

Equivalent Circuit of Induction Motor Referred to Stator Side ...

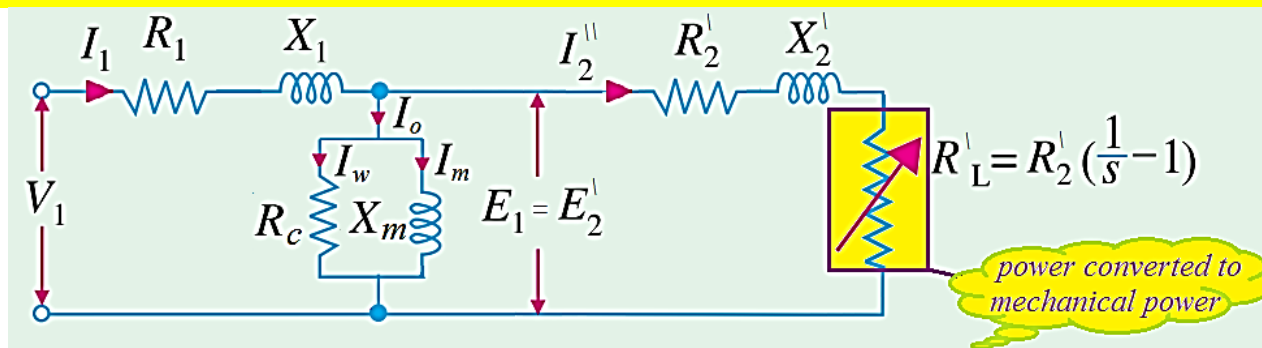


Fig. 3: eqt. circuit of induction motor per phase at any slip referred to stator side
(Fig. 3 is the same as Fig. 2)

Note that: the element R_L' is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

- At no-load (where $N_s \approx N$), the slip is practically zero and the load R_L' is infinite. This condition looks like a transformer whose secondary winding is open-circuited.
- At standstill ($N=0$), the slip is unity and the load R_L' is zero. This condition looks like a transformer whose secondary winding is short-circuited.
- When the motor is running under load, the value of R_L' will depend upon the value of the slip s . This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.
- R_L' depends on slip s . If the slip s increases, the load R_L' decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load.

Power Relations from Equivalent Circuit of Induction Motor

- (i) Stator input power : $P_{in} = 3V_1 I_1 \cos \phi_1$
- (ii) Stator Cu losses : $P_{cu(stator)} = 3I_1^2 R_1$
- (iii) Rotor input power : $P_{rotor} = \frac{3(I_2^{\prime\prime})^2 R_2^{\prime}}{s}$
- (iv) Rotor Cu losses : $P_{cu(rotor)} = 3(I_2^{\prime\prime})^2 R_2^{\prime}$ or $P_{cu(rotor)} = s P_{rotor}$
- (v) Total mechanical power developed by rotor : $P_{mech} = 3(I_2^{\prime\prime})^2 R_2^{\prime} \left(\frac{1}{s} - 1\right)$

$$\text{Or } P_{mech} = P_{rotor} - P_{cu(rotor)} = \frac{3(I_2^{\prime\prime})^2 R_2^{\prime}}{s} - 3(I_2^{\prime\prime})^2 R_2^{\prime} = 3(I_2^{\prime\prime})^2 R_2^{\prime} \left(\frac{1}{s} - 1\right)$$

- (vi) Total or gross torque developed by rotor :

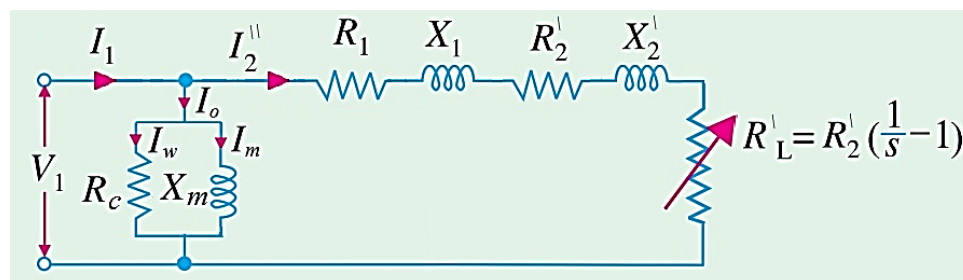
$$T_{gross} = \frac{P_{rotor}}{\omega_s} = \frac{3(I_2^{\prime\prime})^2 \frac{R_2^{\prime}}{s}}{\frac{2\pi N_s}{60}} = 9.55 \frac{3(I_2^{\prime\prime})^2 \frac{R_2^{\prime}}{s}}{N_s} = 9.55 \frac{P_{rotor}}{N_s}$$

N_s is the synchronous speed in rpm

Note: the shaft torque T_{sh} less than T_{gross} by friction and windage losses.

Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch ($R_c \parallel jX_m$) to the input terminals as shown in Figure



(Fig.: approximate eqt. circuit of IM per phase referred to stator side). This step has been taken on the assumption that voltage drop in R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1 . From the above approximate circuit, we note that:

- (i) Unlike a power transformer, the magnetic circuit of induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the transformer. Consequently, the exact equivalent circuit must be used for accurate results.
- (ii) The values of X_1 and X_2' in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit.
- (iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.
- (iv) Despite the above drawbacks of approximate equivalent circuit, this approximate circuit is satisfactory for large motors but not preferred for small motors.

Examples on Equivalent Circuit of Induction Motor

Example 34.55. A 3-phase, star-connected 400 V, 50-Hz, 4-pole induction motor has the following per phase parameters in ohms, referred to the stators.

$$R_1 = 0.15, X_1 = 0.45, R_2' = 0.12, X_2' = 0.45, X_m = 28.5$$

Compute the stator current and power factor when the motor is operated at rated voltage and frequency with $s = 0.04$. **(Elect. Machines, A.M.I.E. Sec. B, 1990)**

[Ans: $\bar{I}_1 = 71.058 \angle -21.47^\circ \text{ A}$, $pf = \cos \phi_1 = \cos 21.47^\circ = 0.93 \text{ lag}$]

Example 34.59. A 115V, 60-Hz, 3-phase, Y-Connected, 6-pole induction motor has an equivalent T-circuit consisting of stator impedance of $(0.07 + j 0.3) \Omega$ and an equivalent rotor impedance at standstill of $(0.08 + j 0.3) \Omega$. Magnetising branch has $G_o = 0.022 \text{ mho}$, $B_o = 0.158 \text{ mho}$. Find (a) secondary current (b) primary current (c) primary p.f. (d) gross power output (e) gross torque (f) input (g) gross efficiency by using approximate equivalent circuit. Assume a slip of 2% .

[Note: T-circuit means approximate circuit, $Y = G + jB$ & $Z = \frac{1}{Y}$]

[Ans: (a) $16.15 \angle -8.4^\circ \text{ A}$ (b) $21.66 \angle -36.38^\circ \text{ A}$ (c) 0.805 lag (d) $P_{gross} = 3I_2'^2 R_L' = 3067 \text{ Watt}$
 (e) 24.9 Nm (f) $P_{in} = 3473 \text{ Watt}$ (g) $\% \eta = \frac{P_{gross}}{P_{in}} \times 100 = 88.3\%$]

EXAMPLE 9.9 *A 3-phase, 400 V, 6-pole, 19 kW induction motor has the following parameters of its approximate circuit model.*

$$\begin{aligned}R_1 &= 1.4 \, \Omega & R'_2 &= 0.6 \, \Omega \\X_1 &= 2 \, \Omega & X'_2 &= 1 \, \Omega \\X_m &= 50 \, \Omega & f &= 50 \text{ Hz}\end{aligned}$$

The rotational loss is 275 W. For a slip of 0.03, determine:

- (a) the line current, pf and power input.*
- (b) the shaft torque, and mechanical output.*
- (c) the efficiency.*

ans: (a) 12.22A, 0.866Lag, 7330Watt.
(b) 62.8Nm, 6650Watt.
(c) 87%.

Methods of Starting 3-Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

- 1- Direct-on-line starting,
- 2- Stator resistance starting,
- 3- Autotransformer starting,
- 4- Star-delta starting,
- 5- Rotor resistance starting.

- Methods (1) to (4) are applicable to all induction motor types.
- Method (5) is applicable only to slip ring (wound rotor) motors.

In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon, the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

Methods of Starting 3-Phase Squirrel-cage Motor

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

- 1- Direct-on-line starting,
- 2- Stator resistance starting,
- 3- Autotransformer starting,
- 4- Star-delta starting,

(in all above methods, we control the applied voltage across **the stator**)

Starting Method Direct-on-line (DOL)

- The motor is started by connecting it directly to 3-phase supply at a rated voltage. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply, the starting current will be high (4 to 10 times the full-load current). Consequently, this method of starting is suitable for relatively small machines (up to 7.5kW).
- The high starting current in DOL method can be controlled by reducing the applied voltage but this way is not allowable here.
- Advantages:
 - simplest, cheapest and most common starting method.
 - lowest temperature rise at starting compared to other starting methods.
 - Motors that start and stop frequently often have control system, which consist of a contactor and overload protection such as a thermal relay.
- Disadvantages:
 - Full voltage is switched directly onto the motor terminals.
 - For small motors, T_{st} is (1.5:3 times) of T_{fl} . Also, I_{st} is (3:8 times) of I_{fl} or even higher.

Relation between T_{st} and T_{fl} for Direct-on-line (DOL) Method

- Rotor input power : $P_{rotor} = \omega_s \cdot T_{gross} = \frac{2\pi N_s}{60} T_{gross} = \text{const.} \cdot T_{gross} \rightarrow 1$
- Rotor Cu losses : $P_{cu(rotor)} = 3(I_2')^2 R_2 = s P_{rotor} \rightarrow \therefore P_{rotor} = \frac{3(I_2')^2 R_2}{s} \rightarrow 2$

From 1 & 2: $\text{const.} \cdot T_{gross} = \frac{3(I_2')^2 R_2}{s} \rightarrow T_{gross} \propto \frac{(I_2')^2}{s}$

Also, $T_{gross} \propto \frac{I_1^2}{s}$ ($I_2' \propto I_1$)

➤ At starting : $T_{st} \propto \frac{I_{st}^2}{s} \quad \because (s = 1 \text{ at starting}) \rightarrow T_{st} \propto I_{st}^2 \rightarrow 3$

➤ At full-load: $T_{fl} \propto \frac{I_{fl}^2}{s_{fl}} \rightarrow 4$

From 3 & 4: $\frac{T_{st}}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}$ at starting: the starting current I_{st} is the short-circuit (blocked-rotor) current I_{sc} .

Relation between T_{st} and T_{fl} for Direct-on-line (DOL) Method ...

$$\frac{T_{st}}{T_{fl}} = \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_{fl}$$

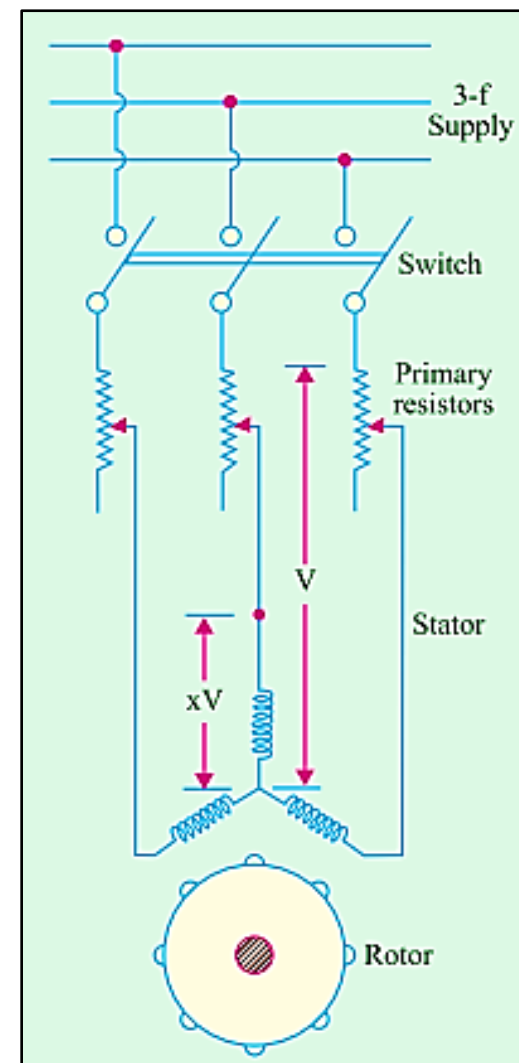
- Let us illustrate the above relation with a numerical example. Suppose $I_{sc} = 5I_{fl}$ and the full-load slip $s_{fl} = 0.04$. Then,

$$\frac{T_{st}}{T_{fl}} = \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_{fl} = \left(\frac{5I_{fl}}{I_{fl}} \right)^2 \times 0.04 = 1 \quad \rightarrow \quad \therefore T_{st} = T_{fl}$$

Note that the starting current is as large as five times the full-load current but the starting torque is just equal to the full-load torque. Therefore, the starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

Stator Resistance Starting Method

- In this method, external resistances (*primary resistors* or *rheostat* or *reactors*) are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor.
- There are two drawbacks.
 - First, the reduced voltage applied to the motor during starting reduces the starting torque. (*because the current varies directly with the applied voltage, but the torque varies as square of the applied voltage*). For example, if the applied voltage is reduced by 50%, the starting current is reduced by 50%, but the torque is reduced by 25% of the full-voltage value.
 - Secondly, a lot of power is wasted in the starting resistances.



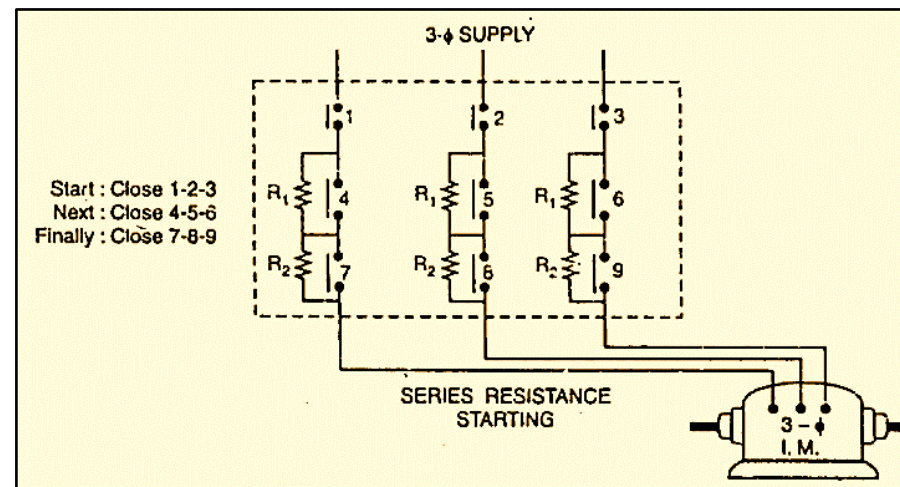
Stator Resistance Starting Method ...

- Operation of the used rheostat is shown in the Figure. The rheostat consists of ($R_1 + R_2$).

➤ Firstly: close the switches 1,2,3 (*i.e.*, close a 3-ph circuit breaker) to make the stator resistance has a high value of ($R_{\text{stator}} + R_1 + R_2$).

➤ Secondly: close the switches 4,5,6 to make the stator resistance has a less value of ($R_{\text{stator}} + R_2$).

➤ Thirdly: close the switches 7,8,9 (by keeping the switches 4,5,6 are closed) to make the stator resistance has a lower value of (R_{stator}).



This means that the stator current gradually increases (also the applied voltage is gradually increases) by a fraction x , where ($x > 1$).

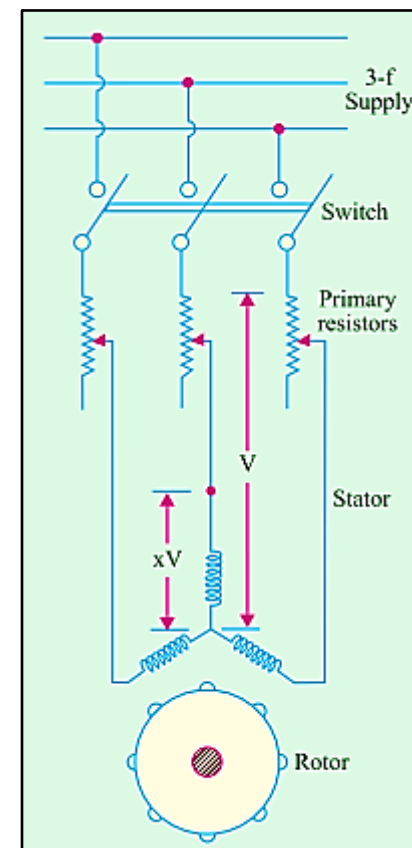
Relation between T_{st} and T_{fl} for Stator Resistance Starting Method

- Let (V) be the rated voltage/phase at the beginning of inserting the rheostat (i.e., instant of motor starting). After get out the rheostat from the circuit, then the voltage applied to the motor/phase will be (xV).

$$\because I_{st} = x I_{sc} \quad \text{Then,} \quad \frac{T_{st}}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}} \right)^2 s_{fl} = x^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_{fl}$$

Thus, at direct switching of the motor (the rheostat is inserted completely to the stator circuit) the current reduces by a fraction x of the rated-voltage starting current (I_{sc}), the starting torque is reduced by a fraction x^2 of that value obtained by the direct switching.

So, the reduced voltage applied to the motor during the direct switching period reduces the current but at the same time increases the accelerating time because of the reduced value of the torque. Therefore, this method is used for smooth starting of small motors only.



A 3- ϕ motor is designed to run at 5% slip on full-load. If motor draws 6 times the full-load current at starting at the rated voltage, estimate the ratio of starting torque to the full-load torque.

Ans.: **1.8**