CHAPTER # 4 SIGNAL FLOW GRAPH (SFG)

After completing this chapter, the students will be able to:

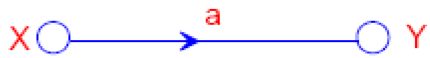
- Convert block diagrams to signal-flow graph,
- Find the transfer function of multiple subsystems using Mason's rule,

1. Introduction

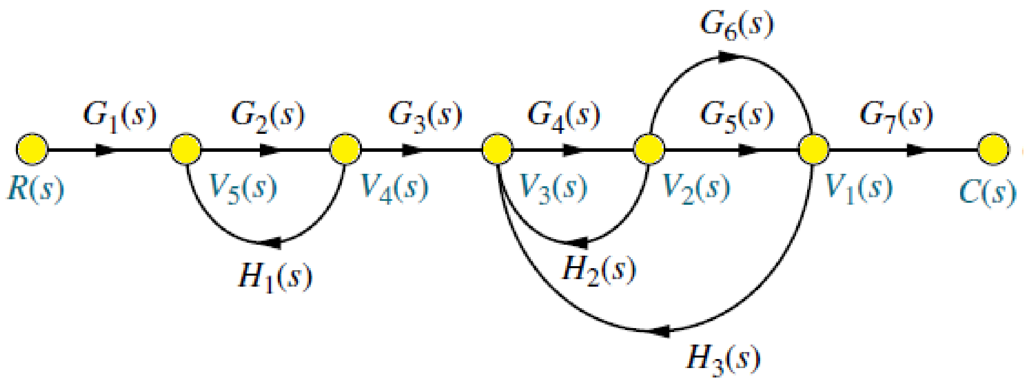
For complex control systems, the block diagram reduction technique is difficult. An alternative method for determining the relationship between system variables has been developed by ***Samuel Jefferson Mason*** **(*1953*)** and is based on a signal flow graph. The block diagram reduction technique requires successive application of fundamental relationships (Cascade, Parallel and/or Canonical) in order to arrive at the system transfer function. On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.

A signal flow graph is a diagram that consists of nodes that are connected by branches. A node is assigned to each variable of interest in the system, and branches are used to relate the different variables. The main advantage for using SFG is that a straight forward procedure is available for finding the transfer function in which it is not necessary to move pickoff point around or to redraw the system several times as with block diagram manipulations. Moreover, Mason's formula has several components that must be evaluated first.

SFG is a diagram that represents a set of simultaneous linear algebraic equations which describe a system. Let us consider an equation, $y = a\,x$. It may be represented graphically as,



2. Terminology



Node: A point that denoting a variable or a signal. (e.g. R(s), C(s), V_1(s), …)

Branch: A unidirectional path that joining two Nodes. Relation between variables is written next to the directional arrow. (e.g. G_1(s), G_2(s), H_1(s), …)

Forward Path: A continuous sequence of branches that can be traversed from input node to the output node without touching any node twice.

e.g. $G_1(s)\ G_2(s)\ G_3(s)\ G_4(s)\ G_5(s)\ G_7(s)$ & $G_1(s)\ G_2(s)\ G_3(s)\ G_4(s)\ G_6(s)\ G_7(s)$

Loop: A closed path that originates at one node and terminates at the same node. Along the loop, no node is touched twice.

e.g. $G_2(s)H_1(s)$, $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, $G_4(s)G_6(s)H_3(s)$

Non-Touching Loops: Loops with no common nodes and/or branches

e.g. $G_2(s)H_1(s)$ & $G_4(s)H_2(s)$, $G_2(s)H_1(s)$ & $G_4(s)G_5(s)H_3(s)$, $G_2(s)H_1(s)$ & $G_4(s)G_6(s)H_3(s)$

Input node (Source): node having only outgoing branches (e.g. R(s))

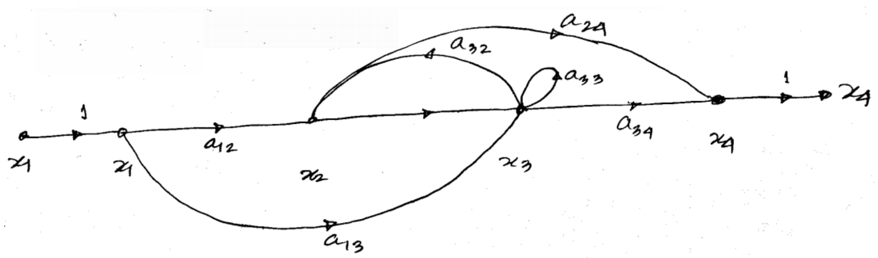
<u>Output node (Sink):</u> node having only incoming branches (e.g. C(s))

<u>Mixed node:</u> A node that has both incoming and outgoing branches. (e.g. V_2(s))

3. Construction of SFG from D.E.

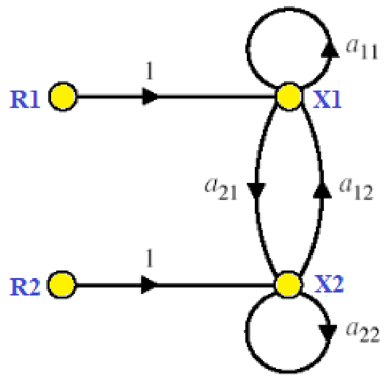
SFG of a single input system can be constructed from the describing equations:

$$x_2 = a_{12}x_1 + a_{32}x_3$$
$$x_3 = a_{13}x_1 + a_{23}x_2 + a_{33}x_3$$
$$x_4 = a_{24}x_4 + a_{34}x_3$$



SFG of a multi input system can be constructed from the describing equations:

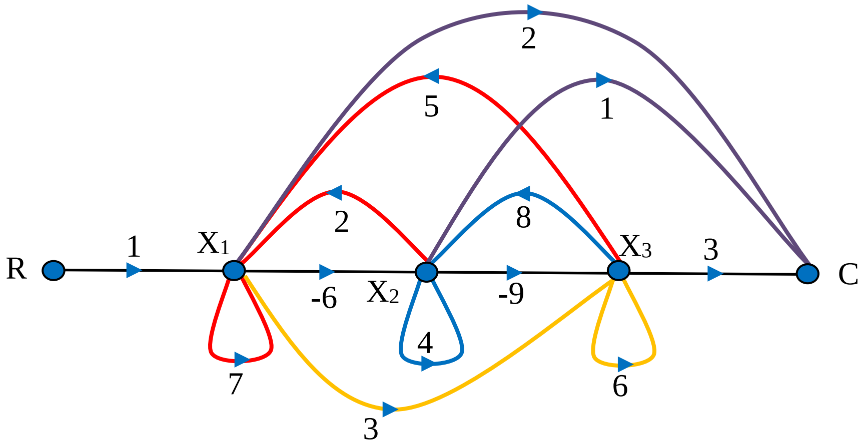
$$X_1 = a_{11}X_1 + a_{12}X_2 + R_1$$
$$X_2 = a_{21}X_1 + a_{22}X_2 + R_2$$



<u>Example (1):</u>

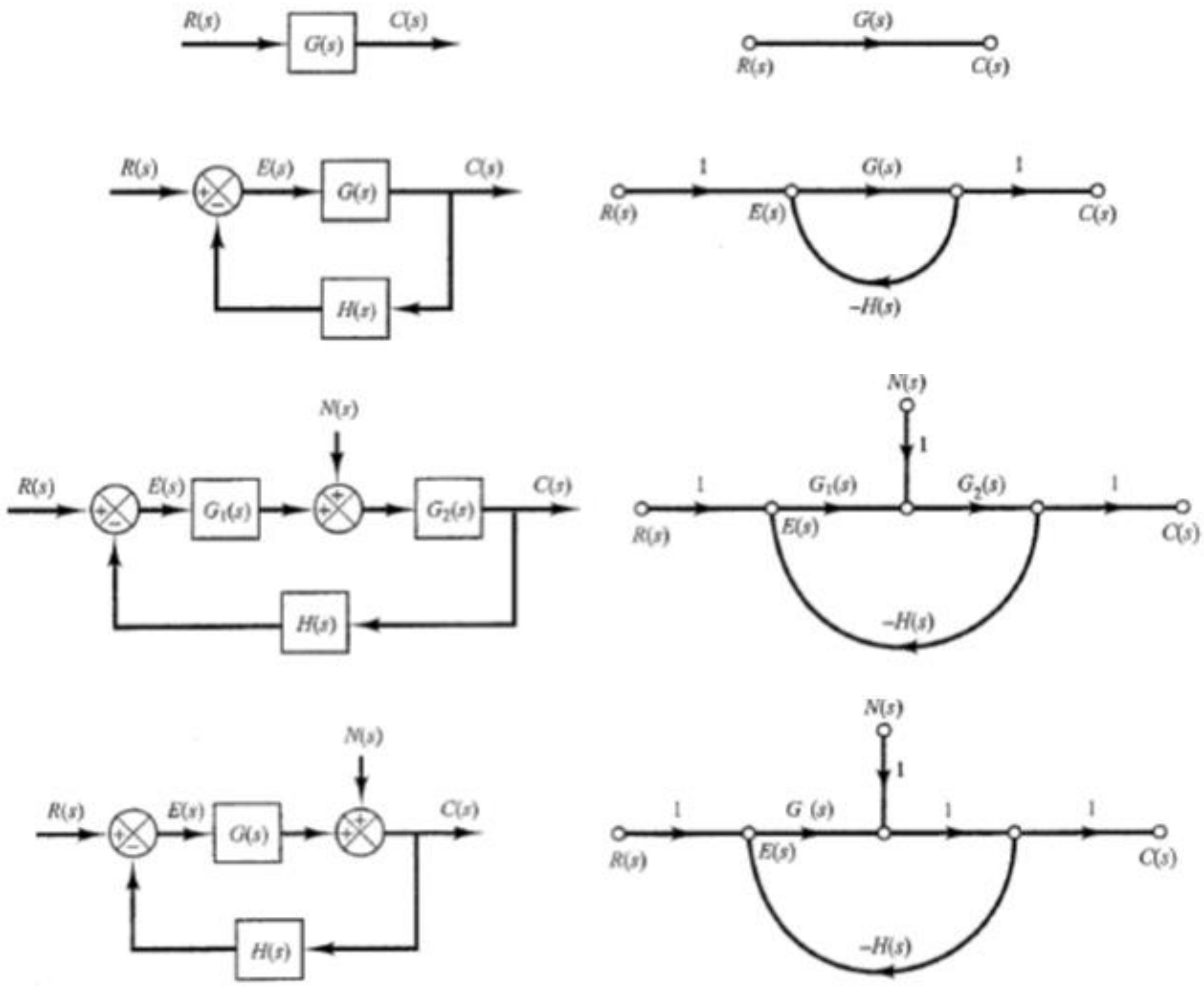
Construct SFG of the system described by the following equations; where R is input & C is output and x_1, x_2, and x_3 are the system nodes.

$$x_1 = R + 7x_1 + 2x_2 + 5x_3$$
$$x_2 = -6x_1 + 4x_2 + 8x_3$$
$$x_3 = 3x_1 - 9x_2 + 6x_3$$
$$C = 2x_1 + x_2 + 3x_3$$



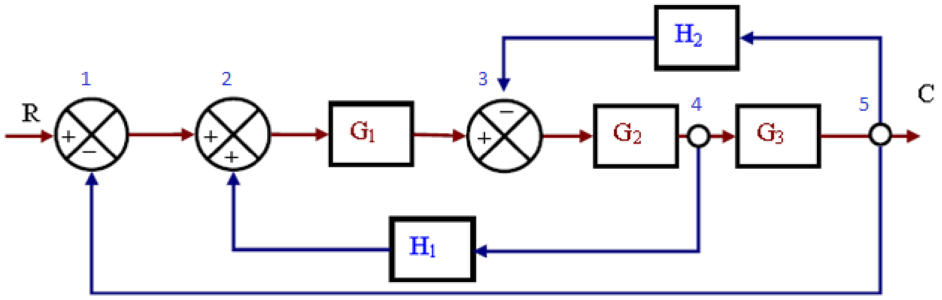
On the other hand, the signal flow graph can be given and the student is asked to obtain the system equations.

4. SFG from Block Diagram

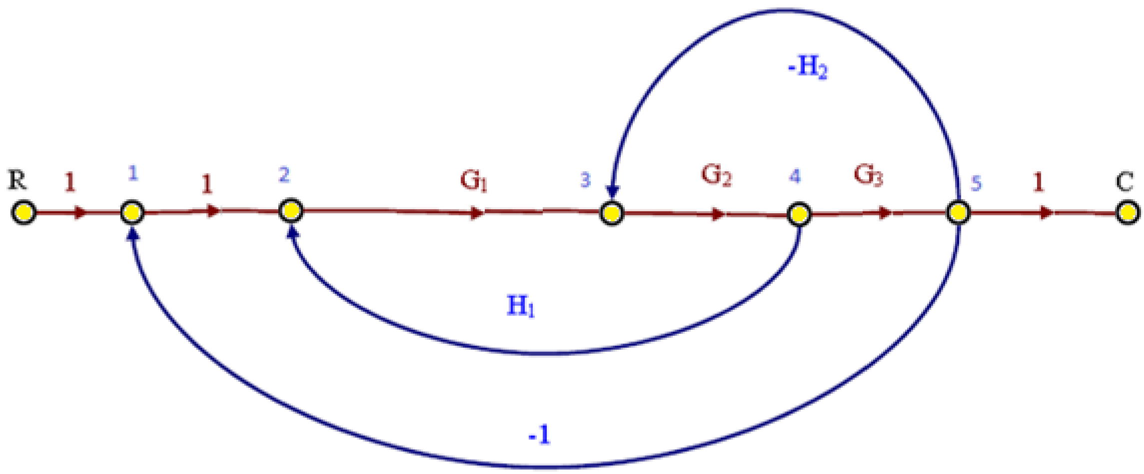


Example (2):

Draw the DFG from the block diagram given below.



Choose the nodes to represent the variables say 1, 2, .. 5 as shown in the block diagram above. Connect the nodes with appropriate gain along the branch. The signal flow graph is shown below.



5. Mason's Formula to Calculate Transfer Function

$$T.F = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$$

Where: N is the number of forward paths from input to output

P_k is the gain of the k^{th} path from input to output

Δ_k is the sub-determinant corresponds the k^{th} path from input to output

Δ is the main determinant of the control system

The main determinate (Δ) can be calculated as:

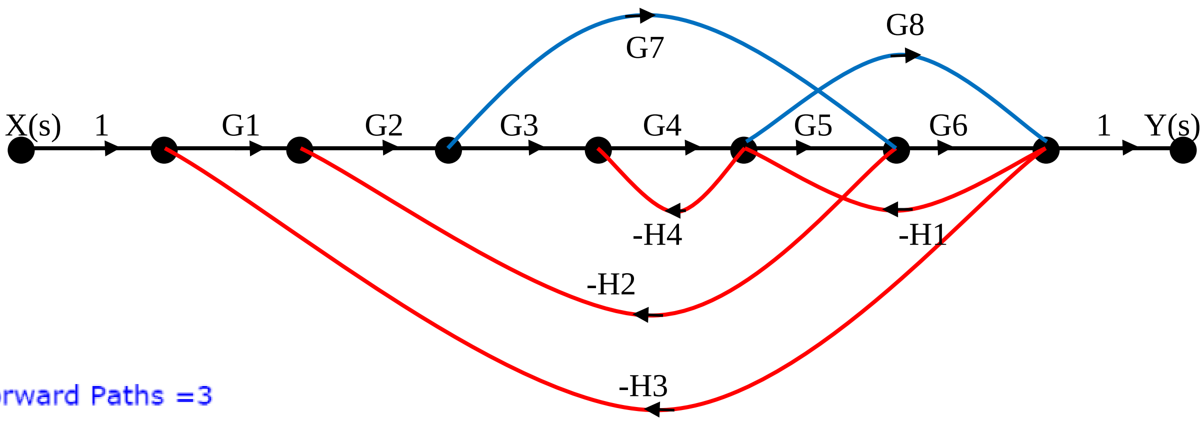
$$\Delta = 1 - \sum Gain\ of\ every\ loop + \sum Gain\ product\ of\ every\ 2\ non\ touching\ loops - \sum Gain\ product\ of\ every\ 3\ non\ touching\ loops + \sum Gain\ product\ of\ every\ 4\ non\ touching\ loops - \ ...etc$$

The sub-determinate (Δ_k) can be calculated as:

$$\Delta_k = 1 - \sum Gain\ of\ every\ loop\ doesn't\ touch\ the\ path\ P_k + \sum Gain\ product\ of\ every\ 2\ non\ touching\ loops\ doesn't\ touch\ the\ path\ P_k - \sum Gain\ product\ of\ every\ 3\ non\ touching\ loops\ doesn't\ touch\ the\ path\ P_k + \sum Gain\ product\ of\ every\ 4\ non\ touching\ loops\ doesn't\ touch\ the\ path\ P_k$$

Example (3):

Using Mason's formula, calculate the T.F. $Y(s)/X(s)$



Forward Paths =3

$P_1 = G_1G_2G_3G_4G_5G_6$; $P_2 = G_1G_2G_7G_6$ $P_3 = G_1G_2G_3G_4G_8$

Feedback loops

$L_1 = -G_2G_3G_4G_5H_2$; $L_2 = -G_5G_6H_1$; $L_3 = -G_8H_1$; $L_4 = -G_7H_2G_2$;
$L_5 = -G_4H_4$; $L_6 = -G_1G_2G_3G_4G_5G_6H_3$; $L_7 = -G_1G_2G_7G_6H_3$
$L_8 = -G_1G_2G_3G_4G_8H_3$

Loop L_5 does not touch loop L_4 or Loop L_7
Loop L_3 does not touch Loop L_4
All other loops touch

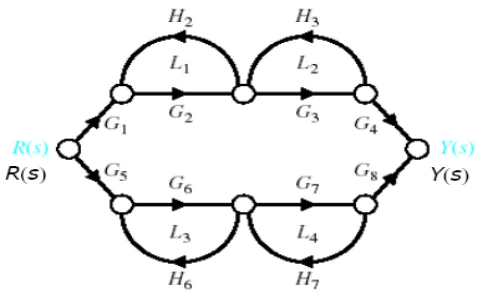
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$

$\Delta_1 = \Delta_3 = 1 ; \Delta_2 = 1 - L_5 = 1 + G_4H_4$

$$\frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$

Example (4):

Find the T.F. $Y(s)/X(s)$



Forward Paths =2

$P_1 = G_1G_2G_3G_4 \;\; ; \; P_2 = G_5G_6G_7G_8$

Feedback loops

$L_1 = G_2H_2 \;\; ; \; L_2 = G_3H_3 \;\; ; \;\; L_3 = G_6H_6 \; ; \; L_4 = G_7H_7 \; ;$

Loops L_1 and L_2 do not touch loop L_3 and L_4

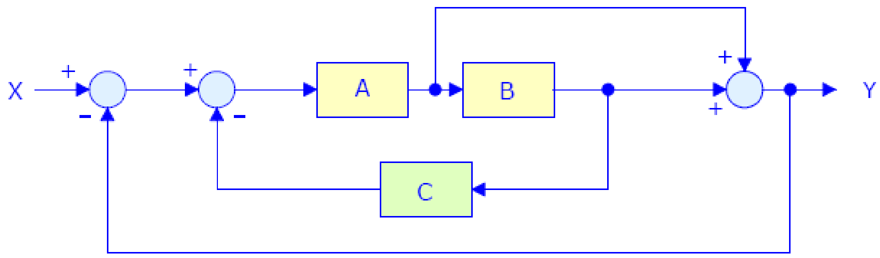
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$

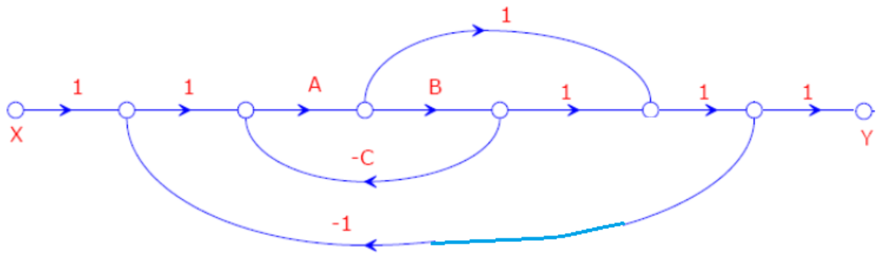
$\Delta_1 = 1 - (L_3 + L_4) \quad ; \quad \Delta_2 = 1 - (L_1 + L_2)$

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)}$$

Example (5):

Using Mason's Formula, Find the T.F. $Y(s)/X(s)$





$P_1 = AB$; $P_2 = A$

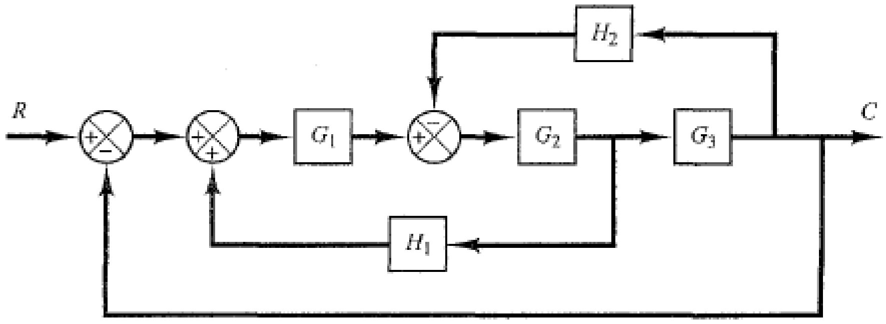
$$\Delta = 1 - (-ABC - AB - A)$$
$$\Delta = 1 + ABC + AB + A$$

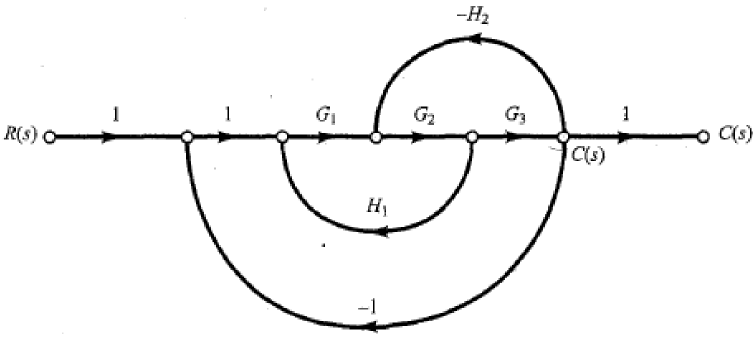
$\Delta_1 = 1$; $\Delta_2 = 1$

$$\frac{Y}{X} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{A(1+B)}{1+ABC+AB+A}$$

Example (6):

Using Mason's Formula, Find the T.F. *C*(*s*)/*R*(*s*)





In this system there is only one forward path between the input *R(s)* and the output *C(s)*. The forward path gain is

$$P_1 = G_1G_2G_3$$

we see that there are three individual loops. The gains of these loops are

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

Note that since all three loops have a common branch, there are no non-touching loops. Hence, the determinant **Δ** is given by

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

The cofactor **Δ** $_1$ of the determinant along the forward path connecting the input node and output node is obtained from **Δ** by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

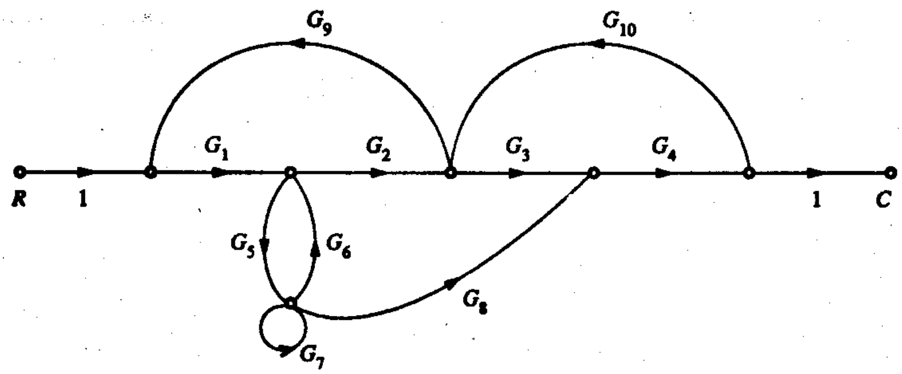
$$\Delta_1 = 1$$

Therefore, the overall gain between the input $R(s)$ and the output $C(s)$, or the closed-loop transfer function, is given by

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

Example (7):

Using Mason's Formula, Find the T.F. $C(s)/R(s)$

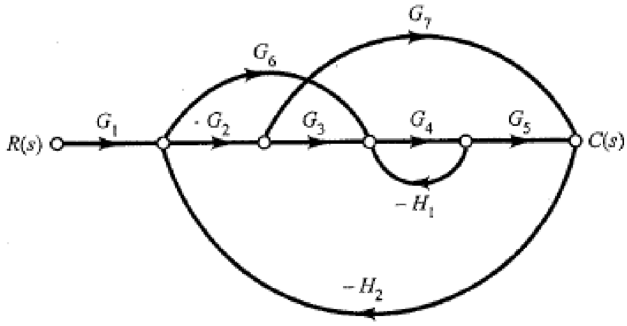


$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4(1 - G_7) + G_1G_5G_8G_4}{1 - [G_1G_2G_9 + G_3G_4G_{10} + G_1G_5G_8G_4G_{10}G_9 + G_5G_6 + G_7] + [G_1G_2G_9G_7 + G_3G_4G_{10}G_5G_6 + G_3G_4G_{10}G_7]}$$

Example (8):

Using Mason's Formula, Find the T.F. $C(s)/R(s)$



In this system, there are three forward paths between the input *R(s)* and the output *C(s)*. The forward path gains are

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

There are four individual loops, the gains of these loops are

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

Loop L_1 does not touch loop L_2; Hence, the determinant **Δ** is given by

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2$$

The cofactor Δ_1, is obtained from Δ by removing the loops that touch path *PI*. Therefore, by removing L_1, L_2, L_3, L_4, and L_1, L_2 from Δ equation, we obtain

$$\Delta_1 = \Delta_2 = 1$$

The cofactor $\mathbf{\Delta_3}$ is obtained by removing L_2, L_3, L_4, and L_1, L_2 from Δ Equation, giving

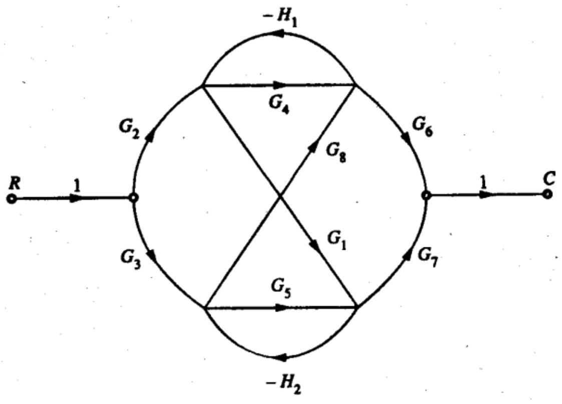
$$\Delta_3 = 1 - L_1$$

The closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_4H_1G_2G_7H_2}$$

Example (9):

Consider the control system whose signal flow graph is shown below. Determine the system transfer function using Mason's formula.



* There are **SIX** Forward Paths:

$P_1 = G_2 G_4 G_6$

$P_2 = G_3 G_5 G_7$

$P_3 = G_2 G_1 \cdot G_7$

$P_4 = G_3 G_8 G_6$

$P_5 = -G_2 G_1 \cdot H_2 G_8 \cdot G_6$

$P_6 = -G_3 G_8 H_1 G_1 G_7$

* There are **THREE** feedback loops:

$P_{11} = -H_1 G_4$

$P_{21} = -H_2 G_5$

$P_{31} = G_1 H_2 G_8 H_1$

* There are **ONE** combination of two-non-touching feedback loops:

$P_{12} = H_1 H_2 G_4 G_5$

$$\Delta = 1 - [-H_1 G_4 - H_2 G_5 + G_1 H_2 G_8 H_1] + [H_1 H_2 G_4 G_5]$$
$$= 1 - G_1 H_2 G_8 H_1 + H_2 G_5 - G_1 H_2 G_8 H_1 + H_1 H_2 G_4 G_5$$

$$\Delta_1 = 1 - (-H_2 G_5) = 1 + H_2 G_5$$

$$\Delta_2 = 1 - (-H_1 G_4) = 1 + H_1 G_4$$

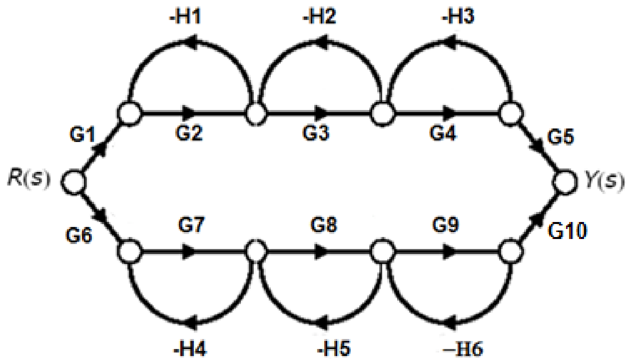
$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

Using Mason's Formula, the system Transfer Function is:

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Example (10):

For the signal flow graph of a certain control system shown below, find the system characteristic equation.



The characteristic equation obtained from mason's formula is Δ=0

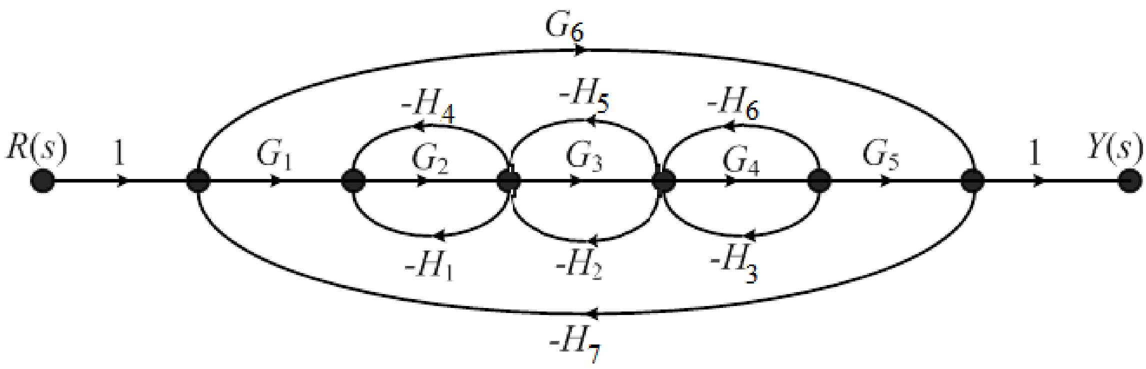
$\Delta = 1$ - ($\sum$ all different loop gains)
+ ($\sum$ gain products of all combinations of 2 non-touching loops)
- ($\sum$ gain products of all combinations of 3 non-touching loops)
+ ($\sum$ gain products of all combinations of 4 non-touching loops)
- .....

Loop Gains	Two non-touching Loops	Three non-touching Loops
	$L_1L_3 = G_2G_4H_1H_3$	
	$L_1L_4 = G_2G_7H_1H_4$	$L_1L_3L_4 = -G_2G_4G_7H_1H_3H_4$
	$L_1L_5 = G_2G_8H_1H_5$	$L_1L_3L_5 = -G_2G_4G_8H_1H_3H_5$
$L_1 = -G_2H_1$	$L_1L_6 = G_2G_9H_1H_6$	$L_1L_3L_6 = -G_2G_4G_9H_1H_3H_6$
$L_2 = -G_3H_2$	$L_2L_4 = G_3G_7H_2H_4$	$L_1L_4L_6 = -G_2G_7G_9H_1H_4H_6$
$L_3 = -G_4H_3$	$L_2L_5 = G_3G_8H_2H_5$	$L_2L_4L_6 = -G_3G_7G_9H_2H_4H_6$
$L_4 = -G_7H_4$	$L_2L_6 = G_3G_9H_2H_9$	$L_3L_4L_6 = -G_4G_7G_9H_3H_4H_6$
$L_5 = -G_8H_5$	$L_3L_4 = G_4G_7H_3H_4$	
$L_6 = -G_9H_6$	$L_3L_5 = G_4G_8H_3H_5$	Four non-touching Loops
	$L_3L_6 = G_4G_9H_3H_6$	$L_1L_3L_4L_6 = G_2G_4G_7G_9H_1H_3H_4H_6$
	$L_4L_6 = G_7G_9H_4H_6$	

$$\Delta = 1 - \{L_1 + L_2 + L_3 + L_4 + L_5 + L_6\}$$
$$+ \{L_1L_3 + L_1L_4 + L_1L_5 + L_1L_6 + +L_2L_4 + L_2L_5 + L_2L_6 + L_3L_4$$
$$+ L_3L_5 + L_3L_6 + L_4L_6\}$$
$$- \{L_1L_3L_4 + L_1L_3L_5 + L_1L_3L_6 + L_1L_4L_6 + L_2L_4L_6 + L_3L_4L_6\}$$
$$+ \{L_1L_3L_4L_6\} = 0$$

Example (11):

Consider the control system whose signal flow graph is shown below. Determine the system transfer function using Mason’s formula.



* There are **TWO** Forward Paths:

P1= G1G2G3G4G5

P2=G6

* There are **EIGHT** feedback loops:

L1= - G2H1	L2= - G3H2
L3= - G4H3	L4= - G2H4
L5= - G3H5	L6= - G4H6
L7= - G6H7	L8= - G1G2G3G4G5H7

* There are **TEN** two-non-touching feedback loops:

L1L3 = G2G4H1H3	L1L6 = G2G4H1H6
L1L7 = G2G6H1H7	L2L7 = G3G6H2H7
L3L4 = G2G4H3H4	L3L7 = G4G6H3H7
L4L6 = G2G4H4H6	L4L7 = G2G6H4H7
L5L7 = G3G6H5H7	L6L7 = G4G6H6H7

* There are **FOUR** three-non-touching feedback loops:

L1L3L7 = - G2G4G6 H1H3H7	L1L6L7 = - G2G4G6H1H6H7
L3L4L7 = - G2G4G6H3H4H7	L4L6L7 = - G2G4G6H4H6H7

Δ = 1 + {G2H1+G3H2+G4H3+G2H4+G3H5+G4H6+G6H7+G1G2G3G4G5H7}+{ G2G4H1H3+G2G4H1H6+G2G6H1H7+G3G6H2H7+G2G4H3H4+G4G6H3H7+ G2G4H4H6+G2G6H4H7+G3G6H5H7+G4G6H6H7}+{G2G4G6H1H3H7+ G2G4G6H1H6H7+G2G4G6H3H4H7+G2G4G6H4H6H7}

Δ1 = 1

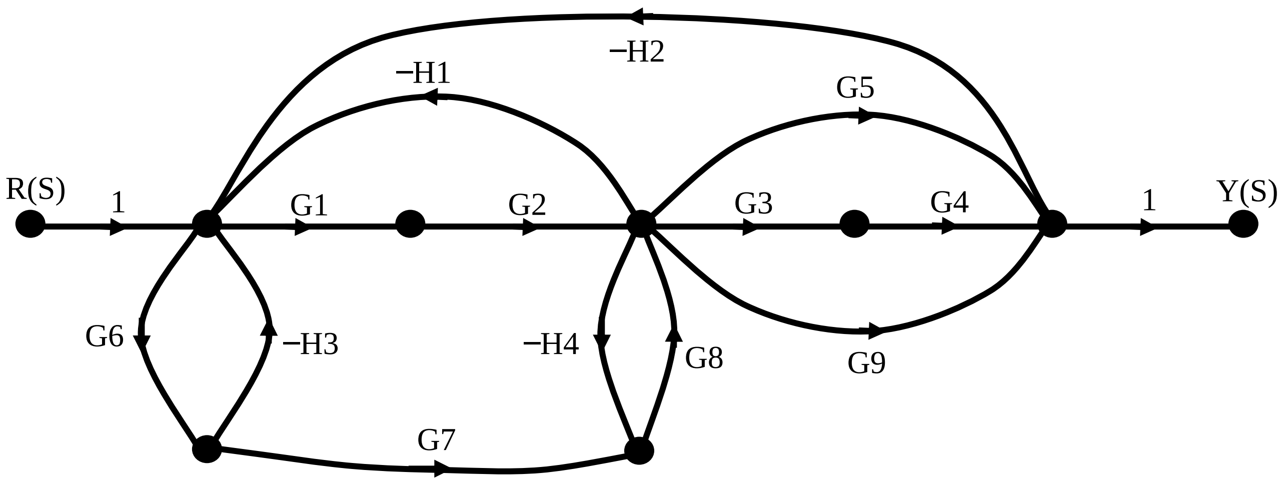
Δ2 = 1 + {G2H1+G3H2+G4H3+G2H4+G3H5+G4H6}+{ G2G4H1H3+G2G4H1H6 +G2G4H3H4+ G2G4H4H6}

Using Mason's Formula, the system Transfer Function is:

$$\frac{Y(S)}{R(S)} = \frac{G1G2G3G4G5 + G6\{1 + \{G2H1 + G3H2 + G4H3 + G2H4 + G3H5 + G4H6\} + \{G2G4H1H3 + G2G4H1H6 + G2G4H3H4 + G2G4H4H6\}\}}{1 + \{G2H1 + G3H2 + G4H3 + G2H4 + G3H5 + G4H6 + G6H7 + G1G2G3G4G5H7\} + \{G2G4H1H3 + G2G4H1H6 + G2G6H1H7 + G3G6H2H7 + G2G4H3H4 + G4G6H3H7 + G2G4H4H6 + G2G6H4H7 + G3G6H5H7 + G4G6H6H7\} + \{G2G4G6H1H3H7 + G2G4G6H1H6H7 + G2G4G6H3H4H7 + G2G4G6H4H6H7\}}$$

Example (12):

For the control system whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).



Forward paths

$P_1 = G_1G_2G_3G_4$ $\quad$ $P_2 = G_1G_2G_5$

$P_3 = G_1G_2G_9$ $\quad$ $P_4 = G_6G_7G_8G_3G_4$

$P_5 = G_6G_7G_8G_5$ $\quad$ $P_6 = G_6G_7G_8G_9$

Feedback Loops:

$L_1 = -G_6H_3$ $\quad$ $L_2 = -G_8H_4$

$L_3 = -G_1G_2H_1$ $\quad$ $L_4 = -G_6G_7G_8H_1$

$L_5 = -G_1G_2G_3G_4H_2$ $\quad$ $L_6 = -G_1G_2G_5H_2$

$L_7 = -G_1G_2G_9H_2$ $\quad$ $L_8 = -G_6G_7G_8G_3G_4H_2$

$L_9 = -G_6G_7G_8G_5H_2$ $\quad$ $L_{10} = -G_6G_7G_8G_9H_2$

Two non-touching Feedback Loops:

$L_1L_2 = G_6G_8H_3H_4$

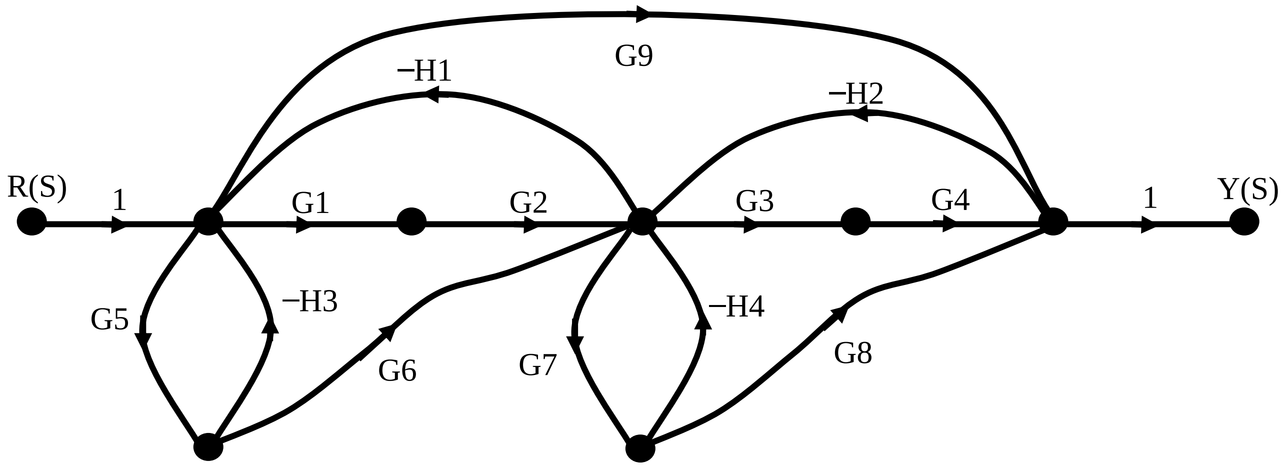
$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$\Delta = 1+\{ G_6H_3+ G_8H_4+ G_1G_2 H_1+ G_6G_7 G_8 H_1+ G_1G_2 G_3G_4 H_2+ G_1G_2 G_5H_2+ G_1G_2 G_9 H_2 +$
$G_6G_7 G_8G_3G_4 H_2 + G_6G_7 G_8G_5 H_2 + G_6G_7 G_8G_9 H_2\}+ G_6G_8H_3H_4$

$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Example (13):

For the control system whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).



Forward Paths:
P1 = G1 G2 G3 G4
P2 = G5 G6 G3 G4
P3 = G1 G2 G7 G8
P4 = G5 G6 G7 G8
P5 = G9

Feedback Loops
L1 = - G5 H3
L2 = - G7 H4
L3 = - G1 G2 H1
L4 = - G3 G4 H2
L5 = - G5 G6 H1
L6 = - G7 G8 H2
L7 = G9 H2 H1

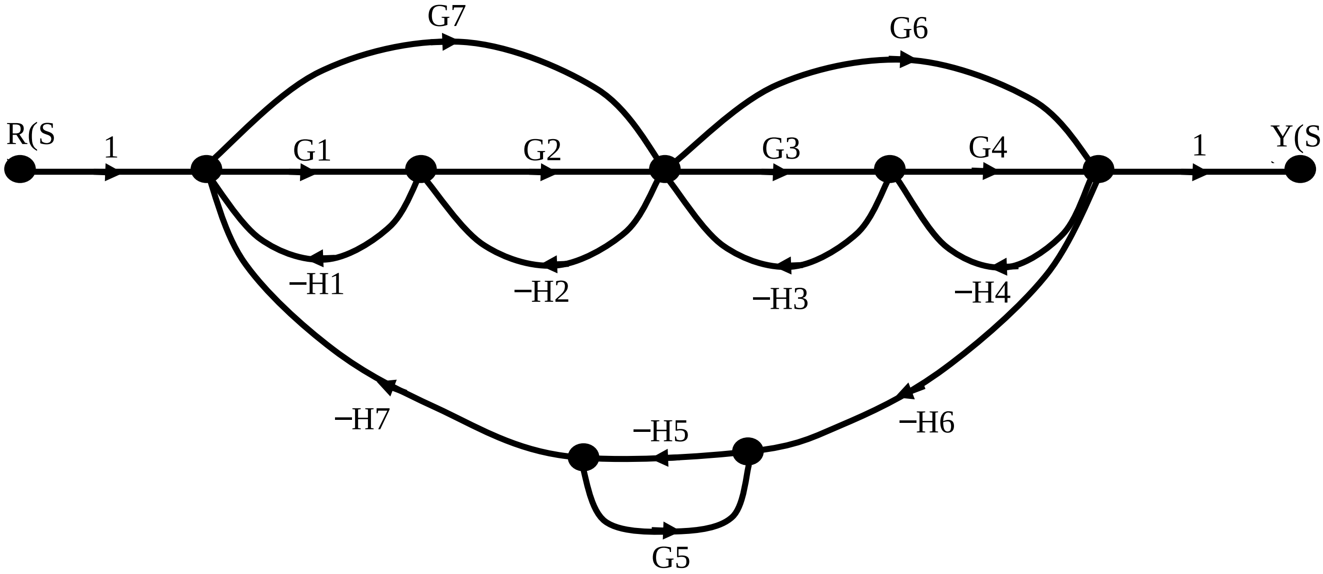
Two non-touching Loops
L1 L2 = G5 H3 G7 H4
L1 L4 = G5 H3 G3 G4 H2
L1 L6 = G5 H3 G7 G8 H2
Δ1 = Δ2 = Δ3 = Δ4 = 1
Δ5 = 1 + G7 H4
Δ = 1 – {L1+L2+L3+L4+L5+L6+L7} + {L1 L2 + L1 L4 + L1 L6}

Using Mason's formula

$$\frac{C(S)}{R(S)} = \frac{P1\Delta1 + P2\Delta2 + P3\Delta3 + P4\Delta4 + P5\Delta5}{\Delta}$$

Example (14):

For the control system whose signal flow graph is shown in Fig. 1, using Mason's formula, find the system transfer function.



Forward paths:

P1 = G1 G2 G3 G4

P2 = G7 G6

P3 = G1 G2 G6

P4 = G7 G3 G4

Feedback Loops:

L1 = − G1 H1

L2 = − G2 H2

L3 = − G3 H3

L4 = − G4 H4

L5 = − G5 H5

L6 = G6 H3 H4

L7 = G7 H1 H2

L8 = − G1 G2 G3 G4 H6 H5 H7

L9 = − G7 G6 H6 H5 H7

L10 = − G1 G2 G6 H6 H5 H7

L11 = − G7 G3 G4 H6 H5 H7

Two non-touching feedback loops:

L1L3 = G1 G3 H1 H3

L1L4 = G1 G4 H1 H4

L1L5 = G1 G5 H1 H5

L1 L6 = − G1 G6 H1 H3 H4

L2L4 = G2 G4 H2 H4

L2L5 = G2 G5 H2 H5

L3L5 = G3 G5 H3 H5

L4L5 = G4 G5 H4 H5

L4L7 = − G4 G7 H4 H1 H2

L5 L6 = − G5 G6 H5 H3 H4

L5L7 = − G5 G7 H5 H1 H2

Three non-touching feedback loops:

L1L3L5 = − G1 G3 G5 H1 H3 H5
L1L4L5 = − G1 G4 G5 H1 H4 H5
L1L5L6 = G1 G5 G6 H1 H5 H3 H4
L2L4L5 = − G2 G4 G5 H2 H4 H5
L4L5L7 = G4 G5 G7 H4 H5 H1 H2

Δ = 1 − { L1 + L2 + L3 + L4 + L5 + L6 + L7 + L8 + L9 + L10 + L11 } + { L1L3 + L1L4 + L1L5 + L1L6 + L2L4 + L2L5 + L3L5 + L4L5 + L4L7 + L5L6 + L5L7 } − { L1L3L5 + L1L4L5 + L1L5L6 + L2L4L5 + L4L5L7 }

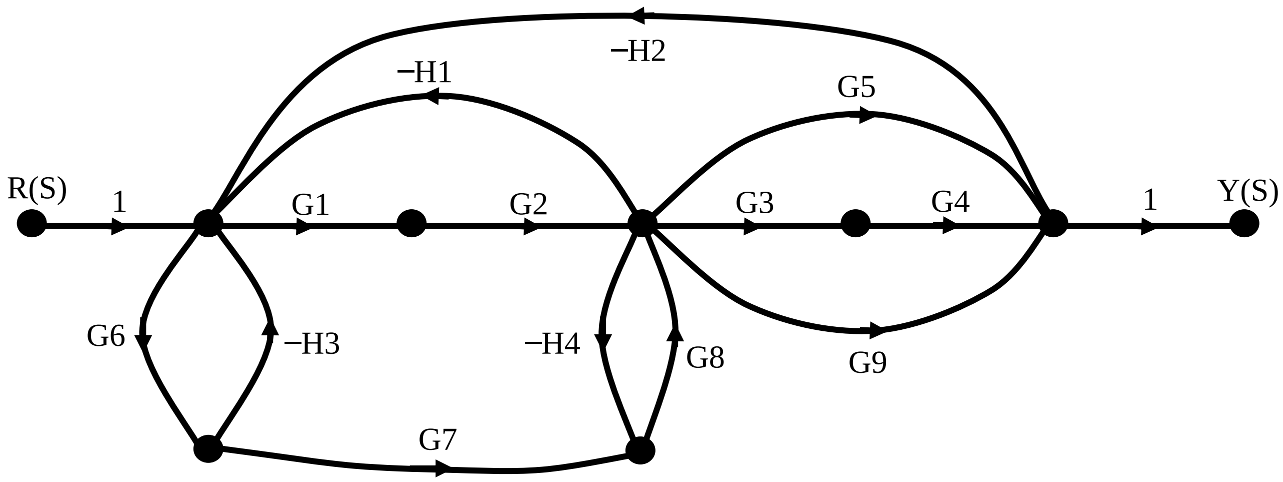
Δ1 = Δ2 = Δ3 = Δ4 = 1 − {L5} = 1 + G5 H5

The system transfer function is obtained by Mason's formula as follows:

Y(S)/R(S) = { P1Δ1 + P2Δ2 + P3Δ3 + P4Δ4 } / Δ

Example (15):

For the control system whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).



Forward paths

$P_1 = G_1G_2G_3G_4$ $P_2 = G_1G_2G_5$

$P_3 = G_1G_2G_9$ $P_4 = G_6G_7G_8G_3G_4$

$P_5 = G_6G_7G_8G_5$ $P_6 = G_6G_7G_8G_9$

Feedback Loops:

$L_1 = -G_6H_3$ $L_2 = -G_8H_4$

$L_3 = -G_1G_2H_1$ $L_4 = -G_6G_7G_8H_1$

$L_5 = -G_1G_2G_3G_4H_2$ $L_6 = -G_1G_2G_5H_2$

$L_7 = -G_1G_2G_9H_2$ $L_8 = -G_6G_7G_8G_3G_4H_2$

$L_9 = -G_6G_7G_8G_5H_2$ $L_{10} = -G_6G_7G_8G_9H_2$

Two non-touching Feedback Loops:

$L_1L_2 = G_6G_8H_3H_4$

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

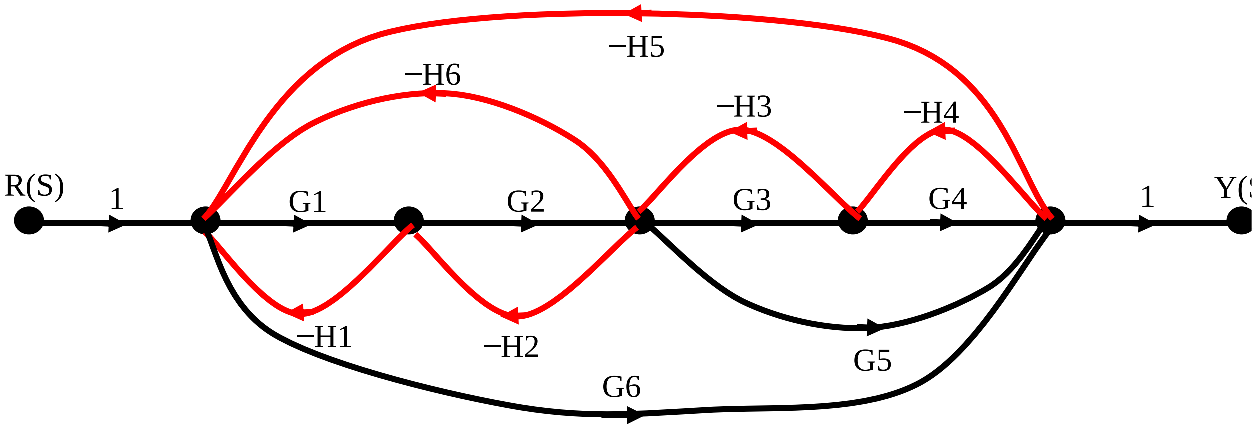
$\Delta = 1 + \{ G_6H_3 + G_8H_4 + G_1G_2H_1 + G_6G_7G_8H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_5H_2 + G_1G_2G_9H_2 +$

$G_6G_7G_8G_3G_4H_2 + G_6G_7G_8G_5H_2 + G_6G_7G_8G_9H_2\} + G_6G_8H_3H_4$

$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Example (16):

Using Mason's formula, find the transfer function of the control system shown below.



* There are **THREE** Forward Paths:

$P_1 = G_1G_2G_3G_4$, $P_2 = G_1G_2G_5$, $P_3 = G_6$,

* There are **ELEVEN** feedback loops:

$L_1 = - G_1 H_1$; $L_2 = - G_2 H_2$;

$L_3 = - G_3 H_3$; $L_4 = - G_4 H_4$;

$L_5 = G_5 H_4 H_1$; $L_6 = - G_1 G_2 H_6$;

$L_7 = - G_1G_2G_3G_4H_5$; $L_8 = - G_1G_2G_5H_5$;

$L_9 = - G_6H_5$; $L_{10} = - G_6H_4H_3H_6$;

$L_{11} = G_6H_4H_3H_2H_1$;

* There are **SEVEN** combination of two-non-touching feedback loops:

$L_1L_3 = G_1 H_1 G_3 H_3$ $L_1L_4 = G_1 H_1 G_4 H_4$

$L_1L_5 = - G_1 H_1 G_5 H_4 H_1$ $L_2L_4 = G_2 H_2 G_4 H_4$

$L_2L_9 = G_2 H_2 G_6H_5$ $L_3L_9 = G_3 H_3 G_6H_5$

$L_4L_6 = G_4 H_4 G_1 G_2 H_6$

$\Delta 1 = \Delta 2 = 1$

$\Delta 3 = 1 - [L_2 + L_3] = 1 + G_2 H_2 + G_3 H_3$

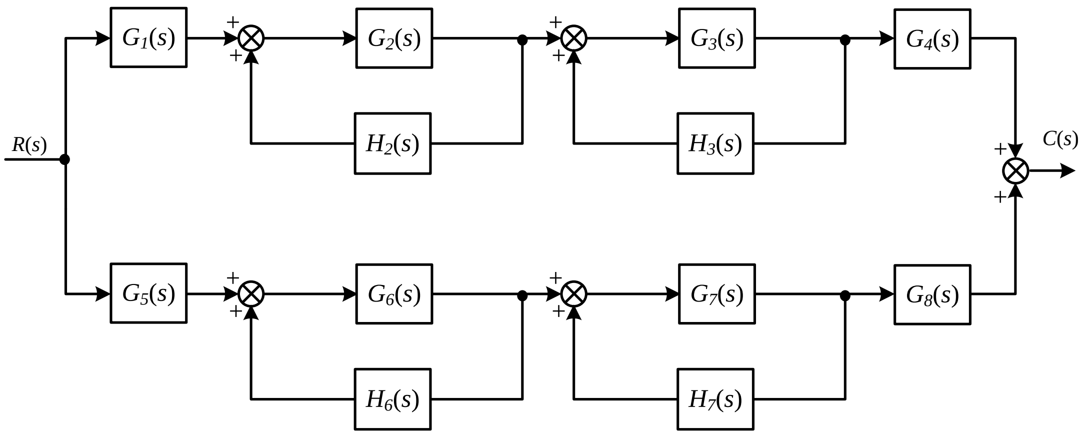
$\Delta = 1 - \{L_1 + L_2 + L_3 + \dots + L_{11}\} + \{L_1L_3 + L_1L_4 + L_1L_5 + \dots + L_4L_6\}$

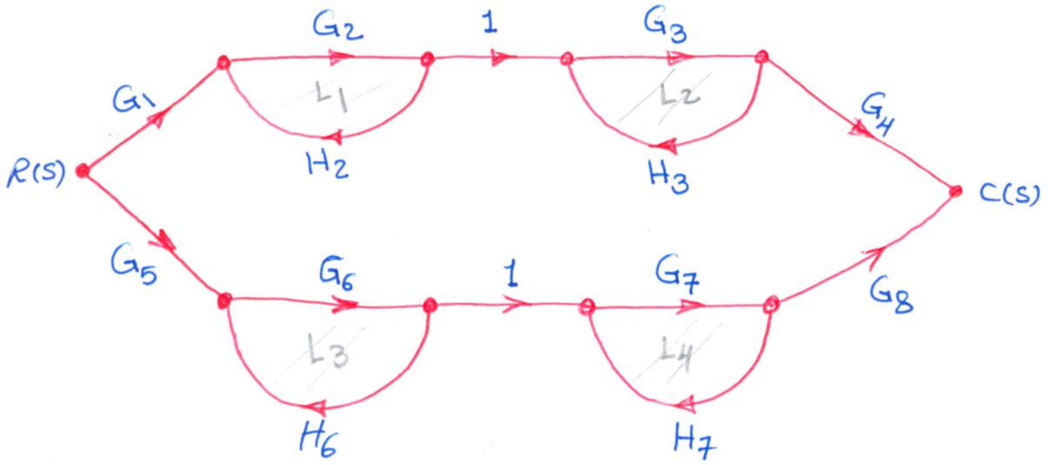
Using Mason's Formula, the system Transfer Function is:

$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

Example (17):

Consider the control system shown below, draw the corresponding signal flow graph, and obtain the closed–loop transfer function using Mason's gain formula.





Forward path (2 paths)

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_5 G_6 G_7 G_8$

Loops (4 loops)

$L_1 = G_2H_2$

$L_2 = G_3H_3$

$L_3 = G_6H_6$

$L_4 = G_7H_7$

two nontouching Loops

$L_1L_2 = G_2G_3H_2H_3$

$L_1L_3 = G_2G_6H_2H_6$

$L_1L_4 = G_2G_7H_2H_7$

$L_2L_3 = G_3G_6H_3H_6$

$L_2L_4 = G_3G_7H_3H_7$

$L_3L_4 = G_6G_7H_6H_7$

three nontouching Loops

$L_1L_2L_3 = G_2G_3G_6H_2H_3H_6$

$L_1L_2L_4 = G_2G_3G_7H_2H_3H_7$

$L_1L_3L_4 = G_2G_6G_7H_2H_6H_7$

$L_2L_3L_4 = G_3G_6G_7H_3H_6H_7$

four non-touching Loops

$L_1L_2L_3L_4 = G_2G_3G_6G_7H_2H_3H_6H_7$

$$\Delta_1 = 1 - [G_6H_6 + G_7H_7] + [G_6G_7H_6H_7]$$

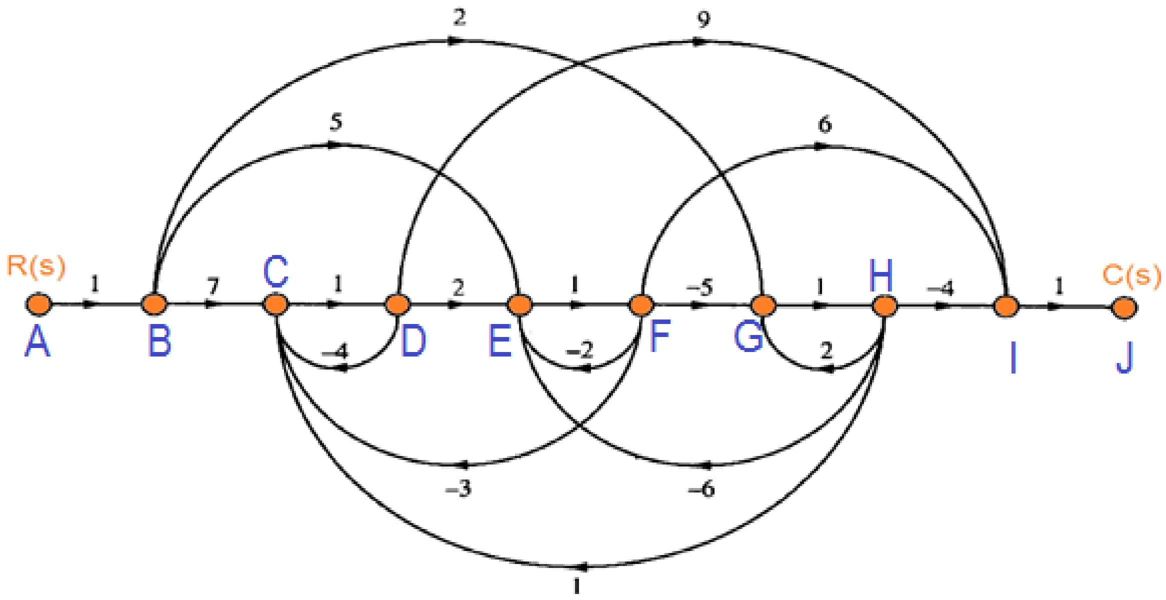
$$\Delta_2 = 1 - [G_2H_2 + G_3H_3] + [G_2G_3H_2H_3]$$

$$\Delta = 1 - [G_2H_2 + G_3H_3 + G_6H_6 + G_7H_7] + [G_2G_3H_2H_3 + G_2G_6H_2H_6 + G_2G_7H_2H_7 + G_3G_6H_3H_6 + G_3G_7H_3H_7 + G_6G_7H_6H_7] - [G_2G_3G_6H_2H_3H_6 + G_2G_3G_7H_2H_3H_7 + G_2G_6G_7H_2H_6H_7 + G_3G_6G_7H_3H_6H_7] + [G_2G_3G_6G_7H_2H_3H_6H_7]$$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4\{1 - (G_6H_6 + G_7H_7) + G_6G_7H_6H_7\} + G_5G_6G_7G_8\{1 - (G_2H_2 + G_3H_3) + G_2G_3H_2H_3\}}{\Delta}$$

Example (18):



Forward paths:

P_1 = ABCDEFGHIJ = 7×2×-5×-4 = 280

P_2 = ABCDEFIJ = 7×2×6 = 84

P_3 = ABEFGHIJ = 5×-5×-4 = 100

P_4 = ABEFIJ = 5×6 = 30

P_5 = ABGHIJ = 2×-4 = -8

P_6 = ABCDIJ = 7×9 = 63

P_7 = ABGHCDIJ = 2×9 = 18

P_8 = ABGHCDEFIJ = 2×2×6 = 24

P_9 = ABGHEFIJ = 2×-6×6 = -72

P_{10} = ABEFCDIJ = 5×-3×9 = -135

P_{11} = ABEFGHCDIJ = 5×-5×9 = -225

P_{12} = ABGHEFCDIJ = 2×-6×-3×9 = 324

Loops:

L_1 = CDC = -4 L_2 = EFE = -2

L_3 = GHG = 2 L_4 = CDEFC = 2×-3 = -6

L_5 = EFGHE = -5×-6 = 30 L_6 = CDEFGHC = 2×-5 = -10

Two Non-touching Loops:

L_1 L_2 = -4×-2 = 8 L_1 L_3 = -4×2 = -8

L_1 L_5 = -4×30 = -120 L_2 L_3 = -2×2 = -4

L_3 L_4 = 2×-6 = -12

Three Non-touching Loops:

L_1 L_2 L_3 = -4×-2×2 = 16

Δ= 1 – {-4 -2 +2 -6 +30 -10} + {8-8-120-4-12} – {16} = -161

Δ1 = 1 – {0} = 1

Δ2 = 1 – {L_3} = 1-2 = -1

Δ3 = 1 – {L_1} = 1 +4 = 5

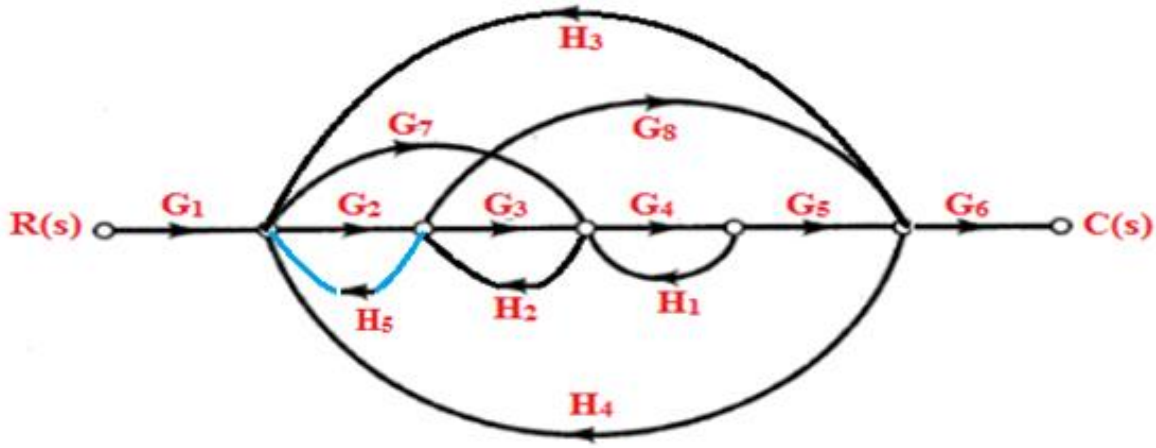
$\Delta 4 = 1 - \{L_1+L_3\} + \{L_1L_3\} = 1 - \{-4+2\} - 8 = -5$
$\Delta 5 = 1 - \{L_1+L_2+L_4\} + \{L_1L_2\} = 1 - \{-4-2-6\} + 8 = 21$
$\Delta 6 = 1 - \{L_2+L_3+L_5\} + \{L_2L_3\} = 1 - \{-2+2+30\} - 4 = -33$
$\Delta 7 = 1 - \{L_2\} = 1 + 2 = 3$
$\Delta 8 = 1 - \{0\} = 1$
$\Delta 9 = 1 - \{L_1\} = 1 - \{-4\} = 5$
$\Delta 10 = 1 - \{L_3\} = 1 - 2 = -1$
$\Delta 11 = 1 - \{0\} = 1$ $\Delta 12 = 1 - \{0\} = 1$

$$\frac{C(s)}{R(s)} = \frac{\{280 \times 1 + 84 \times (-1) + 100 \times 5 + 30 \times (-5) + (-8) \times 21 + 63 \times (-33) + 18 \times 3 + 24 \times 1 - 72 \times 5 - 135 \times (-1) - 225 \times 1 + 324 \times 1\}}{-161}$$

$$\frac{C(s)}{R(s)} = \frac{-1749}{-161} = 10.8634$$

Example (19):

Consider the control system described by the signal flow graph given below.



Obtain the closed–loop transfer function using Mason's gain formula.

In this system, there are three forward paths between the input *R(s)* and the output *C(s)*. There are **FOUR** forward path gains which are:

$P_1 = G_1G_2G_3G_4G_5G_6$ $P_2 = G_1G_7G_4G_5G_6$
$P_3 = G_1G_2G_8G_6$ $P_4 = G_1G_7H_2G_8G_6$

There are **TWELVE** individual loops, the gains of these loops are

$L_1 = G_4H_1$ $L_2 = G_3H_2$
$L_3 = G_2H_5$ $L_4 = G_7H_2H_5$
$L_5 = G_2G_3G_4G_5H_4$ $L_6 = G_7G_4G_5H_4$
$L_7 = G_2G_8H_4$ $L_8 = G_7H_2G_8H_4$
$L_9 = G_2G_3G_4G_5H_3$ $L_{10} = G_7G_4G_5H_3$
$L_{11} = G_2G_8H_3$ $L_{12} = G_7H_2G_8H_3$

There are **THREE** pairs of non-touching loops, the gains of these loops are

$L_1L_3 = G_4H_1G_2H_5$
$L_1L_7 = G_4H_1G_2G_8H_4$
$L_1L_{11} = G_4H_1G_2G_8H_3$

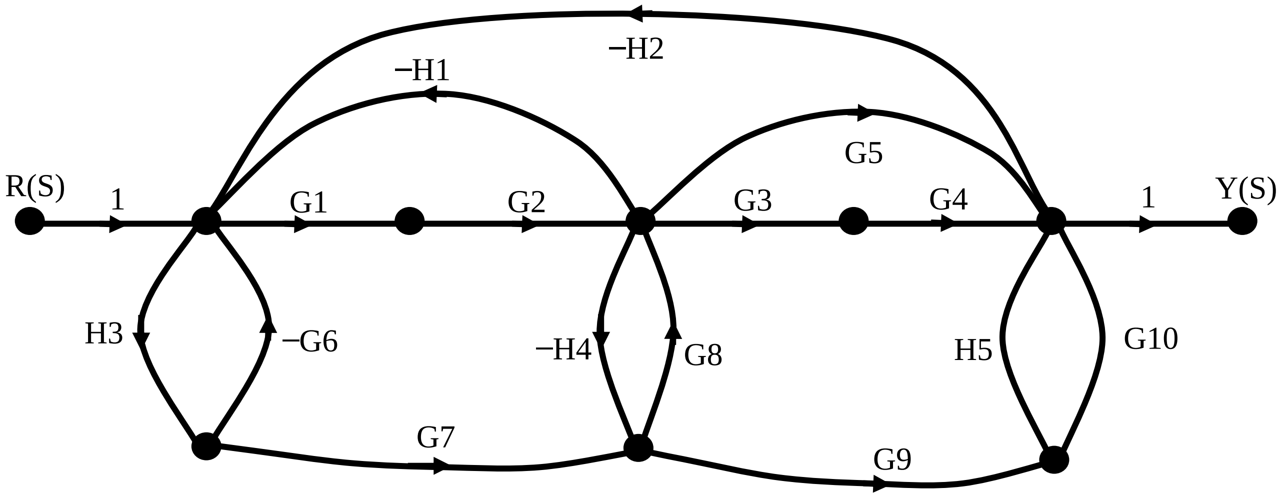
$$\Delta_1 = \Delta_2 = \Delta_4 = 1, \qquad \Delta_3 = 1 - L_1$$

$$\Delta = 1 - \{L_1 + \cdots + L_{12}\} + \{L_1L_3 + L_1L_7 + L_1L_{11}\}$$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

Example (20):

For the control system, whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).



forward paths:
$P_1 = G_1 G_2 G_3 G_4$
$P_2 = G_1 G_2 G_5$
$P_3 = G_1 G_2 H_4 G_9 G_{10}$
$P_4 = H_3 G_7 G_8 G_3 G_4$
$P_5 = H_3 G_7 G_8 G_5$
$P_6 = H_3 G_7 G_9 G_{10}$

6 paths

feedback loops: (11 loops)

$L_1 = G_6 H_3$

$L_2 = G_8 H_4$

$L_3 = G_{10} H_5$

$L_4 = G_1 G_2 H_1$

$L_5 = H_3 G_7 G_8 H_1$

$L_6 = G_1 G_2 G_3 G_4 H_2$

$L_7 = G_1 G_2 G_5 H_2$

$L_8 = G_1 G_2 H_4 G_9 G_{10} H_2$

$L_9 = H_3 G_7 G_8 G_3 G_4 H_2$

$L_{10} = H_3 G_7 G_8 G_5 H_2$

$L_{11} = H_3 G_7 G_9 G_{10} H_2$

two non-touching loops:

$L_1 L_2$, $L_1 L_3$, $L_2 L_3$, $L_3 L_4$, $L_3 L_5$ } 5 pairs

three non-touching loops

$L_1 L_2 L_3$

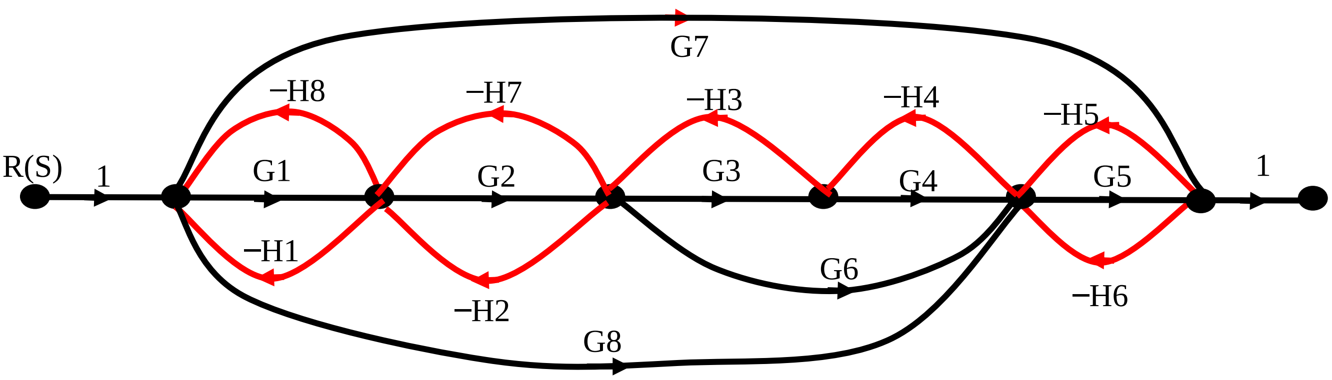
$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

$$\Delta = 1 - [L_1 + L_2 + \cdots + L_{11}] + [L_1 L_2 + L_1 L_3 + L_2 L_3 + L_3 L_4 + L_3 L_5] - [L_1 L_2 L_3]$$

$$T.F = \frac{Y(s)}{R(s)} = \frac{P_1 + P_2 + P_3 + P_4 + P_5 + P_6}{\Delta} \quad \#$$

Example (21):

For the signal flow graph of a control system shown below, using Mason's formula, find the system transfer function and the system characteristic equation.



* There are **FOUR** Forward Paths:

$P_1 = G_7$ $P_2 = G1G2G3G4$

$P_3 = G_1G_2G_6G_5$ $P_4 = G_8G_5$

* There are **TWENTY ONE** feedback loops:

$L_1 = G_1 H_1$; $L_2 = G_2 H_2$;

$L_3 = G_3 H_3$; $L_4 = G_4 H_4$;

$L_5 = G_5 H_5$; $L_6 = G_5 H_6$;

$L_7 = G_2H_7$; $L_8 = G_1H_8$;

$L_9 = G_6H_4H_3$; $L_{10} = G_8H_4H_3H_7H_8$;

$L_{11} = G_8H_4H_3H_7H_1$; $L_{12} = G_8H_4H_3H_2H_1$;

$L_{13} = G_8H_4H_3H_2H_8$; $L_{14} = G_7H_5H_4H_3H_7H_8$;

$L_{15} = G_7H_5H_4H_3H_7H_1$; $L_{16} = G_7H_5H_4H_3H_2H_1$;

$L_{17} = G_7H_5H_4H_3H_2H_8$; $L_{18} = G_7H_6H_4H_3H_7H_8$;

$L_{19} = G_7H_6H_4H_3H_7H_1$; $L_{20} = G_7H_6H_4H_3H_2H_1$;

$L_{21} = G_7H_6H_4H_3H_2H_8$;

* There are **EIGHTEEN** combination of two-non-touching feedback loops:

$L_1L_3 = G_1 H_1 G_3 H_3$ $L_1L_4 = G_1 H_1 G_4 H_4$

$L_1L_5 = G_1 H_1 G_5 H_5$ $L_1L_6 = G_1 H_1 G_5 H_6$

$L_1L_9 = G_1 H_1 G_6 H_4 H_3$ $L_2L_4 = G_2 H_2 G_4H_4$

$L_2L_5 = G_2\, H_2\, G_5\, H_5$ $L_2L_6 = G_2\, H_2\, G_5\, H_6$

$L_3L_5 = G_3\, H_3\, G_5\, H_5$ $L_3L_6 = G_3\, H_3\, G_5\, H_6$

$L_3L_8 = G_3\, H_3\, G_1\, H_8$ $L_4L_7 = G_4\, H_4\, G_2\, H_7$

$L_4L_8 = G_4\, H_4\, G_1\, H_8$ $L_5L_7 = G_5\, H_5\, G_2\, H_7$

$L_5L_8 = G_5\, H_5\, G_1\, H_8$ $L_6L_7 = G_5\, H_6\, G_2\, H_7$

$L_6L_8 = G_5\, H_6\, G_1\, H_8$ $L_8L_9 = G_1\, H_8\, G_6\, H_4\, H_3$

* There are **FOUR** combination of three-non-touching feedback loops:

$L_1L_3\, L_5 = G_1\, H_1\, G_3\, H_3\, G_5\, H_5$ $L_1L_3\, L_6 = G_1\, H_1\, G_3\, H_3\, G_5\, H_6$

$L_3L_5\, L_8 = G_3\, H_3\, G_5\, H_5\, G_1\, H_8$ $L_3L_6\, L_8 = G_3\, H_3\, G_5\, H_6\, G_1\, H_8$

$\Delta = 1 – [\, L_1 + L_2 + \ldots + L_{15}] – [\, L_1L_3 + L_1L_4 + \ldots + L_8\, L_9] + [\, L_1L_3L_5 + \ldots + L_3L_6L_8]$

$\Delta 1 = 1 – [L_2 + L_3 + L_4 + L_7 + L_9] + [L_2L_4 + L_4L_7]$

$\Delta 2 = \Delta 3 = 1$

$\Delta 4 = 1 - [L_2 + L_3 + L_7]$

$\Delta = 1 - \{L_1 + L_2 + L_3 + ... + L_{21}\} + \{L_1L_3 + L_1L_4 + L_1L_5 + ...+ L_4L_6\}$

Using Mason's Formula, the system Transfer Function is:

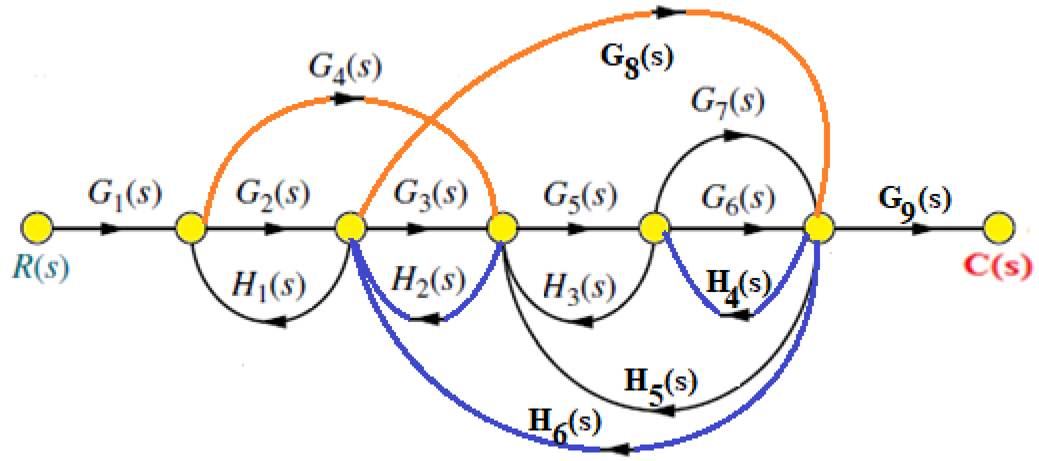
$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

The characteristic equation is:

$\Delta = 1 – [\, L_1 + L_2 + \ldots + L_{15}] – [\, L_1L_3 + L_1L_4 + \ldots + L_8\, L_9] + [\, L_1L_3L_5 + \ldots + L_3L_6L_8] = 0$

Example (22):

For the signal flow graph of a control system shown below, using Mason’s formula, find the system transfer function and the system characteristic equation.



There are **SIX** forward path gains which are:

$P_1 = G_1G_2G_3G_5G_6\,G_9$ $\quad$ $P_2 = G_1G_2G_3G_5G_7G_9$

$P_3 = G_1G_4G_5G_6G_9$ $\quad$ $P_4 = G_1G_4G_5G_7G_9$

$P_5 = G_1G_4H_2G_8G_9$ $\quad$ $P_6 = G_1G_2G_8G_9$

There are **Fifteen** individual loops, the gains of these loops are

$L_1 = G_2H_1$

$L_2 = G_3H_2$

$L_3 = G_4H_2H_1$

$L_4 = G_5H_3$

$L_5 = G_6H_4$

$L_6 = G_7H_4$

$L_7 = G_5G_6H_5$

$L_8 = G_5G_7H_5$

$L_9 = G_3G_5G_6H_6$

$L_{10} = G_3G_5G_7H_6$

$L_{11} = G_8H_6$

$L_{12} = G_8H_4H_3H_2$

$L_{13} = G_8H_5H_2$

$L_{14} = G_4G_5G_6H_6H_1$

$L_{15} = G_4G_5G_7H_6H_1$

There are **Ten** pairs of non-touching loops, the gains of these loops are

$L_1L_4 = G_2H_1\,G_5H_3$

$L_1L_5 = G_2H_1\,G_6H_4$

$L_1L_6 = G_2H_1\,G_7H_4$

$L_1L_7 = G_2H_1\ G_5G_6H_5$
$L_1L_8 = G_2H_1\ G_5G_7H_5$
$L_2L_5 = G_3H_2\ G_6\ H_4$
$L_2L_6 = G_3H_2\ G_7\ H_4$
$L_3L_5 = G_4H_2H_1\ G_6\ H_4$
$L_3L_6 = G_4H_2H_1\ G_7\ H_4$
$L_4L_{11} = G_5H_3\ G_8\ H_6$

There is only **<u>ONE</u>** three non-touching loops, the gains of this loops are
$L_1L_4L_{11} = G_2H_1\ G_5H_3\ G_8H_6$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = 1$$

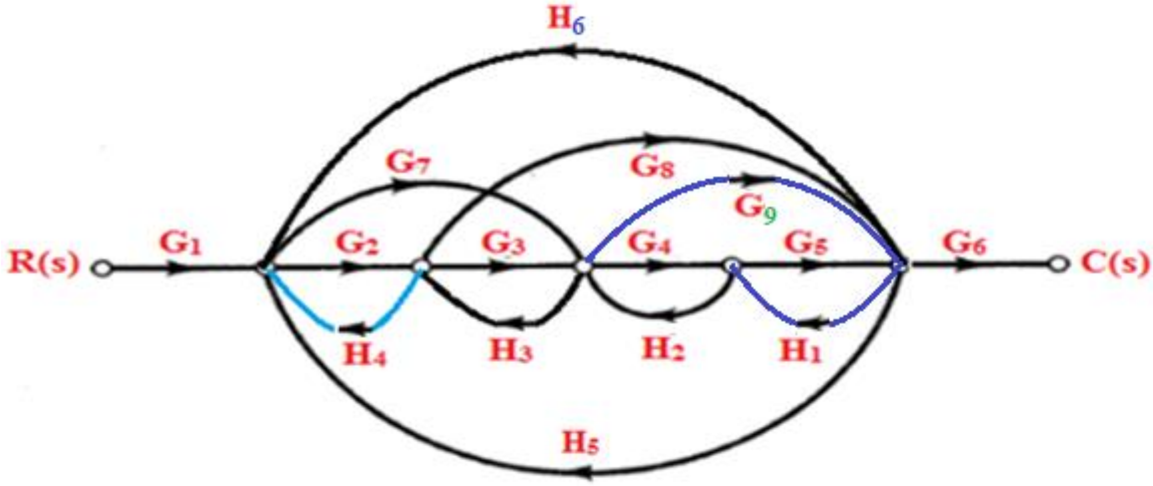
$$\Delta_6 = 1 - L_4 = 1 - L_4 = 1 - G_5H_3$$

$$\Delta = 1 - \{L_1 + \cdots + L_{13}\} + \{L_1L_4 + L_1L_5 + \cdots + L_4L_{11}\} - \{L_1L_4L_{11}\}$$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

<u>Example (23):</u>

For the control system, whose signal flow graph is shown below, using Mason's formula, find the system transfer function C(s)/R(s).



There are **<u>SIX</u>** forward path gains which are:

$P_1 = G_1G_2G_3G_4G_5G_6$
$P_2 = G_1G_7G_4G_5G_6$
$P_3 = G_1G_7H_3G_8G_6$
$P_4 = G_1G_2G_8G_6$
$P_5 = G_1G_2G_3G_9G_6$
$P_6 = G_1G_7G_9G_6$

There are **<u>Nineteen</u>** individual loops, the gains of these loops are

$L_1 = G_2H_4$
$L_2 = G_3H_3$
$L_3 = G_4H_2$
$L_4 = G_5H_1$
$L_5 = G_7\ H_3H_4$
$L_6 = G_8H_1H_2\ H_3$

$L_7 = G_9H_1H_2$
$L_8 = G_2G_3G_4G_5H_5$
$L_9 = G_2G_3G_4G_5H_6$
$L_{10} = G_7G_4G_5H_5$
$L_{11} = G_7G_4G_5H_6$
$L_{12} = G_7H_3G_8H_5$
$L_{13} = G_7H_3G_8H_6$
$L_{14} = G_2G_8H_5$
$L_{15} = G_2G_8H_6$
$L_{16} = G_2G_3G_9H_5$
$L_{17} = G_2G_3G_9H_6$
$L_{18} = G_7G_9H_5$
$L_{19} = G_7G_9H_6$

There are **<u>Seven</u>** pairs of non-touching loops, the gains of these loops are

$L_1L_3 = G_2H_4G_4H_2$
$L_1L_4 = G_2H_4G_5H_1$
$L_1L_7 = G_2H_4G_9H_1H_2$
$L_2L_4 = G_3H_3G_5H_1$
$L_3L_{14} = G_4H_2G_2G_8H_5$
$L_3L_{15} = G_4H_2G_2G_8H_6$
$L_4L_5 = G_5H_1G_7H_3H_4$

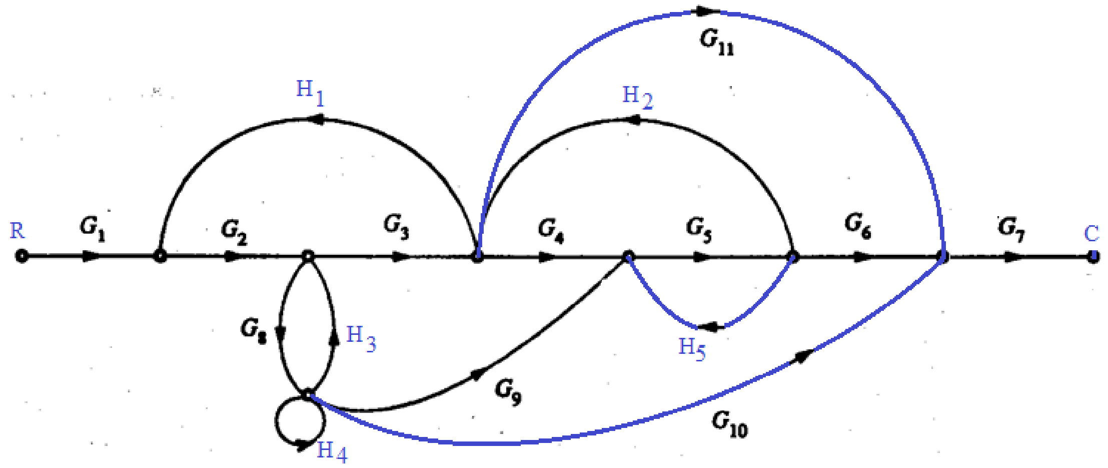
$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_5 = \Delta_6 = 1, \qquad \Delta_4 = 1 - L_3$$

$$\Delta = 1 - \{L_1 + \cdots + L_{19}\} + \{L_1L_3 + L_1L_4 + \cdots + L_3L_{15}\}$$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

<u>Example (24):</u>

Consider the control system described by the signal flow graph given below. Obtain the closed–loop transfer function using Mason's gain formula.



There are **<u>FIVE</u>** forward path gains which are:

$P_1 = G_1G_2G_3G_4G_5G_6G_7$
$P_2 = G_1G_2G_3G_{11}G_7$
$P_3 = G_1G_2G_8G_9G_5G_6G_7$
$P_4 = G_1G_2G_8G_9G_5H_2G_{11}G_7$

$P_5 = G_1G_2G_8G_{10}G_7$

There are **SIX** individual loops, the gains of these loops are

$L_1 = G_8H_3$
$L_2 = H_4$
$L_3 = G_2\ G_3H_1$
$L_4 = G_4G_5H_2$
$L_5 = G_2G_8G_9G_5H_2H_1$
$L_6 = G_5\ H_5$

There are **SIX** pairs of non-touching loops, the gains of these loops are

$L_1L_4 = G_8H_3\ G_4G_5H_2$
$L_1L_6 = G_8H_3\ G_5\ H_5$
$L_2L_3 = H_4\ G_2\ G_3H_1$
$L_2L_4 = H_4\ G_3G_5H_2$
$L_2L_6 = H_4\ G_5\ H_5$
$L_3L_6 = G_2\ G_3H_1\ G_5\ H_5$

There is only **ONE** three non-touching loops, the gains of this loops are
$L_2L_3L_6 = H_4\ G_2\ G_3H_1\ G_5\ H_5$

$$\Delta_1 = 1 - L_2$$

$$\Delta_2 = 1 - \{L_2 + L_6\} + \{L_2L_6\}$$

$$\Delta_3 = \ \Delta_4 = 1$$

$$\Delta_5 = 1 - L_6 - L_4$$

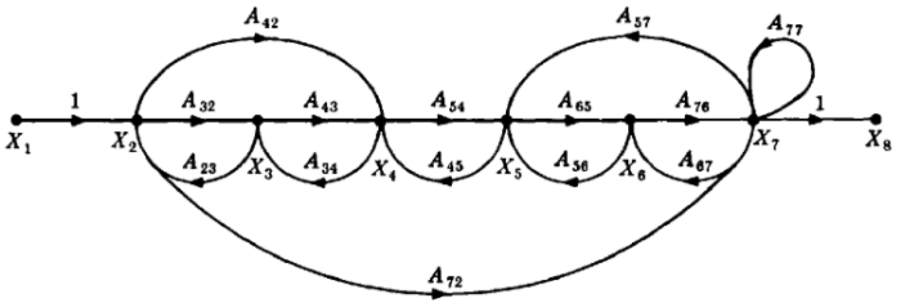
$$\Delta = 1 - \{L_1 + \cdots + L_6\} + \{L_1L_4 + L_1L_6 + \cdots + L_36\} - \{L_2L_3L_6\}$$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5}{\Delta}$$

Sheet 3 (Signal Flow Graph)

Problem #1

Consider the signal flow graph shown below, identify the following:

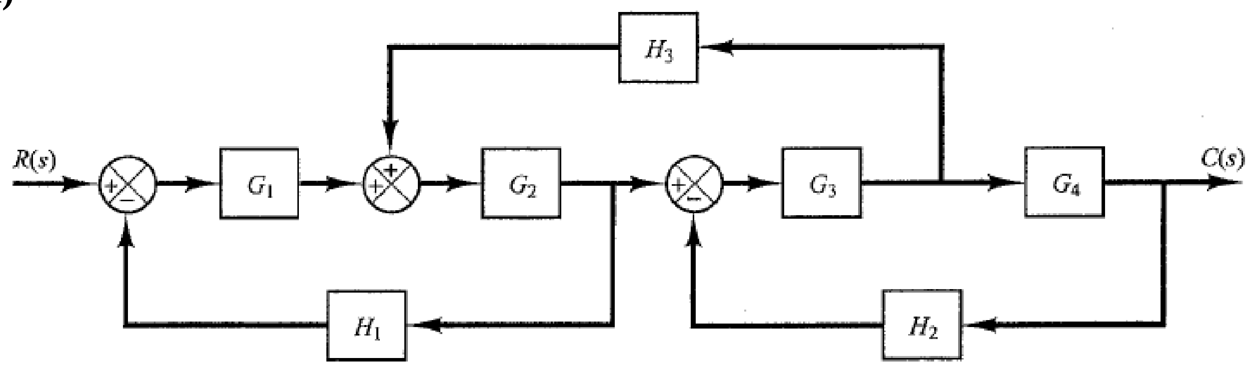


a) Input node.
b) Output node.
c) Self loop.
d) Determine the loop gains of the feedback loops.
e) Determine the path gains of the forward paths

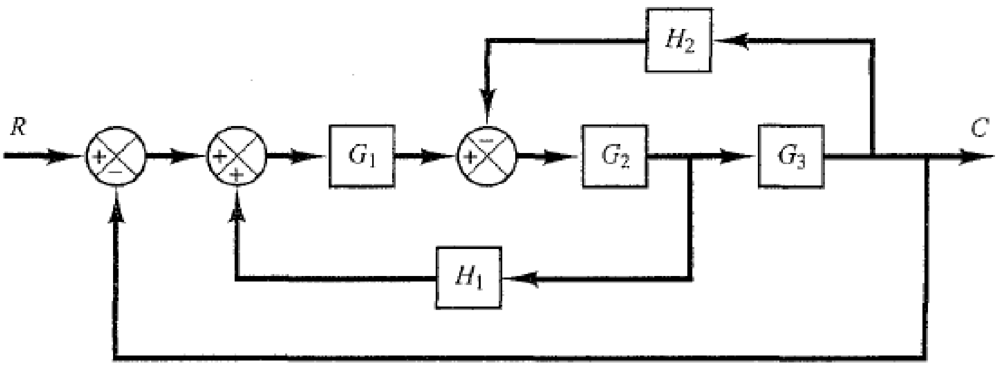
Problem #2

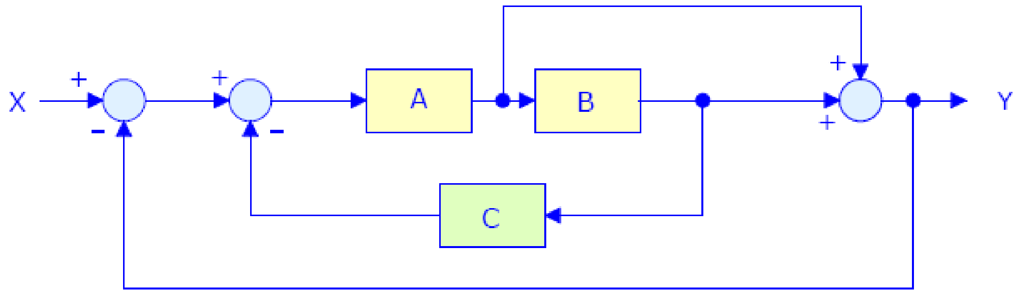
For the control systems represented by block diagrams shown in figures below, Draw the corresponding signal flow graph (SFG), then using Mason's rule to obtain the system transfer function.

a)

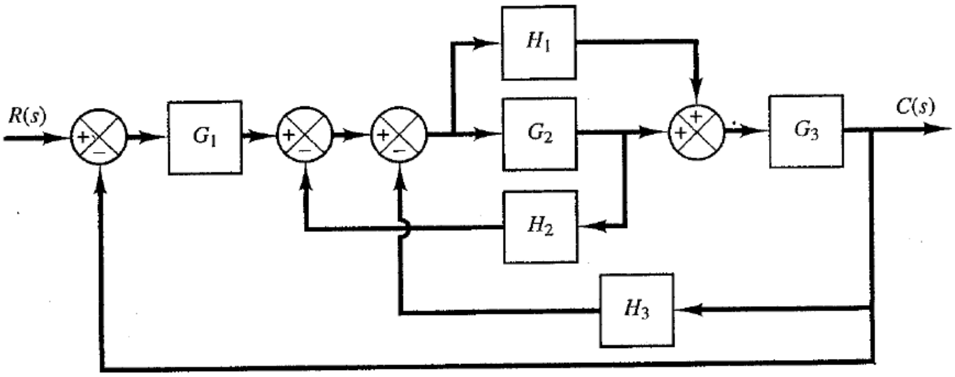


b)

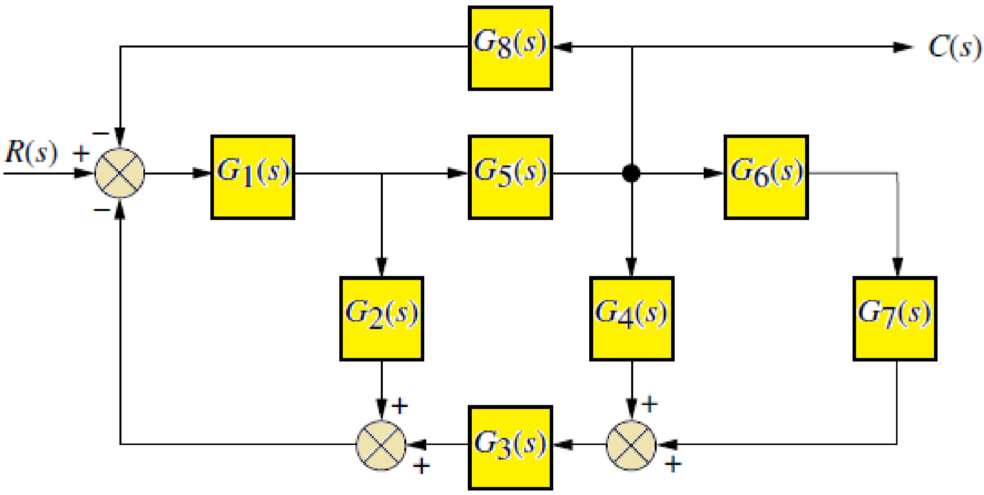




c)



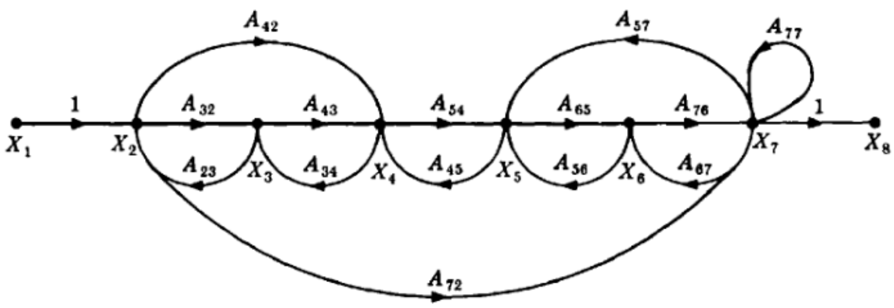
d)



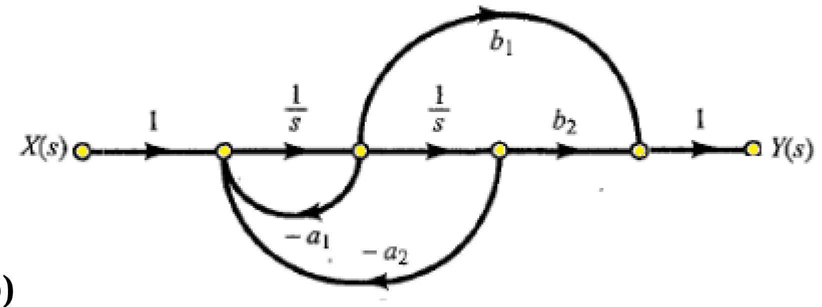
e)

<u>Problem #3</u>

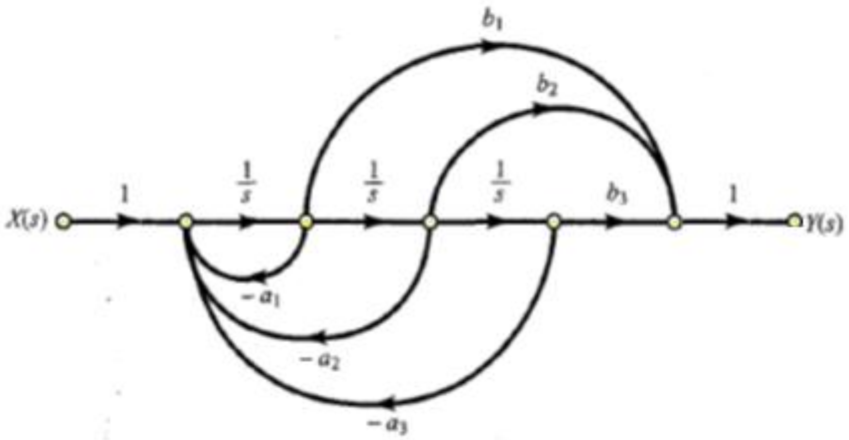
Using Mason's Rule, find the transfer function for the following SFG's



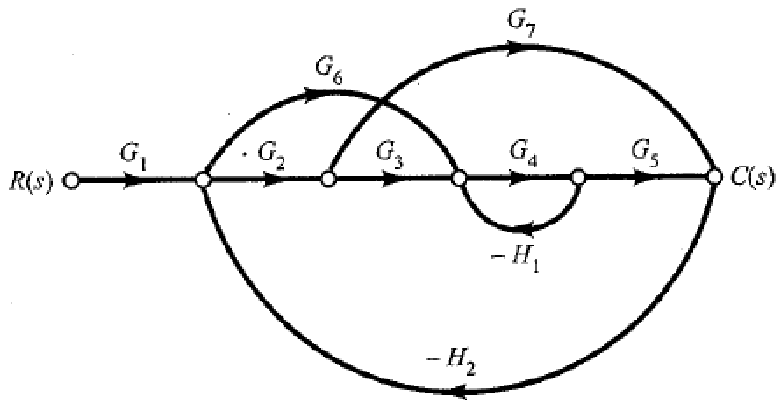
a)


b1
1
1/s
1/s
b2
1
X(s)
Y(s)
− a1
− a2

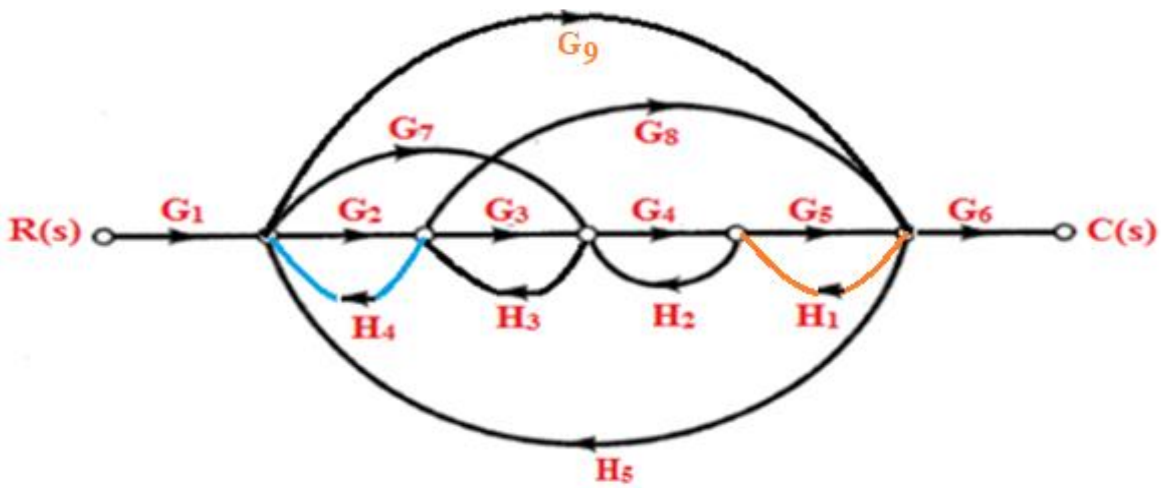
b)


b1
b2
1
1/s
1/s
1/s
b3
1
X(s)
Y(s)
− a1
− a2
− a3

c)

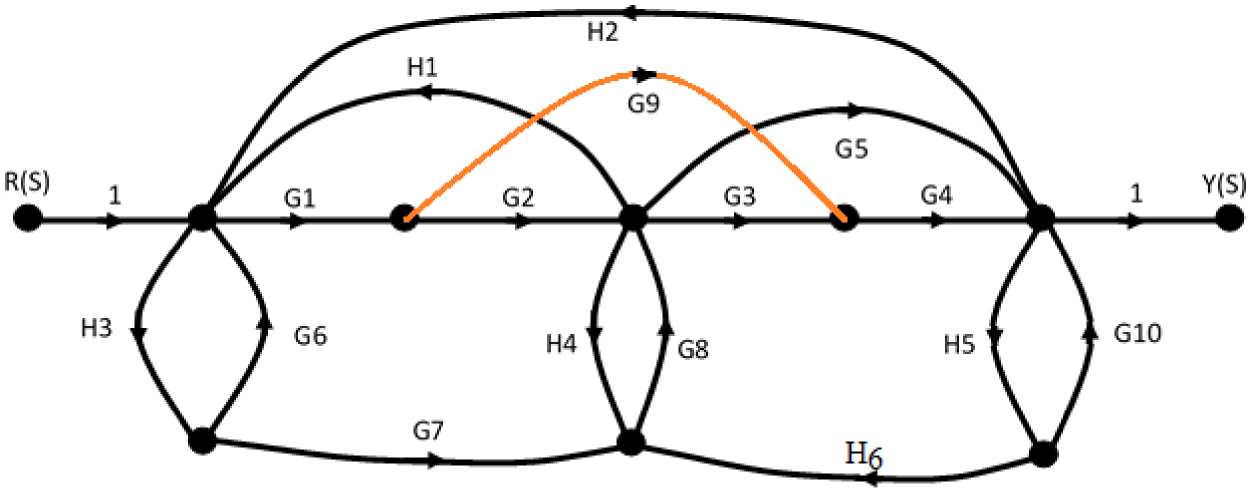

G7
G6
R(s)
G1
G2
G3
G4
G5
C(s)
− H1
− H2

d)


G9
G7
G8
R(s)
G1
G2
G3
G4
G5
G6
C(s)
H4
H3
H2
H1
H5

e)

f)



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