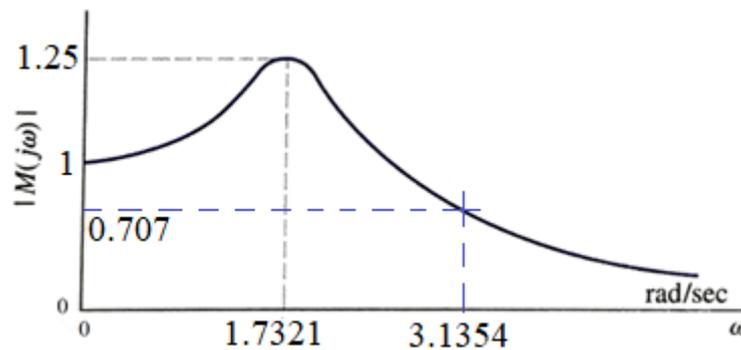
$$\omega_n^2 = 5 \rightarrow \omega_n = \sqrt{5}$$

$$2\zeta\omega_n = 2 \rightarrow \zeta = 1/\sqrt{5}$$

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2} = \sqrt{5} \sqrt{1 - 2 \times 0.2} = \sqrt{3} = 1.7321$$

$$M_p = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} = \frac{1}{\frac{2}{\sqrt{5}}\sqrt{1 - 0.2}} = \frac{1}{0.8} = 1.25$$

$$\omega_{BW} = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = \sqrt{5} \sqrt{0.8 + \sqrt{1.36}} = 3.1354$$

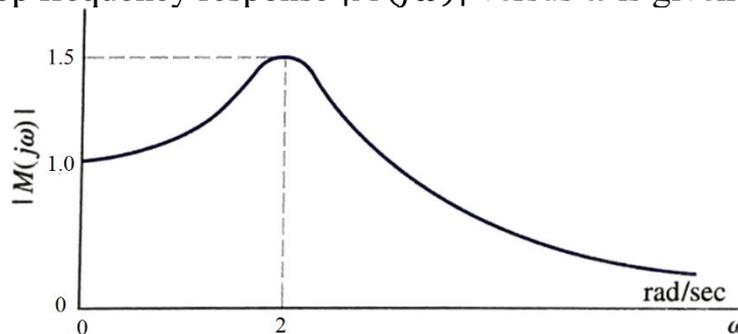


Example (8)

The open-loop T.F of a unity-feedback control system is

$$G(s) = \frac{K}{S(S + a)}$$

And the closed-loop frequency response $|M(j\omega)|$ versus ω is given in figure below.



Find the value of K and a, then calculate the band width (BW)

The closed-loop T.F. is:

$$\frac{C(s)}{R(s)} = \frac{K}{S^2 + aS + K}$$

By comparing the characteristic eqn. of the given system with the standard one:

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$



We obtain that:

$$\begin{aligned}\omega_n^2 &= K \\ 2\zeta\omega_n &= a\end{aligned}$$

From the closed-loop frequency response, $M_p = 1.5$ and $\omega_p = 2$ rad/s

$$\begin{aligned}M_p = 1.5 &= \frac{1}{2\zeta\sqrt{1-\zeta^2}} \\ 3\zeta\sqrt{1-\zeta^2} &= 1 \\ 9\zeta^2(1-\zeta^2) &= 1 \\ 9\zeta^4 - 9\zeta^2 + 1 &= 0\end{aligned}$$

Let $\zeta^2 = x$, therefore, $\zeta = \sqrt{x}$

$$9x^2 - 9x + 1 = 0$$

$$x = 0.87278, \rightarrow \zeta = 0.93423$$

$$x = 0.127322, \rightarrow \zeta = 0.3568$$

$$\omega_p = 2 = \omega_n\sqrt{1-2\zeta^2}$$

At $\zeta = 0.93423$,

$$\omega_n = \frac{2}{\sqrt{1-2\zeta^2}} = \frac{2}{\sqrt{1-2 \times 0.87278}} \text{ this gives imaginary value}$$

Therefore, $\zeta = 0.93423$ is rejected.

$$\omega_n = \frac{2}{\sqrt{1-2\zeta^2}} = \frac{2}{\sqrt{1-2 \times 0.127322}} = 2.3166 \text{ rad/s}$$

$\zeta = 0.3568$ and $\omega_n = 2.3166$

$$K = \omega_n^2 = 5.3665$$

$$a = 2\zeta\omega_n = 2 \times 0.3568 \times 2.3166 = 1.6531$$

$$\omega_{BW} = \omega_n\sqrt{1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 2.3166\sqrt{0.745356 + \sqrt{1.5556}} = 3.27 \text{ rad/s}$$

Example (9)

The open-loop T.F of a unity-feedback control system is,

$$G(s) = \frac{60}{S(S+2)(S+6)}$$

Calculate the peak amplitude (M_p), frequency at which M_p is occurred (ω_p) and the band width (BW) of the closed-loop frequency response.

The closed-loop T.F. is:

$$\frac{C(s)}{R(s)} = \frac{60}{S(S+2)(S+6) + 60} = \frac{60}{S^3 + 8S^2 + 12S + 60}$$



This is a 3rd order system, so we can't use the equations given as a function of ζ and ω_n

$$\frac{C(j\omega)}{R(j\omega)} = \frac{60}{(j\omega)^3 + 8(j\omega)^2 + 12(j\omega) + 60} = \frac{60}{(60 - 8\omega^2) + j(12\omega - \omega^3)}$$

$$M(j\omega) = \left| \frac{C(j\omega)}{R(j\omega)} \right| = \frac{60}{\sqrt{(60 - 8\omega^2)^2 + (12\omega - \omega^3)^2}}$$

$$\frac{dM(j\omega)}{d\omega} = \frac{-60 \times 0.5(\dots)^{-0.5} \{256\omega^3 - 1920\omega + 288\omega + 6\omega^5 - 96\omega^3\}}{(60 - 8\omega^2)^2 + (12\omega - \omega^3)^2} = 0$$

$$256\omega^3 - 1920\omega + 288\omega + 6\omega^5 - 96\omega^3 = 0$$

$$6\omega^5 + 160\omega^3 - 1632\omega = 0$$

$$\omega^4 + 26.6667\omega^2 - 272 = 0$$

Let $\omega^2 = x$, $\rightarrow \omega = \sqrt{x}$

$$x^2 + 26.6667x^2 - 272 = 0$$

$x = -34.5413$ (rejected)

$x = 7.874632$ (accepted) $\rightarrow \omega_p = 2.8062$

$$\left| \frac{C(j\omega)}{R(j\omega)} \right|_{\omega_p} = \frac{60}{\sqrt{(60 - 8 \times 7.874632)^2 + 7.874632(12 - 7.874632)^2}} = 5.0175$$

8. Relation Between Open-& Closed-Loop Frequency Responses (Nichole Chart)

The Bode plot is generally constructed for an open loop transfer function of a system. In order to draw the Bode plot for a closed loop system, the transfer function has to be developed, and then factorized to its poles and zeros. This process is tedious and cannot be carried out without the aids of a powerful calculator or a computer.

Nichols, developed a simple process by which the unity feedback closed-loop frequency response of a system can be easily deduced from the open-loop transfer function. This approach is outlined as follows:

Consider a unity feedback control system given in Fig. 16.

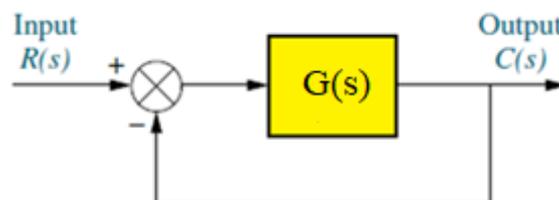


Fig. 16, Unity feedback control system

The closed-loop T.F. is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

The polar plot of $G(j\omega)$ in the GH plane is shown in Fig. 17. Where the vector OA represents the magnitude and angle of $G(j\omega)$ in G-plane.

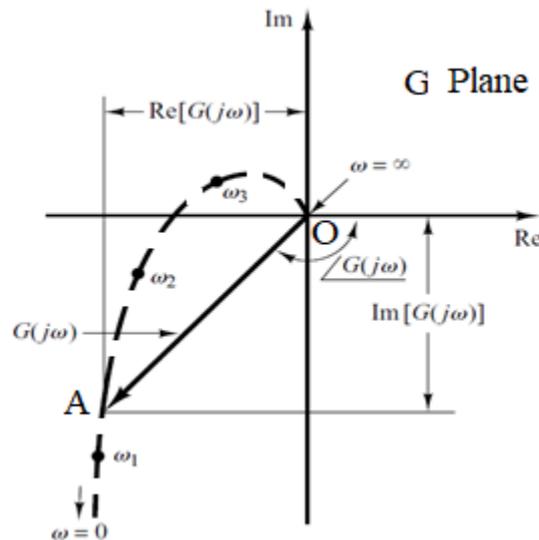


Fig. 17, Polar plot for $G(j\omega)$ represented in G-plane

Assuming that the real part of $G(j\omega)$ is $x(\omega)$ and the imaginary part of $G(j\omega)$ is $y(\omega)$.

Therefore, the closed-loop T.F. is expressed as:

$$\frac{C(\omega)}{R(\omega)} = \frac{x(\omega) + jy(\omega)}{1 + x(\omega) + jy(\omega)} = M \angle \phi$$

$$M = \frac{\sqrt{x^2 + y^2}}{\sqrt{(1 + x)^2 + y^2}}$$

$$M^2 = \frac{x^2 + y^2}{(1 + x)^2 + y^2}$$

$$M^2(1 + x)^2 + M^2y^2 = x^2 + y^2$$

Expanding and collecting similar terms:

$$(1 - M^2)x^2 - 2M^2x - M^2 + (1 - M^2)y^2 = 0$$

At $M = 1$; x (real part of G) = -0.5

At $M \neq 1$; the above equation is divided by $(1 - M^2)$ and is rewritten as:

$$x^2 + \frac{2M^2}{M^2 - 1}x + \frac{M^2}{M^2 - 1} + y^2 = 0$$



Completing square;

$$\left(x + \frac{M^2}{M^2 - 1}\right)^2 + y^2 = \frac{M^2}{M^2 - 1}$$

This is an equation of a circle with radius

$$\frac{M}{M^2 - 1}$$

And its center at

$$x = -\frac{M^2}{M^2 - 1}, \quad y = 0$$

The representation of the circle equation given above at different values of M is given in Fig. 18. Thus, if the polar frequency response of an open-loop function, G(s), is plotted and superimposed on top of the constant M circles, the closed-loop magnitude frequency response is determined by each intersection of this polar plot with the constant M circles.

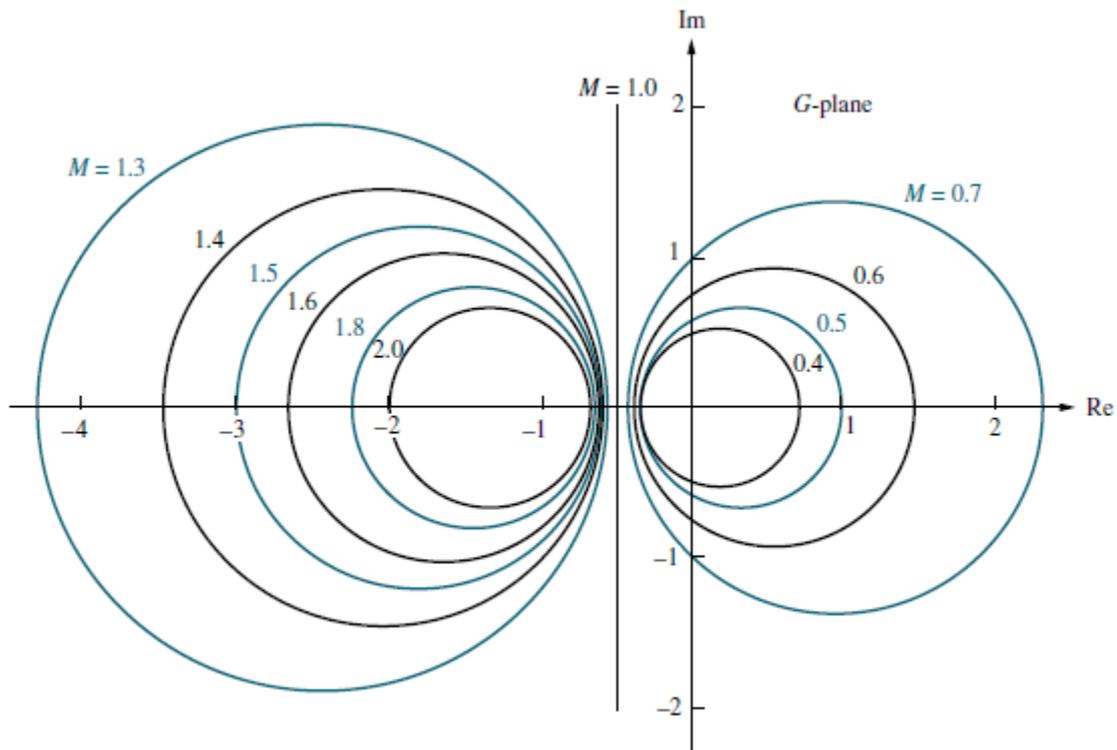


Fig. 18, Constant M circles

The phase angle of the closed-loop T.F is given by:

$$\phi = \tan^{-1} \frac{y}{x} - \tan^{-1} \frac{y}{x + 1}$$



Taking tan for both sides:

$$\tan(\phi) = N = \frac{\frac{y}{x} - \frac{y}{x+1}}{1 + \frac{y}{x} \cdot \frac{y}{x+1}} = \frac{y(x+1) - y(x)}{x(x+1) + y^2} = \frac{y}{x^2 + x + y^2}$$

$$x^2 + x + y^2 - \frac{1}{N}y = 0$$

Completing squares:

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2N}\right)^2$$

This is an equation of a circle with radius

$$\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2N}\right)^2}$$

And its center at

$$x = \frac{1}{2}, \quad y = \frac{1}{2N}$$

The representation of the circle equation given above at different values of N is given in Fig. 19. Superimposing a unity feedback, open-loop frequency response over the constant N circles yields the closed-loop phase response of the system.

Both M and N circles are represented in one figure to get the magnitude and angle easily as shown in Fig. 20. The main disadvantage of using the M and N circles is that changes of gain in the open-loop transfer function, G(s), cannot be handled easily. For example, in the Bode plot, a gain change is handled by moving the Bode magnitude curve up or down an amount equal to the gain change in dB scale. Since the M and N circles are not dB plots, changes in gain require each point of G(j ω) to be multiplied in length by the increase or decrease in gain.

Another presentation of the M and N circles, called a Nichols chart, displays the constant M circles in dB, so that changes in gain are as simple to handle as in the Bode plot. A Nichols chart is shown in Fig. 21.

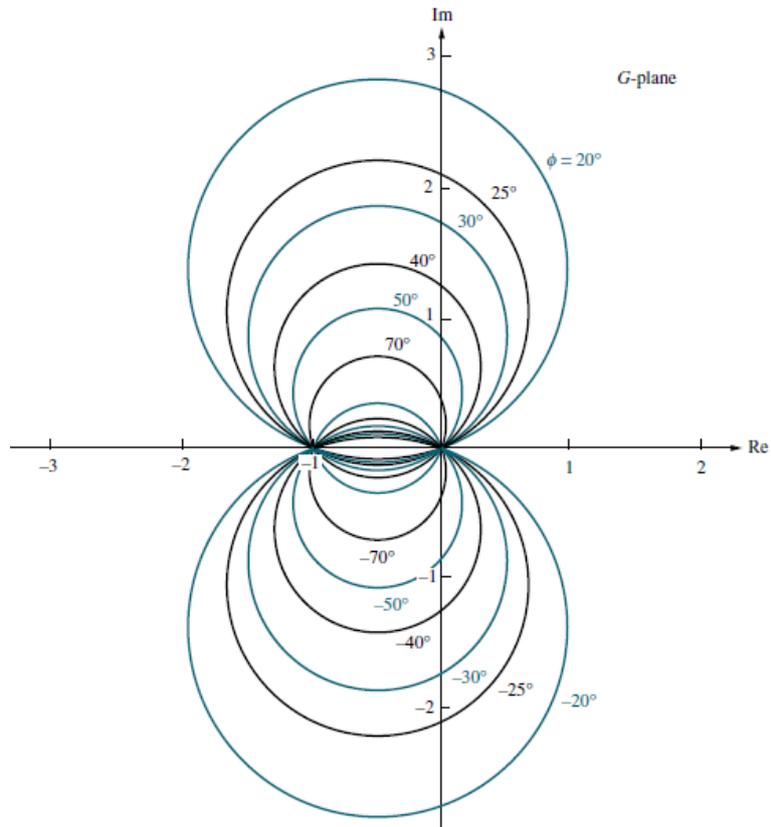


Fig. 19, Constant N circles

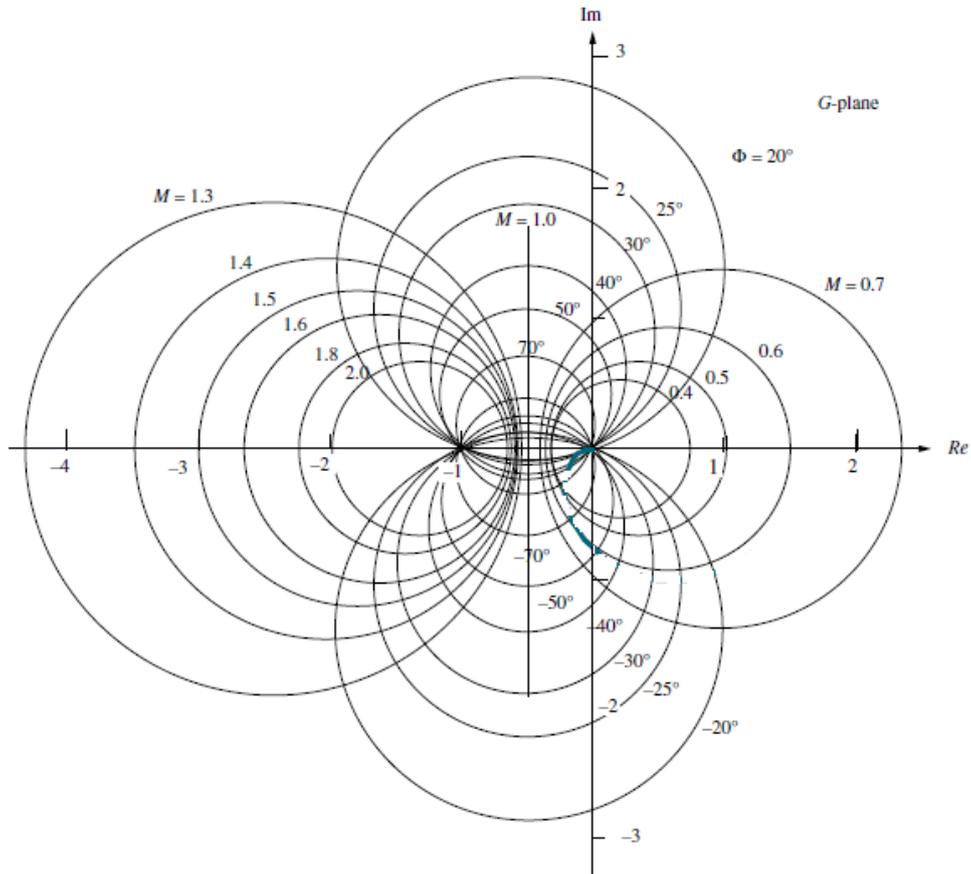


Fig. 20, Constant M and N circles

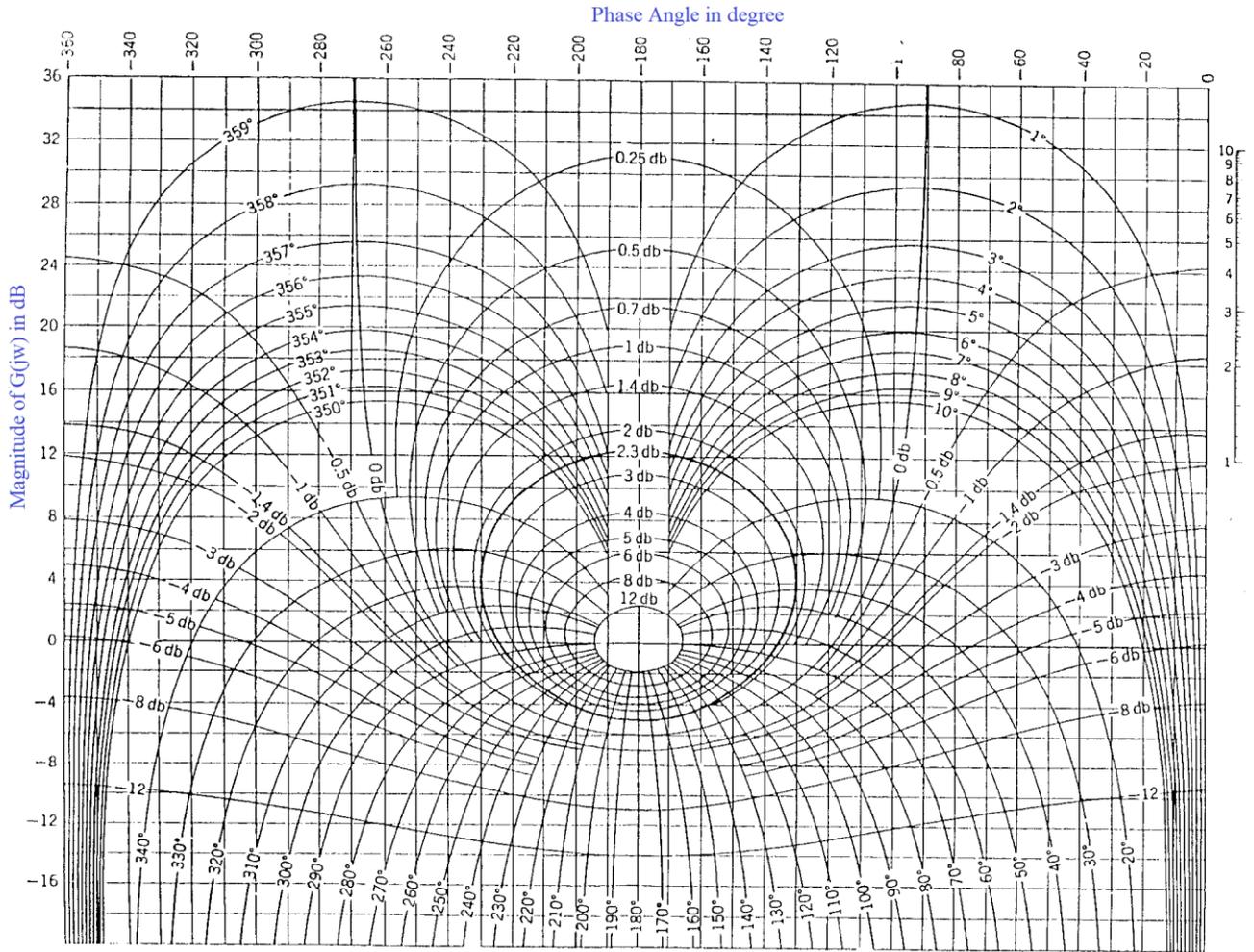


Fig. 21, Nichole Chart



Sheet 10 (Bode Plots)

(1) For the following transfer functions, sketch the Bode plots:

$G(s) = \frac{1}{(s+1)(s+10)}$	$G(s) = \frac{(s+10)}{(s+1)(s+20)}$
$G(s) = \frac{1}{(s^2+2s+50)}$	$G(s) = \frac{10(s+10)}{s(s+1)(s+20)}$
$G(s) = \frac{(s-5)}{(s+1)(s^2+12s+50)}$	$G(s) = \frac{1(s+5)}{s^2(s+1)(s+10)}$

(2) For each of the following transfer functions, sketch the Bode diagram and determine the gain crossover frequency (that is, the frequency at which the magnitude of $G(j\omega) = 20 \log |G(j\omega)| = 0$ dB, and the phase crossover frequency (that is, the frequency at which the phase angle of $G(j\omega) = -180^\circ$):

$G(s) = \frac{1000}{(s+10)(s+2)}$	$G(s) = \frac{100}{(s+0.1)(s^2+s+20)}$
$G(s) = \frac{50(s+10)}{(s+1)(s+100)}$	$G(s) = \frac{1000(s^2+14s+50)}{(s+5)(s+80)(s+500)}$

(3) Consider the non-unity feedback system in Figure (1), where the controller gain is $K=2$. Sketch the Bode plot of the open loop transfer function. Determine the phase angle of the open loop transfer function when the magnitude equals to 0 dB.

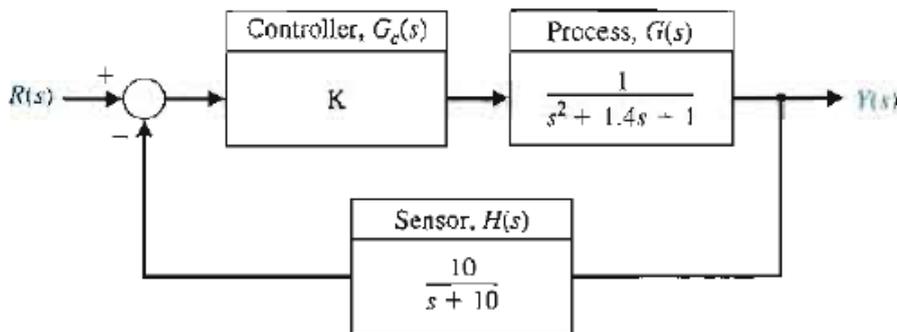


Figure (1) Non-unity feedback system with controller gain K

(4) Consider the system given in Figure (2) where

- $G_c(s) = \frac{K(s+5)}{(s+10)}$
- $G_p(s) = \frac{1}{s(s^2 + 2s + 2)}$
- $H(s) = 1$

- Find K such that the velocity error coefficient $K_v=10$
- Draw the Bode plot of the open-loop system
- From the Bode plot, find the frequency corresponding to 0 dB gain
- From the Bode plot, find the frequency corresponding to -180° phase

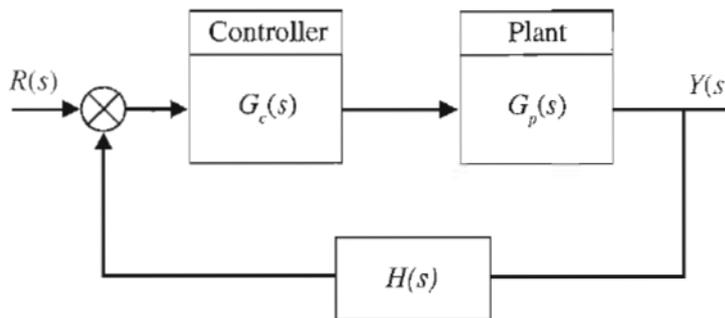


Figure (2) A closed-loop control system

- The asymptotic log-magnitude curves for two transfer functions are given in Figure (3). Sketch the corresponding asymptotic phase angle curves for each system. Determine the transfer function for each system.

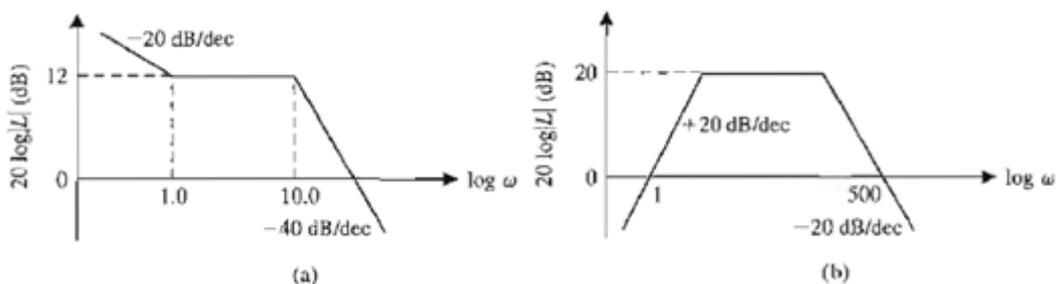


Figure (3) Log-magnitude curves

- A position control system may be constructed by using an AC motor and AC components, as shown in Figure (4-a). To measure the open-loop frequency response, we simply disconnect X from Y and X' from Y' and then apply a sinusoidal modulation signal generator to the $Y - Y'$ terminals and measure the response at $X - X'$. The resulting frequency response of the loop transfer function $L(j\omega) = G_c(j\omega)G(j\omega)H(j\omega)$, is shown in Figure (4-b). Determine the transfer function $L(j\omega)$.

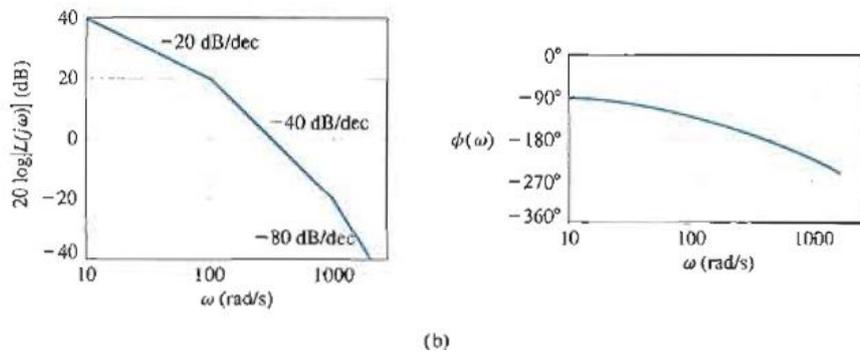
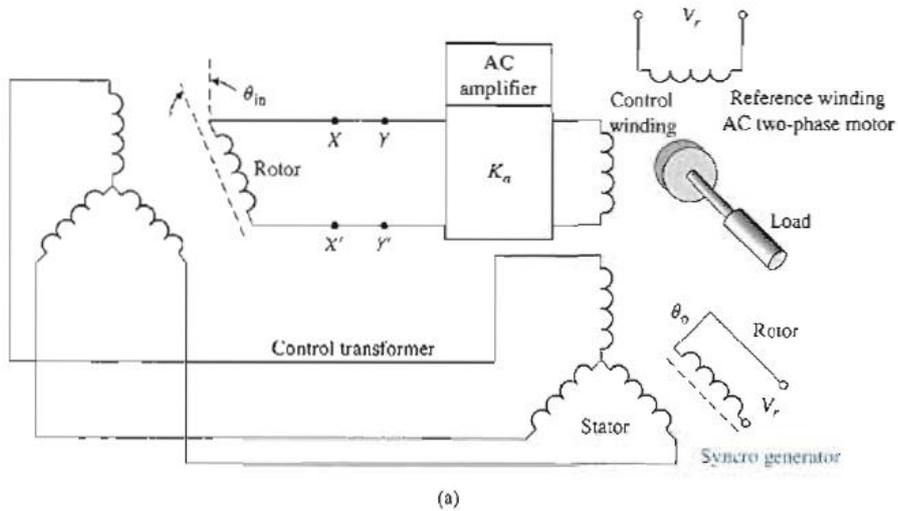


Figure (4) (a) AC motor control, (b) Frequency response

(7) A helicopter with a load on the end of a cable is shown in Figure (5-a). The position control system is shown in Figure (5-b), where the visual feedback is represented by $H(s)$. Sketch the Bode diagram of $G(j\omega)H(j\omega)$.

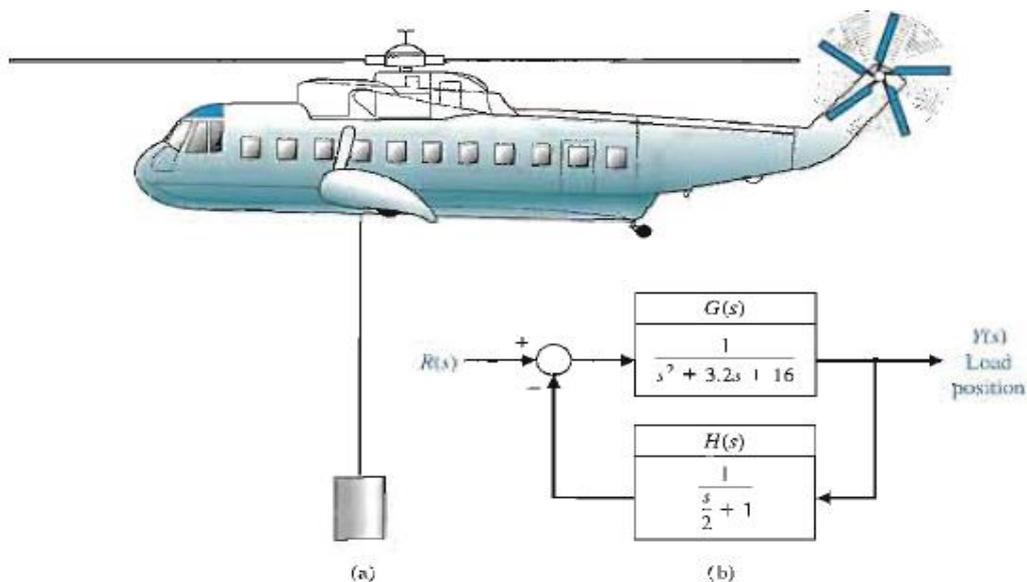


Figure (5) A helicopter feedback control system



(8) Sketch Bode plot of a system with transfer function

$$G(s) = \frac{K(1 + 0.1s)}{s(1 + 2s)(s^2 + 2s + 2)}$$

- (a) $K=10$
- (b) $K=20$
- (c) Compare plots obtained in (a) and (b)

(9) A system has the loop transfer function

$$G(s) = \frac{2.5(1 + s/5)}{s(1 + 2s)(1 + s/7 + s^2/49)}$$

Plot the Bode diagram. Show that the phase margin is approximately 28° and that the gain margin is approximately 21 dB.

(10) Consider a unity feedback system with the loop transfer function

$$G(s)H(s) = \frac{10(1 + 0.4s)}{s(1 + 2s)(1 + 0.24s + 0.04s^2)}$$

- (a) Plot the Bode diagram,
- (b) Find the gain margin and phase margin.

(11) The experimental Oblique Wing Aircraft (OWA) has a wing that pivots, as shown in Figure (6). The wing is in the normal unskewed position for low speeds and can move to a skewed position for improved supersonic flight. The aircraft control system loop transfer function is

$$G(s)G_c(s) = \frac{K(1 + 0.5s)}{s(1 + 2s)[1 + (s/20) + (s/8)^2]}$$

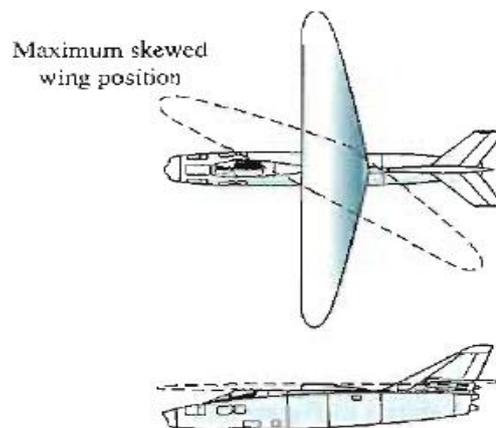


Figure (6) The Oblique Wing Aircraft, top and side

- (a) Sketch the Bode diagram when $K=4$,
 (b) Find the gain margin and phase margin,
 (c) Find the value of K for critical stable system.

(12) Consider a unity feedback system with the following open-loop transfer function:

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

- (a) For $K=4$, show that the gain margin is 3.5 dB,
 (b) If we wish to achieve a gain margin equal to 16 dB, determine the value of the gain K .

(13) A closed-loop system, as shown in Figure (7) has $H(s)=1$ and

$$G(s)G_c(s) = \frac{K}{s(1+0.02s)(1+0.02s)}$$

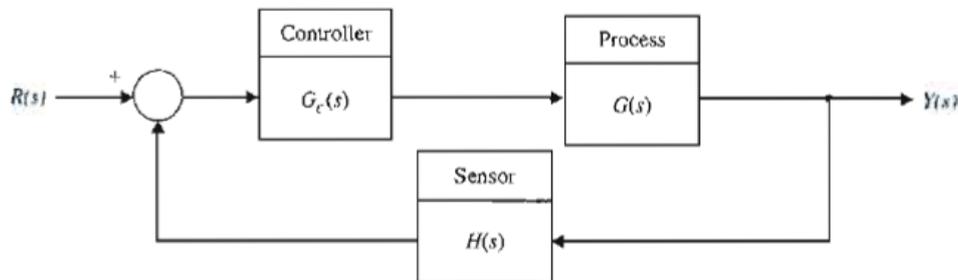


Figure (7) Feedback control system

- (a) Select a gain K so that the steady-state error for a ramp input is 10% of the magnitude of the ramp function A , where $r(t) = At, t > 0$,
 (b) Plot the Bode plot of $G_c(s)G(s)$, and determine the phase and gain margins.

(14) An electric carrier that automatically follows a tape track laid out on a factory floor. Closed-loop feedback systems are used to control the guidance and speed of the vehicle. The block diagram of the steering system is shown in Figure (8). Select a gain K so that the phase margin is approximately 30° .

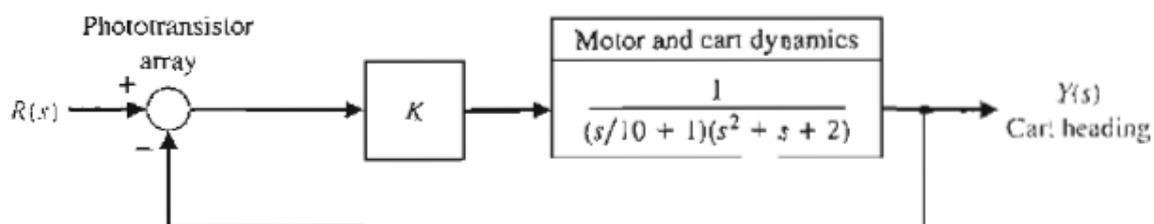


Figure (8) Feedback steering control system

(15) Consider the system shown in Figure (9).

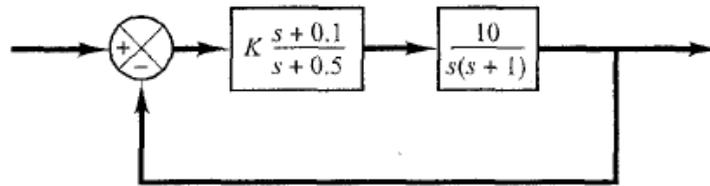


Figure (9) Feedback control system

- Draw a Bode diagram of the open-loop transfer function,
- Determine the value of the gain K such that the phase margin is 50° ,
- What is the gain margin of this system with this gain K ?

References:

- [1] Bosch, R. GmbH. *Automotive Electrics and Automotive Electronics*, 5th ed. John Wiley & Sons Ltd., UK, 2007.
- [2] Franklin, G. F., Powell, J. D., and Emami-Naeini, A. *Feedback Control of Dynamic Systems*. Addison-Wesley, Reading, MA, 1986.
- [3] Dorf, R. C. *Modern Control Systems*, 5th ed. Addison-Wesley, Reading, MA, 1989.
- [4] Nise, N. S. *Control System Engineering*, 6th ed. John Wiley & Sons Ltd., UK, 2011.
- [5] Ogata, K. *Modern Control Engineering*, 5th ed ed. Prentice Hall, Upper Saddle River, NJ, 2010.
- [6] Kuo, B. C. *Automatic Control Systems*, 5th ed. Prentice Hall, Upper Saddle River, NJ, 1987.