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A ROBUST NONLINEAR CONTROLLER FOR AUTONOMOUS HELICOPTER LANDING

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ABSTARCT

We consider the problem of controlling the vertical motion of a nonlinear model of a helicopter during landing, while stabilizing the lateral and longitudinal position and maintaining a constant attitude. The dynamics of the helicopter main and tail rotors are often neglected to simplify the control design, but this reduction has the disadvantage of an unreal choice of the control inputs. We have solved this contradiction by choosing the main controller required for performing the secure autonomous landing by neglecting the complex rotors dynamics, and then we have included these important dynamics in a closed loop model having real controls determined from an auxiliary controller. Simulation results show the effectiveness of the method to cope with uncertainties on the plant and actuator model.

KEYWORDS

Helicopter model- Nonlinear systems- Rotors dynamics- Saturated controls

نقوم في هذا البحث بدراسة التحكم في الحركة الرأسية لنموذج غير خطي لطائرة هليكوبتر أثناء الهبوط مع تثبيت الوضع الطولي و العرضي و الحفاظ علي موضع ثابت. إن نظم الحركة الخاصة بدوار الهليكوبتر الرئيسي أو دوار الذيل غالبا ما يتم إهمالها بغرض تبسيط تصميم نظام التحكم و لكن هذا التبسيط يعيبه اختيار مدخلات تحكم غير واقعية. نقدم في هذا البحث حلا للتناقض السابق عن طريق اختيار نظام التحكم الرئيسي– اللازم لتنفيذ الهبوط الآمن– بإهمال نظم حركة الدوارات المعقدة. بعد ذلك قمنا بتضمين نظم الحركة في نموذج مغلق ذي مدخلات واقعية تم حسابها بواسطة نظام تحكم إضافي مساعد. تظهر نتائج البحث كفاءة هذه الطريقة في التعامل مع مجهولات النموذج و الهدف المراد متابعته.

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1. INTRODUCTION

Autopilot design for helicopters is a challenging testbed in nonlinear feedback design, due to the nonlinearity of the dynamics and the strong coupling between the forces and torques produced by the vehicle actuators. Describing the flight behavior of the helicopter presents a difficult challenge to mathematical modeling because a complete model of a helicopter consists of many subsystems which are rich of dynamics, such as actuator dynamics, main and tail rotor dynamics, rigid body dynamics and force and moment generation processes [1]. Being complex and somewhat unmanageable for the purpose of control, the dynamics of the main and tail rotors are often neglected in many similar studies, hence the helicopter model in it was considered as a rigid body incorporating with a force and moment generation process. While the reduction of the helicopter model eases the design of the controller, it results in unreal control inputs. We propose a solution to the previous contradiction by choosing the main controller required for the task of autonomous landing by neglecting the complex rotor dynamics and then we insert these dynamics in a complete model with real control inputs. The added dynamics requires another auxiliary controller to force the states of the main and tail rotors systems to follow their reference values determined from the main controller.

The resultant force and moment is the summation of the contributions from the helicopter components and systems which can be broken down into a smaller number of major areas; the main rotor, tail rotor, fuselage and empennage, the engine and transmission, the fuel system, the landing gear, the controls, electric and hydraulic power, instrumentation and avionics [1],[2]. The complexity of the helicopter model presented here has the advantage of approximating the reality with higher percent which is a necessary matter in the simulation process. The goal of the control task is the design of an autopilot able to secure autonomous landing of the helicopter on an oscillating platform. In this paper, we address the application of an internal-model based autopilot for a helicopter choosing a solution which combines recent results on nonlinear adaptive regulation and robust stabilization of systems in feedforward form by means of saturated controls[3]. The paper is organized as follows: in section 2 the helicopter model is introduced. In section 3 we describe the statement of the design problem and the main controller introducing the stabilization of the vertical error dynamics, the attitude dynamics and the lateral and longitudinal dynamics. Section 4 deals with the design of the auxiliary controller. Simulation results are illustrated in section 5. Finally, we draw some conclusions in section 6.

2. THE HELICOPTER MODEL

A complete model of a helicopter can be divided into four different subsystems, which are actuator dynamics, main and tail rotor dynamics, force and moment generation processes and rigid body dynamics, as shown in Fig.1, where all symbols are defined at the end of the paper. Because the dynamics of the rotors are quite complex and unmanageable for the purpose of control, they were neglected in many papers such as [3] and [5], for example, and the helicopter model was considered as a rigid body incorporating with a force and moment generation process.

Although this model reduction has the advantage of choosing the control vector to be $(T_M \quad \beta_{1c} \quad \beta_{1s} \quad T_T)$ thus simplifying the design of the control system, it has neglected the most important dynamics of the main and tail rotors. The main disadvantage of the previous modeling is the choice of the control vector which is not a real one because the real control inputs are those of the pilot $(\eta_0 \quad \eta_{1c} \quad \eta_{1s} \quad \eta_{0T})$ or $(\theta_0 \quad \theta_{1c} \quad \theta_{1s} \quad \theta_{0T})$ neglecting the actuator dynamics. We have used the main controller presented in [3] for performing secure landing, thus gaining the benefit of the model simplification, then we have included the complex rotors dynamics to the overall system in order to overcome its deficiency. The added dynamics required the design of an auxiliary controller to force the states of the main and tail rotors systems to follow their reference values determined from the main controller. The block diagram in Fig.2 clarifies this procedure and the following subsections explain its components.

2.1 The Main and Tail Rotors Dynamics

The main rotor is the most important component of any helicopter because it is the primary source providing thrust and direction control. It consists mainly of the rotor blades and the hub. The rotor responds to collective and cyclic inputs by flapping as a disc, in coning and tilting modes. The dynamics of the main rotor flapping angles have been derived from [1] as follows.

Let $\beta_R = (\beta_0 \quad \beta_{1c} \quad \beta_{1s})^T$ and $\Theta = (\theta_0 \quad \theta_{1c} \quad \theta_{1s})^T$

$$\dot{\beta}_R = \Omega_R \left(A_\beta \beta_R + B_\beta \Theta + C\right) \tag{1}$$

such that

$$A_\beta = -C_\beta^{-1} D_\beta,\ B_\beta = C_\beta^{-1} H_1,\ C = C_\beta^{-1} H_2 \tag{2}$$

where

$$C_\beta = \begin{bmatrix} 1 & 0 & 3\mu/2 \\ 0 & 1 & 16/\gamma \\ 4\mu/3 & -16/\gamma & 1 \end{bmatrix},\ D_\beta = \frac{\gamma}{8}\begin{bmatrix} 8\lambda_\beta^2/\gamma & 0 & 0 \\ 4\mu/3 & 8(\lambda_\beta^2 - 1)/\gamma & 1+\mu^2/2 \\ 0 & -1+\mu^2/2 & 8(\lambda_\beta^2 - 1)/\gamma \end{bmatrix} \tag{3}$$

$$H_1 = \frac{\gamma}{8}\begin{bmatrix} 1+\mu^2 & 0 & 4\mu/3 \\ 0 & 1+\mu^2/2 & 0 \\ 8\mu/3 & 0 & 1+3\mu^2/2 \end{bmatrix},$$

$$H_2 = \frac{\gamma}{8}\begin{bmatrix} 4\theta_{tw}\left(1/5+\mu^2/6\right)+4/3(\mu_z-\lambda_0)+2/3\mu+p/\Omega_R \\ 16\left(q/\Omega_R+\dot{p}/2\Omega_R^2\right)/\gamma+q/\Omega_R \\ -16\left(q/\Omega_R-\dot{p}/2\Omega_R^2\right)/\gamma+2\mu\theta_{tw}+2\mu(\mu_z-\lambda_0)+p/\Omega_R \end{bmatrix} \quad (4)$$

Although the normalized velocity μ in Eqs. (3), (4) is a time-varying parameter, we have considered it as a constant value in the simulation because of the slow velocity of the helicopter in the present task of landing. The coning of the rotor is considered the primary source of thrust, and that is why we have included a block in Fig.2 converting from the coning angle β_0 to the main rotor thrust T_M as given below

$$Q_{\theta_0} = \frac{1}{1+\mu^2}\left(8\lambda_\beta\beta_0/\gamma-4\theta_{tw}\left(1/5+\mu^2/6\right)-4\theta_{1s}\mu-4(\mu_z-\lambda_0)/3-2\mu\dot{P}/3\Omega_R\right) \quad (5)$$

$$T_M = \frac{1}{2}\pi\rho s a_0\Omega_R^2R^4(Q_{\theta_0}\left(1/3+\mu^2/2\right)+\mu\left(\theta_{1s}+\dot{P}/2\Omega_R\right)/2+ (\mu_z-\lambda_0)/2+\theta_{tw}\left(1+\mu^2\right)/4) \quad (6)$$

We have derived the dynamics of the tail rotor flapping angle β_{0T} using the theories given in [1] concerning the main rotor.

$$\dot{\beta}_{0T} = A_T\beta_{0T}+B_T\theta_{0T}+C_T \quad (7)$$

where

$$A_T = \left(8\left(\lambda_\beta^2/\gamma\right)_T\left(1+256/\gamma_T^2\right)-128\mu_T^2/9\gamma_T\right)/\left(64\left|C_\beta\right|/\left(\Omega_{TR}\gamma_T^2\right)\right) \quad (8)$$

$$B_T = \left(\left(1+256/\gamma_T^2\right)\left(1+\mu_T^2\right)-16\mu_T^2/9\right)/\left(64\left|C_\beta\right|/\left(\Omega_{TR}\gamma_T^2\right)\right) \quad (9)$$

$$C_T = 4/3\left((\mu_{zT}-\lambda_{0T})\left(1+256/\gamma_T^2\right)-\mu_T^2(\mu_{zT}-\lambda_{0T})\right)/\left(64\left|C_\beta\right|/\left(\Omega_{TR}\gamma_T^2\right)\right) \quad (10)$$

and

$$\left|C_\beta\right| = \gamma_T^2\left(1+256/\gamma_T^2-8\mu_T^2/9\right)/64 \quad (11)$$

Once β_{0T} has been determined, the tail rotor thrust can be calculated as

$$T_T = \frac{1}{2}\pi\rho s a_{0T}\Omega_T^2R_T^4\left(\left(\left(\lambda_\beta^2/\gamma\right)_T\beta_{0T}+\theta_{0T}\right)\left(1/+3\mu_T^2/2\right)/3+(\mu_{zT}-\lambda_{0T})/2\right) \quad (12)$$

2.2 The Force and Moment Generation Process

The basic helicopter subsystems, which have contributions in the force and moment generation, are the main rotor, tail rotor, fuselage, vertical fin and horizontal tailplane [1], [2], [5] . In hover or forward flight with slow velocity, the velocity is so slow that we can ignore the drag contributed from the horizontal, vertical stabilizers, and the fuselage , hence the eternal wrench can be written as [5]

$$f^b = \begin{pmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix} \tag{13}$$

$$\tau^b = \begin{pmatrix} L_M \\ M_M + M_T \\ N_M \end{pmatrix} + \begin{pmatrix} Y_M h_M + Z_M y_M + Y_T h_T \\ -X_M h_M + Z_M l_M \\ -Y_M l_M - Y_T l_T \end{pmatrix} \tag{14}$$

where *X, Y, Z* are the force components along the body axes, *L, M, N* are the moment components, and the subscripts *M* and *T* stand for the contributions of the main and tail rotors respectively. The forces and moments can be expressed as

$$X_M = -T_M \sin\beta_{1c}\ ,\ Y_M = T_M \sin\beta_{1s},\ Z_M = -T_M \cos\beta_{1c}\cos\beta_{1s},\ Y_T = T_T \tag{15}$$

$$L_M = \frac{\partial L_M}{\partial \beta_{1s}}\beta_{1s} - Q_M \sin\beta_{1c}\ ,\ M_M = \frac{\partial M_M}{\partial \beta_{1c}}\beta_{1c} + Q_M \sin\beta_{1s},\ M_T = -Q_T \tag{16}$$

$$N_M = -Q_M \cos\beta_{1c}\cos\beta_{1s} \tag{17}$$

where the rotor torque equations are approximated in [5] as $Q_i \approx C_i^Q T_i^{1.5} + D_i^Q$ for $i = M,T$ and the constants C_i^Q, D_i^Q are defined in Table 1.

The external torque τ^b can be written in the form [3]

$$\tau^b = A(T_M)v + B(T_M) \qquad ,\ v = (\beta_{1c} \quad \beta_{1s} \quad T_T) \tag{18}$$

in which $A(T_M)$ and $B(T_M)$ are , respectively, a matrix and a vector functions of the thrust T_M, whose coefficients depend on the geometry of the helicopter and on the coefficients characterizing the aerodynamic forces. Following [3] and [5] the overall control input is provided by the vector $(T_M \quad \beta_{1c} \quad \beta_{1s} \quad T_T)$ where T_M and T_T are the thrusts generated by the main rotor and tail rotor, respectively. The flapping angles β_{1c}, β_{1s} are the angles of inclination of the tip path plane of the main rotor about y^b axis and x^b axis, respectively.

2.3 The Rigid Body Dynamics

A mathematical model of the helicopter can be derived from Newton-Euler equations of motion of a rigid body which are applicable only in an inertial frame of reference, i.e., neither accelerating nor rotating [6]. Fix an inertial coordinate frame F^i in the Euclidean space, and fix a coordinate frame F^b attached to the body. Let $p^i = col(x, y, z) \in R^3$ denote the position of the center of mass of the rigid body with respect to the origin of F^i, and let $R \in SO(3)$ denote the rotation matrix mapping vectors expressed in F^b coordinates into vectors expressed in F^i coordinates [3].

The following equations describe the model of the helicopter

$$M\ddot{p}^i = Rf^b \tag{19}$$

$$J\dot{\omega}^b = -S(\omega^b)J\omega^b + \tau^b \tag{20}$$

$$\dot{q}_0 = -\frac{1}{2} q^T \omega^b \tag{21}$$

$$\dot{q} = \frac{1}{2}[q_0 I + S(q)]\omega^b \tag{22}$$

where $v^b \in R^3$ is the translational velocity of the center of mass of the body, $\omega^b \in R^3$ is its angular velocity, *M* and *J* are the mass and the inertia tensors of the body and *S(.)* is the skew symmetric matrix. The attitude of the helicopter is its orientation in free space represented by Euler angles or by the quaternions method. The angular velocity ω^b in Eqs. (20)- (22) is the rate of change of the Euler angles. The rotation matrix *R* is represented using the quaternions $\boldsymbol{q} = (q_0, q)$ where q_0 is the scalar part and $q = (q_1 \quad q_2 \quad q_3)$ is the vector part of the quaternion [7].

3. THE MAIN CONTROLLER

The goal of the control task is the design of an autopilot able to secure smooth autonomous landing of the helicopter on an oscillating platform while stabilizing also the lateral and horizontal position and maintaining a constant attitude. The considered setup represents a possible scenario in which a helicopter is required to perform a smooth landing on a deck of a ship which, due to high sea wave motion, is subject to large vertical oscillation and deviations. In general, uncertainties may affect the mechanical parameters of the air vehicle as well as its aerodynamical characteristics, and-in some cases-also the landing maneuver to be performed [8]. One of the main goals of the design is to let the center of mass of the helicopter asymptotically track, as accurately as possible, the reference motion

$$(x^{ref}(t), y^{ref}(t), z^{ref}(t)) = (0,0, H + z^*(t)) \tag{23}$$

where, referring to [3] and [8], *H* is a vertical offset and the vertical trajectory and $z^*(t)$ is modeled as the sum of a fixed number of sinusoidal signals, whose frequencies, amplitudes and phases are unknown constants, as

$$z^*(t) = \sum_{i=1}^{N} A_i \cos(\Omega_i t + \phi_i) \tag{24}$$

The constant reference attitude is $R^{ref}(t) = I$, which corresponds to the following possible choice of the quaternion

$$q_0^{ref}(t) = 1 \qquad\qquad q^{ref}(t) = (0,0,0)^T \tag{25}$$

The signal $z^{ref}(t)$ is generated by a linear time-invariant exosystem.

$$\dot{w} = S(\sigma) w \tag{26}$$

The exosystem depends on a vector σ of unknown parameters, assumed to range over a known compact set [4], and its initial condition *w(0)* is assumed to range over a known compact set *W* [8], namely

$$\sigma = col(\Omega_1, ..., \Omega_N)$$

$$S(\sigma) = diag(S_1, ..., S_N) \tag{27}$$

with

$$S_i = \begin{pmatrix} 0 & \Omega_i \\ -\Omega_i & 0 \end{pmatrix}, \qquad i = 1,...,N . \tag{28}$$

The expression for $z^{ref}(t)$ can now be given as [3]

$$z^{ref}(t) = H + r(w)$$
$$r(w)=Q\,w \tag{29}$$

where $Q = [Q_1 \quad Q_2 \quad ... \quad Q_N]$ with $Q_i = (1 \quad 0)$, $i=1,\dots,N$
and the vertical error and its derivative are defined as

$$e_z = z - z^{ref} \ , \ \ \dot{e}_z = \dot{z} - \dot{r} \tag{30}$$

The control objective can be divided into two tasks: the first is the synchronization of the vertical motion of the helicopter with that of the deck at a given distance *H*. Once synchronization has been achieved, the second task is to provide a smooth landing, letting the vertical offset *H* decay to zero[3].

3.1 Stabilization of the Vertical Error Dynamics

We have mentioned that the main rotor thrust T_M is used to control the vertical dynamics, and we choose the control law from [3] to choose the reference value T_M^* as

$$T_M^* = \frac{gM_0 - u}{1 - sat_c\left(2q_1^2 + 2q_2^2\right)} \tag{31}$$

where M_0 is the nominal value of the helicopter mass and the additional control input *u* is the sum of a stabilizing control and the output of an internal model, i.e. $u = u_{st} + u_{im}$. Consider, as an internal model, the system

$$\dot{\xi} = (F + G\hat{\Psi})\xi + Gu_{st} - FGM_0\dot{e}_z \tag{32}$$
$$u_{im} = \hat{\Psi}\xi \tag{33}$$

where

$$\xi = (\xi_1 \quad \xi_2)^T, \ \hat{\Psi} = (1 \quad \hat{\Psi}_2), \ G = (0 \quad G_2) \text{ and } F = \begin{pmatrix} 0 & H_2 \\ -G_2 & F_2 \end{pmatrix} \tag{34}$$

in which $\xi_1 \in R,\ \xi_2 \in R^{2N}$ and $\hat{\Psi}_2$ is a $1\times 2N$ row vector. In Eq. (33) F_2 is $2N \times 2N$ Hurwitz matrix, G_2 is $2N \times 1$ vector such that the pair *(F_2, G_2)* is controllable. H_2 is $1\times 2N$ matrix such that the pair *(F, G)* is controllable and the matrix *F* is Hurwitz. In case the vector σ is unknown, and this is already the case, we consider $\hat{\Psi}_2$ to be a vector of parameter estimates to be adapted, so that an adaptive controller can be implemented. The adaptation algorithm "tunes" a parameterized family of internal models to the one which reproduces the signal generated by the exosystem and the nonlinearities of the plant and we choose the update law [4]

$$\dot{\hat{\Psi}}_2 = -\gamma_2 \xi_2^T\left(\dot{e}_z + k_1 e_z\right) \tag{35}$$

with k_1 and γ_2 positive design parameters. The control law u is then completed choosing the high-gain stabilizing feedback

$$u_{st} = -k_2(\dot{e}_z + k_1 e_z) \tag{36}$$

where $k_2 > 0$ is a design parameter.

To conclude this work, we must convert from the reference value of the thrust T_M^* to its equivalent value of the flapping angle β_0^* using the following relations which we have derived from [1]

$$C_T^* = T_M^* / \left(\pi\rho\Omega_R^2 R^4\right) \tag{37}$$

$$Q_{\theta_0^*} = \frac{1}{1/3 + \mu^2/2}\left(2C_T^*/a_0 s - \mu\left(\theta_{1s} + \dot{P}/2\Omega_R\right)/2 - \left(\mu_z - \lambda_0\right)/2 - \theta_{tw}\left(1+\mu^2\right)/4\right) \tag{38}$$

$$\beta_0^* = \frac{\gamma}{8\lambda_\beta}\left(Q_{\theta_0^*}\left(1+\mu^2\right) + 4\theta_{tw}\left(1/5 + \mu^2/6\right) + 4\theta_{1s}\mu/3 + 4\left(\mu_z - \lambda_0\right)/3\right) \tag{39}$$

3.2 Stabilization of the Attitude Dynamics

In this section, we deal with the problem of the proper design of $v^* = \left(\beta_{1c}^* \quad \beta_{1s}^* \quad T_T^*\right)$ to stabilize the attitude dynamics represented by $\boldsymbol{q}, \omega^b$. First of all, we use a preliminary control law which is meant to remove the nominal part of $B(T_M)$ from Eq. (18), i.e., we choose

$$v^* = A_0(T_M)^{-1}[\tilde{v} - B_0(T_M)] \tag{40}$$

in which $\tilde{v}$ is an additional control input chosen as

$$\tilde{v} = -k_4\omega^b - k_4 k_3 q + k_4 k_3 u_2 \tag{41}$$

where $k_3 > 0$ and $k_4 > 0$ are design parameters, and u_2 is an additional control input which is assumed to be bounded by a positive number λ_2, i.e.,

$$\|u_2(t)\| \le \lambda_2 \qquad \text{for all } t \ge 0 \tag{42}$$

The bound of Eq. (42) will be enforced by choosing u_2 as a saturated function of the x-y states as shown in [3]. The choice of $\tilde{v}$ in Eq. (42) is proved in [3] to stabilize the trajectories of Eqs. (20-22). As done in the previous section with T_M^* we give here the algebraic equations transferring from T_T^* to β_{0T}^*.

$$C_{T_T}^* = T_T^* / \left(\pi\rho\Omega_T^2 R_T^4\right) \tag{43}$$

$$Q_{\theta_{0T}^*} = \frac{3}{1 + 3\mu_T^2/2}\left(2C_{T_T}^*/a_{0T} s_T - \left(\mu_{zT} - \lambda_{0T}\right)/2\right) \tag{44}$$

$$K_{T1} = \left(1 + 256/\gamma_T^2\right)\left(1 + \mu_T^2\right) - 16\mu_T^2/9 \tag{45}$$

$$K_{T2} = 4\left(\mu_{zT} - \lambda_{0T}\right)\left(1 + 256/\gamma_T^2 - \mu_T^2\right)/3 \tag{46}$$

$$\beta_{0T}^* = \frac{-K_{T1}Q_{\theta_{0T}^*} - K_{T2}}{K_{T2} - K_{T1}\left(\lambda_\beta^2/\gamma\right)_T} \tag{47}$$

3.3 Stabilizing the Lateral and Longitudinal Dynamics

The goal is now the deign of u_2 in order to stabilize the interconnected system between the attitude and the lateral/longitudinal dynamics. The controller will be designed using q_1 and q_2 as virtual controls for the x-y dynamics, with the states $(x, \dot{x}, y, \dot{y})$, and then propagating the resulting control law through the attitude dynamics, with the states (q_0, q, ω^b). To remove drifts in the lateral and longitudinal position due to any constant bias, we begin by augmenting the system dynamics with the bank of integrators[3]

$$\dot{\eta}_x = x \qquad \dot{\eta}_y = y \qquad \dot{\eta}_q = q_3 . \tag{48}$$

Consider the following states, which we group in a manner suitable for explaining the choice of the control law

$$x_1 = \begin{pmatrix} \eta_y \\ \eta_x \end{pmatrix} \qquad x_2 = \begin{pmatrix} y \\ x \end{pmatrix} \qquad x_3 = \begin{pmatrix} \dot{y} \\ \dot{x} \\ \eta_q \end{pmatrix} \tag{49}$$

To ensure the stability of this system we choose the nested saturated control law given in [3] which can be written as

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 + \lambda_1 \sigma(\frac{k_1 z_1}{\lambda_1}) \\ z_3 &= x_3 + \lambda_2 \sigma(\frac{k_2 z_2}{\lambda_2}) \\ u_2 &= -p_2 \lambda_3 \sigma(\frac{k_3 z_3}{\lambda_3}) \end{aligned} \tag{50}$$

where λ_i, k_i, $i = 1,2,...,n$ are positive design parameters, the saturation functions $\sigma(.)$, on which the control law u_2 described below is based, are defined in [3],[9]and $p_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ [3] .

4. THE AUXILIARY CONTROLLER

The task of this section is to design an auxiliary controller to determine the values of the control angles $\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0T}$ in order to enforce the rotor states $\beta_0, \beta_{1c}, \beta_{1s}, \beta_{0T}$ to follow the values of their reference variables $\beta_0^*, \beta_{1c}^*, \beta_{1s}^*, \beta_{0T}^*$ output from the main controller. Let $\beta_R^* = \begin{pmatrix} \beta_0^* & \beta_{1c}^* & \beta_{1s}^* \end{pmatrix}$ and define $\tilde{\beta}_R$ as the error between the state vector β_R and the reference β_R^* such that $\tilde{\beta}_R = \beta_R - \beta_R^*$. The error dynamics can be derived using Eq. (1) as

$$\dot{\tilde{\beta}}_R = \dot{\beta}_R - \dot{\beta}_R^* = A_R \tilde{\beta}_R + A_R \beta_R^* + B_R \Theta + C_R - \dot{\beta}_R^* \tag{51}$$

where $A_R = \Omega_R A_\beta, B_R = \Omega_R B_\beta, C_R = \Omega_R C$.

Let the control angles $\Theta = \theta_1 + \theta_2$ and choose

$$\theta_1 = B_R^{-1}\left(-A_R\beta_R^* - C_R + \dot{\beta}_R^*\right) \quad ,\theta_2 = -K\tilde{\beta}_R \tag{52}$$

This results in the error dynamics to be

$$\dot{\tilde{\beta}}_R = (A_R - B_R K)\tilde{\beta}_R \tag{53}$$

The gain matrix K has been designed to make $\tilde{\beta}_R \to 0$ and $\beta_R \to \beta_R^*$.

The expression for θ_{0T} could be given similarly as

$$\theta_{0T} = B_T^{-1}\left(-A_T\beta_{0T}^* - C_T + \dot{\beta}_{0T}^*\right) - K_T\left(\beta_{0T} - \beta_{0T}^*\right) \tag{54}$$

To calculate the time derivative of the reference flapping angles $\dot{\beta}_R^*, \dot{\beta}_{0T}^*$ in Eqs. (52, 55) we use the following linearized relation [10]

$$\dot{\beta}^* = \frac{\beta_{t+1}^* - \beta_t^*}{\Delta t} \tag{55}$$

This concludes the design of the auxiliary controller.

5. SIMULATION RESULTS

We present in this section simulation results concerning the helicopter model presented in section 2. The nominal values of the plant parameters are given in Table 1. The oscillatory deck motion is assumed to be generated by a four-dimensional exosystem, with parameters $\sigma = (1, 1.5)$ and initial conditions $w(0) = (3,\ 1,\ 2,\ 3)$. The control parameters have been selected as in Table 2. The vertical bias has been chosen as 5m. The reported simulation refers to the vehicle initially at rest, with initial attitude and position given by q (0) = (0.98, 0.138, 0.138, 0) and (x(0), y(0), z(0)) = (10, 10, 10) meters respectively. Fig.3 shows the vertical error $e_z + 5$. Fig.4 shows the longitudinal and lateral displacements. Fig.5 shows the time history of the attitude parameters. The flapping angles and their reference star values are shown together in Fig.6 which shows the coincidence between the state and its desired trajectory. The four control angles are shown in Fig.7 and, finally, the thrusts and their references are given in Fig.8. The simulation of the flapping angles of the main and tail rotors in Fig.6 is an addition to the work in either [3] or [5] which neglected the important rotor dynamics and, hence, resulted in unreal control inputs compared with the four control angles we have simulated in Fig.7. The simulations indicate the effectiveness of the proposed controller.

6. CONCLUSIONS

We have presented an application of nonlinear robust regulation to the challenging problem of designing an autopilot for helicopters landing under uncertain conditions. The main controller is given by a vertical regulator yielding the main rotor thrust T_M^* and an attitude/lateral/longitudinal stabilizer computing the input vector $v^* = \left(\beta_{1c}^* \quad \beta_{1s}^* \quad T_T^*\right)$. The auxiliary controller was used to choose the real control vector $(\theta_0 \quad \theta_{1c} \quad \theta_{1s} \quad \theta_{0T})$ provided that each state of the rotors systems tends to its desired value output from the main controller. We have shown that, given arbitrary large compact sets of initial conditions, of uncertain model parameters and of data characterizing the vertical motion of the landing deck, it is possible to tune the design parameters in order to achieve the desired control objective. .

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LIST OF SYMBOLS

a_0, a_{0T} main and tail rotors blade lift curve slope

c rotor blade chord

C_T , C_{T_T} main and tail rotors thrust coefficient

f^b resultant force acting on the helicopter body

g acceleration due to gravity

h_M height of main rotor hub above fuselage reference point
h_T height of tail rotor hub above fuselage reference point
l_M distance of main rotor hub aft of fuselage reference point
l_T distance of tail rotor hub aft of fuselage reference point
p, q the angular velocity components of helicopter about fuselage *x*- and *y*-axis
R, R_T main and tail rotor blade radius
s, s_T main and tail rotors solidity
V flight speed
y_M offset of the main rotor hub from the fuselage reference point
$\beta_0, \beta_{1c}, \beta_{1s}$ the rotor blade coning, longitudinal and lateral flapping angles
β_{0T} the tail rotor coning angle
γ Lock number$= \rho c a_0 R^4 / I_\beta$
$\eta_0, \eta_{1c}, \eta_{1s}$ pilot's collective lever and cyclic stick positions
η_{0T} tail rotor control run variable
λ_0, λ_{0T} main and tail rotors uniform inflow velocity
λ_β flap frequency ratio
μ advance ratio $= V / \Omega_R R$
μ_T normalized velocity at tail rotor
μ_z velocity of the rotor hub along the hub/shaft *z*-axis (normalized by $\Omega_R R$)
μ_{zT} total normalized tail rotor inflow velocity
θ_0, θ_{0T} main and tail rotor collective pitch angles
θ_{1s}, θ_{1c} longitudinal and lateral cyclic pitch
θ_{tw} main rotor blade linear twist
ρ air density
τ^b resultant torque acting on the helicopter body
Ω_R, Ω_{TR} main and tail rotor speed speeds

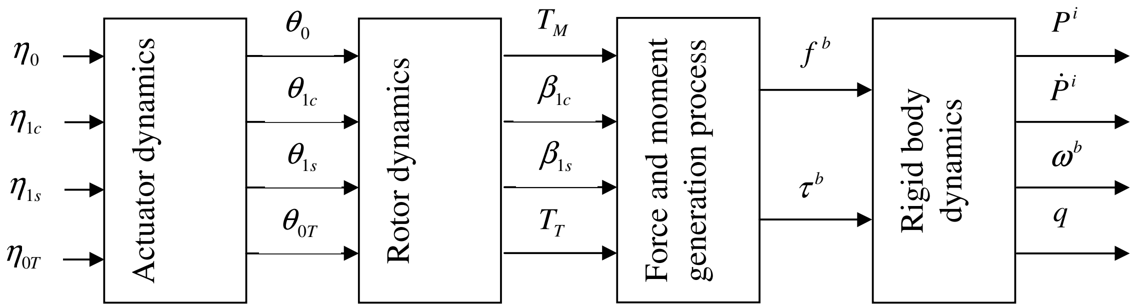

Fig.1. The helicopter dynamics

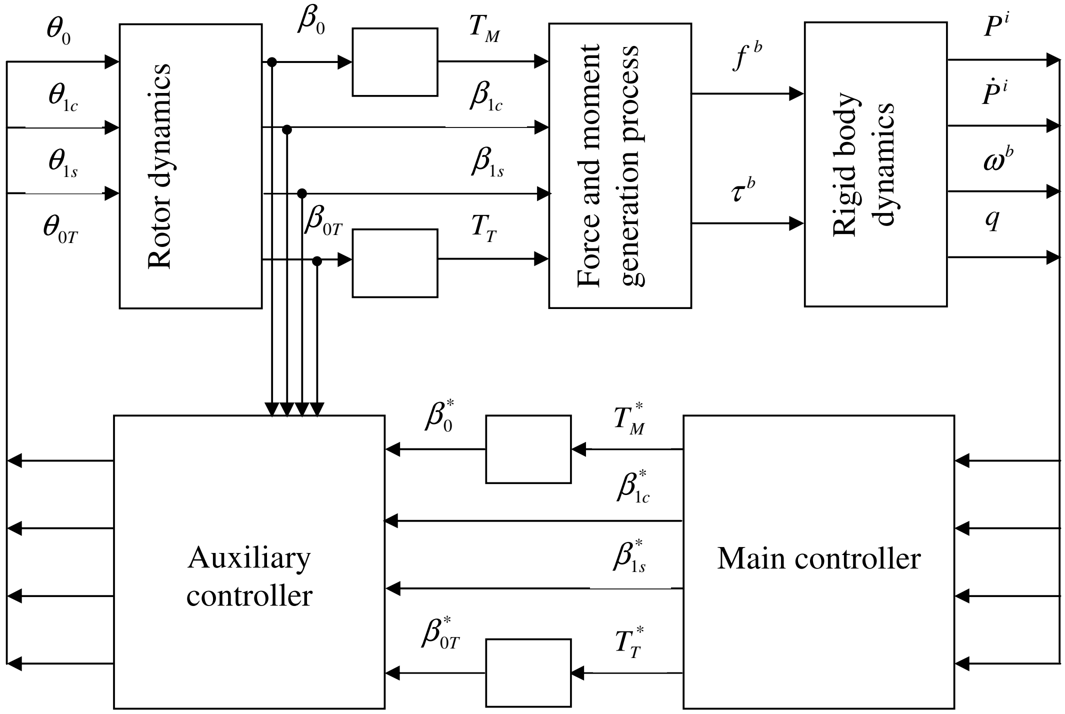

Fig.2. The overall closed loop model

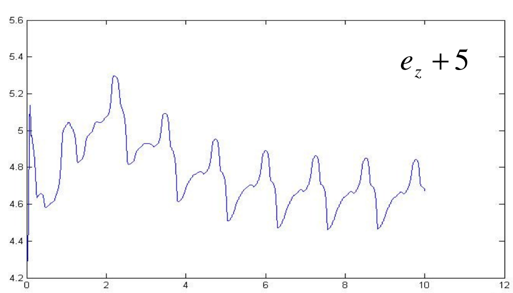

Fig.3. Tracking error

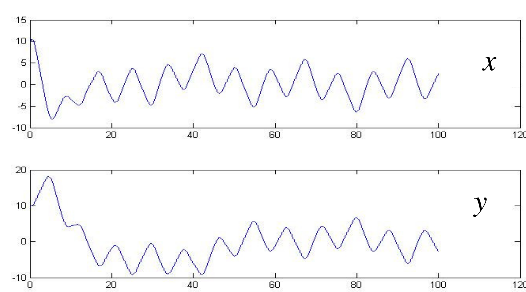

Fig.4. Longitudinal and lateral displacements

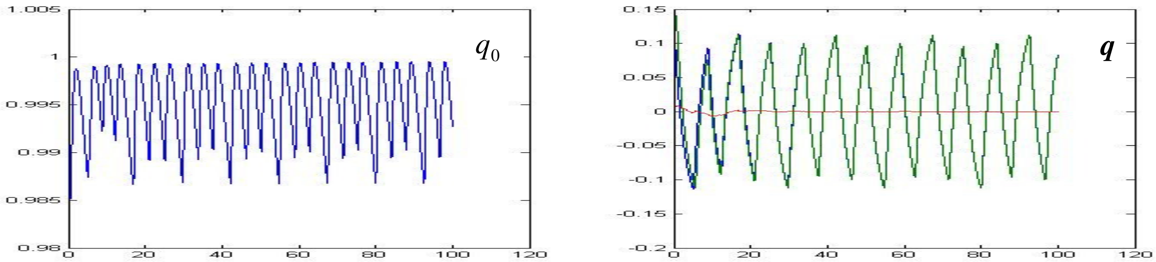

Fig.5. Quaternions

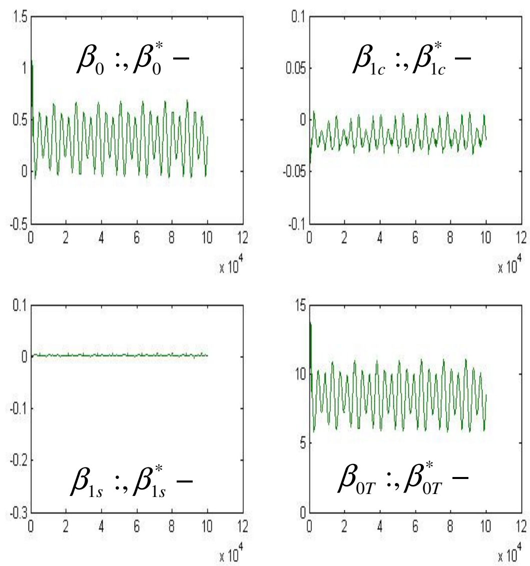

Fig.6 Flapping angles and their reference values

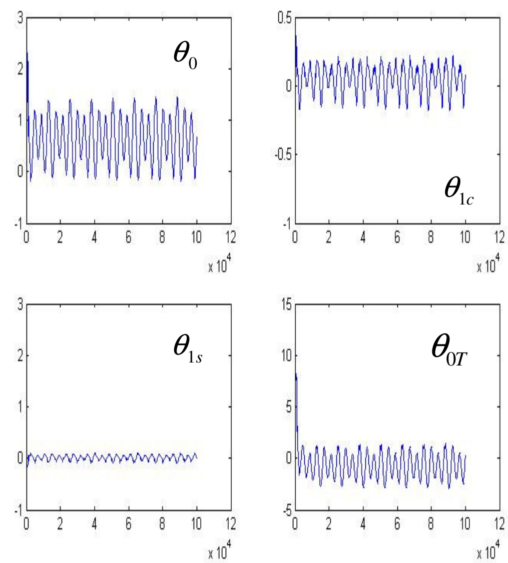

Fig.7. Control angles

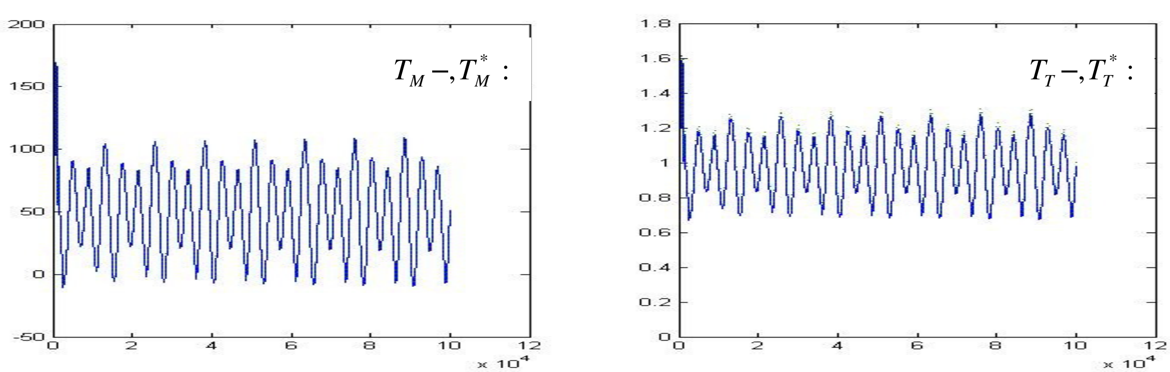

Fig.8. The main and tail rotor thrusts with their reference values

Table1. the parameters of the helicopter model [3], [5]

$h_m = 0.2943$	$h_t = 0.1154$	$\gamma = 0.712$	$\lambda_0 = 0.01$	$R_T = 0.2$	$\rho = 1.2$
$R = 0.5$	$\theta_{tw} = -0.14$	$\gamma_T = 0.02$	$\lambda_{0T} = 0.01$	$a_0 = 6$	$s = 0.77$
$C_M^Q = 0.004452$	$D_M^Q = 0.6304$	$\mu = 0.05$	$\mu_z = 0.05$	$a_{0T} = 2$	$s_T = 0.1$
$C_T^Q = 0.005066$	$D_T^Q = 0.008488$	$\mu_T = 0.033$	$\mu_{zT} = 0.05$	$\Omega_{TR} = 25$	$\Omega_R = 20$
$J_x = 0.142413$	$J_y = 0.271256$	$J_x = 0.271492$	$M = 4.9$	$\frac{\partial L_M}{\partial \beta_{1s}} = 25.23$	$\frac{\partial M_M}{\partial \beta_{1c}} = 25.23$

Table 2. Controller parameters [3]

Vertical dynamics	$k_1 = 0.1$	$K_2 = 45$	$\gamma_2 = 1$
Lat./long. dynamics	$K_0 = 0.09$	$K_1 = 0.081$	$K_2 = 0.75$
Attitude dynamics	$K_3 = 0.8$	$K_4 = 30$	$\varepsilon = 0.1$
Saturation levels	$\lambda_0 = 2000$	$\lambda_1 = 8.1$	$\lambda_2 = 0.295$
Auxiliary controller	$K = I$	$K_T = 1$	