SPC318: System Modeling and Linear Systems

Lecture 5: Closed Loop Control Systems
Block Diagram Reduction & Manipulation

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Lecture Outline:

1. Block Diagram Representation.
2. Block Diagram Reduction Techniques.
3. Superposition of Multiple Inputs.

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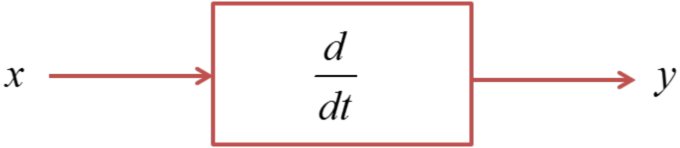
Block Diagram Representation:

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

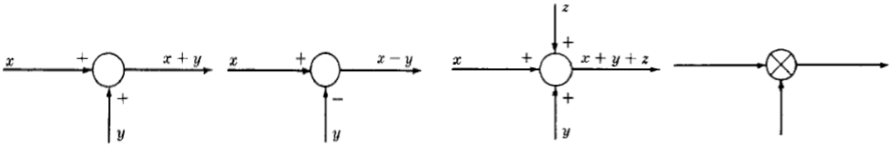


- The interior of the rectangle representing the block usually contains a description of or the name of the element, gain, or the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information or signal flow.

Block Diagram Representation:

Addition and Substraction:

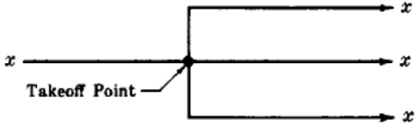
- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- The output is the algebraic sum of the inputs.
- Any number of inputs may enter a summing point.
- We may put a cross in the circle.

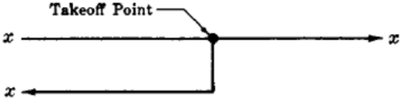


Block Diagram Representation:

takeoff (pickoff) point:

- In order to have the same signal or variable be an input to more than one block or summing point, a takeoff (or pickoff) point is used.
- This permits the signal to proceed unaltered along several different paths to several destinations.





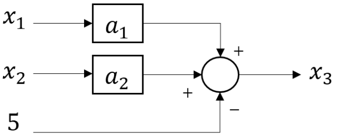
Block Diagram Representation:

Examples:

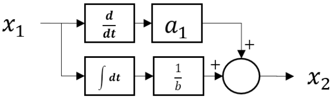
Example 1

Draw the block diagram for the following equations:

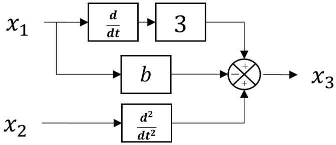
1. $x_3 = a_1 x_1 + a_2 x_2 - 5$
2. $x_2 = a_1 \dfrac{dx_1}{dt} + \dfrac{1}{b} \int x_1 dt$
3. $x_3 = a_1 \dfrac{d^2 x_2}{dt^2} + 3 \dfrac{dx_1}{dt} - b x_1$



(1)



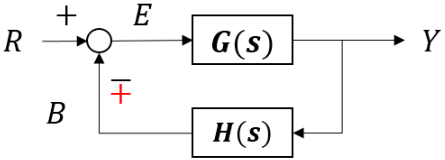
(2)



(3)

Block Diagram Representation:

Canonical Form of A Feedback Control System:



Forward transfer function $= G(s)$

Feedback transfer function $= H(s)$

Open-loop transfer function $= G(s)H(s)$

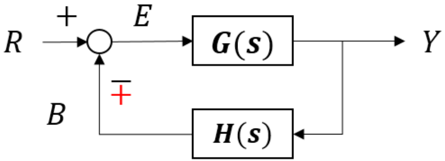
Closed-loop transfer function $= \dfrac{Y(s)}{R(s)} = \dfrac{G(s)}{1 \pm G(s)H(s)}$ (- for +ve feedback) (+ for -ve feedback)

Error ratio $= \dfrac{E(s)}{R(s)} = \dfrac{1}{1 \pm G(s)H(s)}$

Primary feedback ratio $= \dfrac{B(s)}{R(s)} = \dfrac{G(s)H(s)}{1 \pm G(s)H(s)}$

Block Diagram Representation:

Canonical Form of A Feedback Control System:



Closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

The denominator of closed loop transfer function determines the **characteristic equation of the system**.

$$\boxed{1 \pm G(s)H(s) = 0} \qquad \text{characteristic equation}$$

Block Diagram Representation:

Canonical Form of A Feedback Control System:

Example:

Forward transfer function $= G(s) = \dfrac{K}{(1+K)s+1}$

Feedback transfer function $= H(s) = 0.1$

Open-loop transfer function $= G(s)H(s) = \dfrac{0.1K}{(1+K)s+1}$

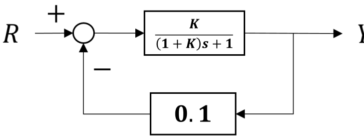
Closed-loop transfer function $=$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{(1+K)s+(1+0.1K)}$$

Characteristic equation: $1+G(s)H(s)=0$ $\boxed{(1+K)s+1+0.1K=0}$

Open loop poles and zeros if $K=10$: (no zero) $(p_o = -\dfrac{1}{1+K})$

Closed loop poles and zeros if $K=10$: (no zero) $(p_c = -\dfrac{1+0.1K}{1+K})$



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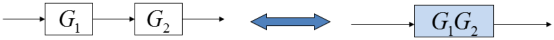
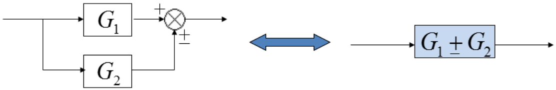
Block Diagram Reduction Techniques:

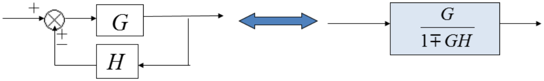
1. Combining blocks in cascade:

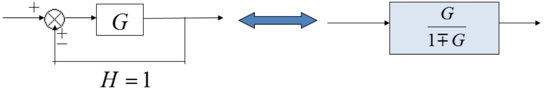


2. Combining blocks in parallel:



3. Eliminating a feedback loop:

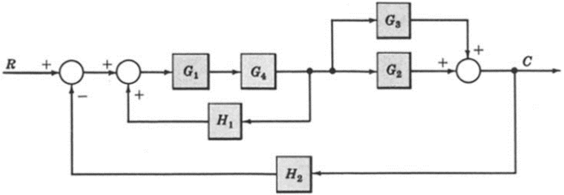




Block Diagram Reduction Techniques:

Example:

Reduce the Block Diagram to Canonical Form:

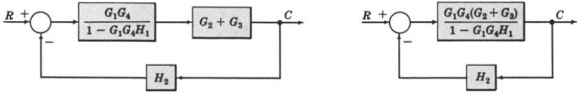


(1) Cascaded:

(2) Parallel:

(3) Feedback:

(4) Canonical Form:

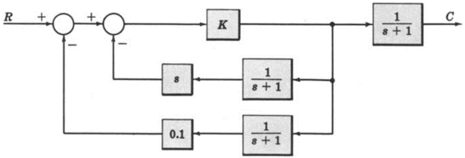


Block Diagram Reduction Techniques:

Lecture Assignment:

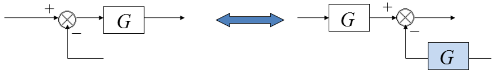
For the system represented by the following block diagram determine:

1. Open loop transfer function.
2. Feed Forward Transfer function.
3. Control ratio.
4. Feedback ratio.
5. Error ratio.
6. Closed loop transfer function.
7. Characteristic equation.
8. Closed loop poles and zeros if $K = 100$.

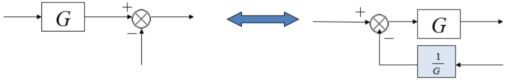


Block Diagram Reduction Techniques:

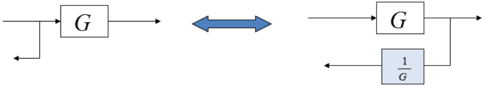
4. Moving a summing point after a block:



5. Moving a summing point before a block:

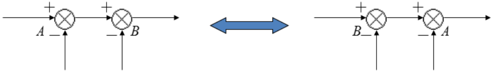


6. Moving a pickoff point after a block:



7. Moving a pickoff point before a block:

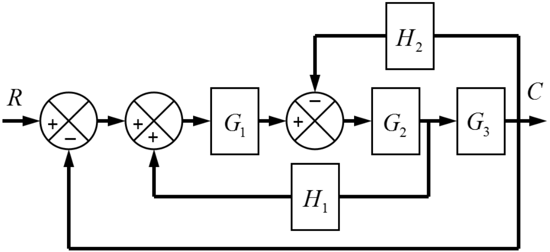
8. Swap with two neighboring summing points:



Block Diagram Reduction Techniques:

Example:

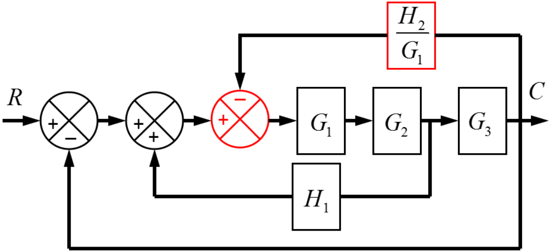
Reduce the Block Diagram to Canonical Form:



Block Diagram Reduction Techniques:

Example:

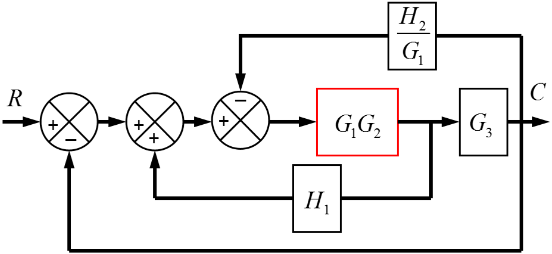
(1) Move summing point before a block:



Block Diagram Reduction Techniques:

Example:

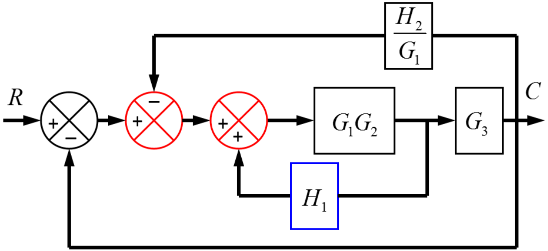
(2) Cascaded blocks:



Block Diagram Reduction Techniques:

Example:

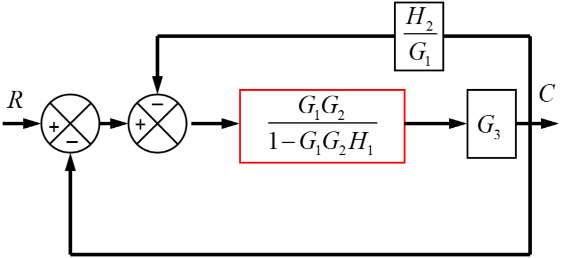
(3) Swap neighbor summing points:



Block Diagram Reduction Techniques:

Example:

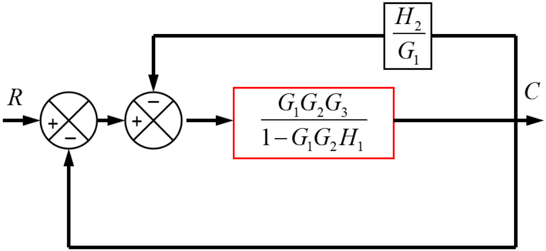
(4) Eliminate feedback:



Block Diagram Reduction Techniques:

Example:

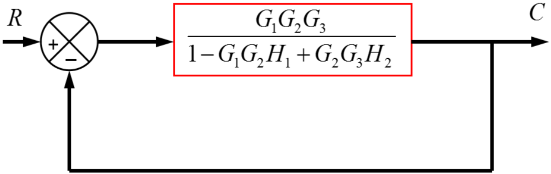
(5) Cascaded blocks:



Block Diagram Reduction Techniques:

Example:

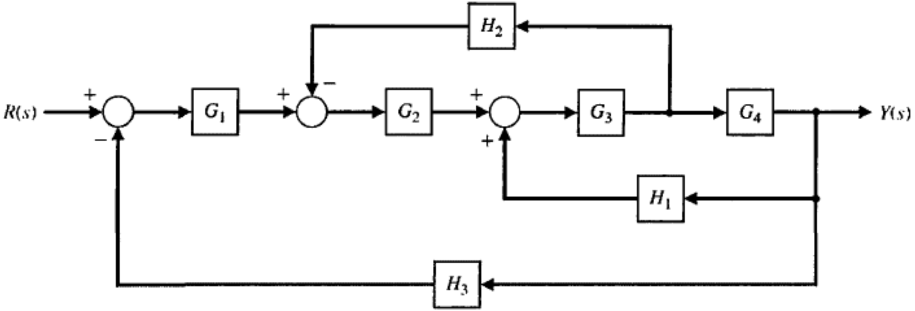
(6) Eliminate feedback:



Block Diagram Reduction Techniques:

Example:

Try it yourself!



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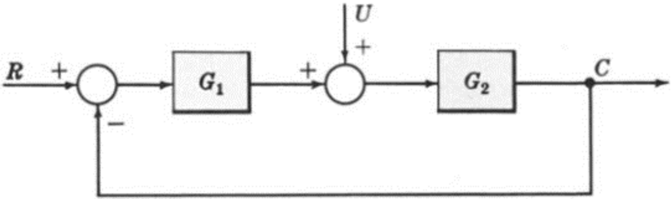
Superposition of Multiple Inputs:



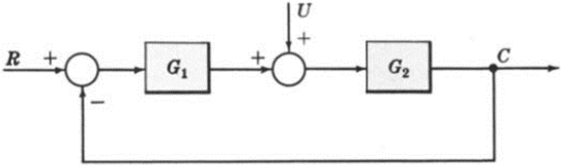
To find the total response in the case of **multiple inputs**:

1. Set all inputs except one equal to zero.
2. Transform the block diagram to canonical form, using the reduction techniques.
3. Calculate the response due to the chosen input acting alone.
4. Repeat step 1 to 3 for the other remaining inputs.
5. Algebraically add all of the responses (output) obtained in steps 1 to 4. This sum is the total output of the system with all inputs acting in the same time.

Superposition of Multiple Inputs:

Example:

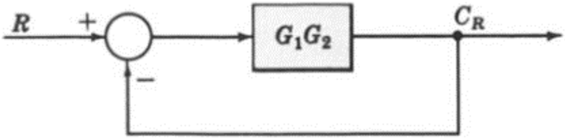
Determine the output C due to inputs R and U using the Superposition Method.



Superposition of Multiple Inputs:

Example:

(1) Put $U = 0$, the system reduces to:

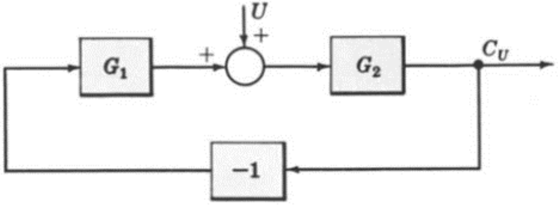


$$C_R = \frac{G_1 G_2}{1 + G_1 G_2} R$$

Superposition of Multiple Inputs:

Example:

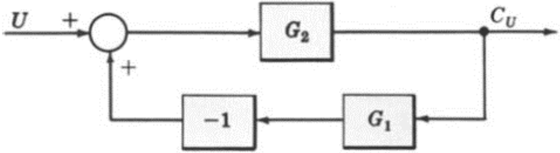
(2) Put $R = 0$, replace the negative feedback with -1 block:



Superposition of Multiple Inputs:

Example:

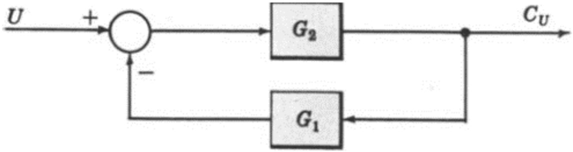
(3) Rearrange the block diagram:



Superposition of Multiple Inputs:

Example:

(4) Convert the -1 block into the summing point:

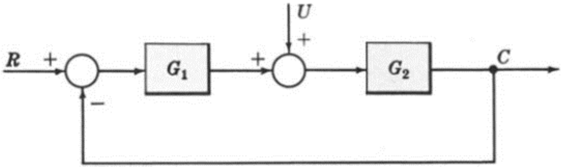


$$C_U = \frac{G_2}{1 + G_1 G_2} U$$

Superposition of Multiple Inputs:

Example:

(5) The total response is:



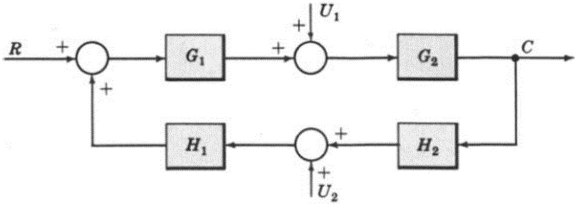
$$C = C_R + C_U = \frac{G_1 G_2}{1 + G_1 G_2} R + \frac{G_2}{1 + G_1 G_2} U$$

Superposition of Multiple Inputs:

Example:

Do it yourself!

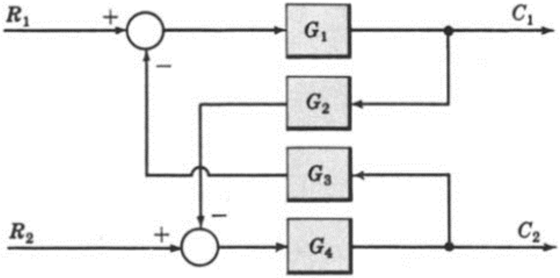
Find the total system response C due to inputs R,U_1 and U_2.



Superposition of Multiple Inputs Multiple Outputs:

Example:

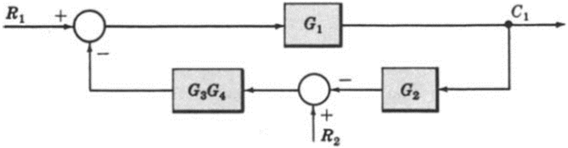
Determine the output C_1 and C_2 due to inputs R_1 and R_2 using the Superposition Method.



Superposition of Multiple Inputs:

Example:

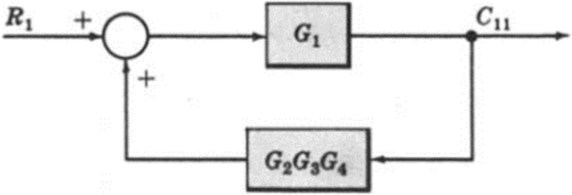
(1) Ignoring the output C_2:



Superposition of Multiple Inputs:

Example:

(2) Put $R_2 = 0$, combining the summing points:

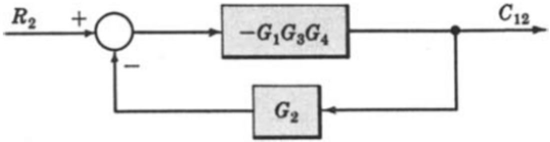


$$C_{11} = \frac{G_1 R_1}{1 - G_1 G_2 G_3 G_4}$$

Superposition of Multiple Inputs:

Example:

(3) Put $R_1 = 0$:



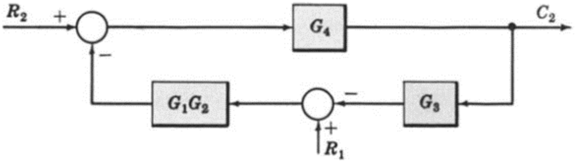
$$C_{12} = -\frac{G_1 G_3 G_4 R_1}{1 - G_1 G_2 G_3 G_4}$$

So, the total output $C_1 = C_{11} + C_{12}$

Superposition of Multiple Inputs:

Example:

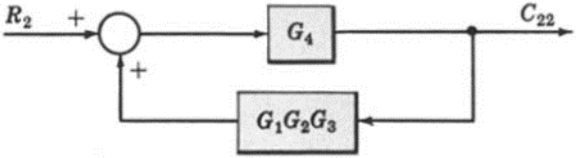
(4) From the original block ignore C_1:



Superposition of Multiple Inputs:

Example:

(5) Put $R_1 = 0$:

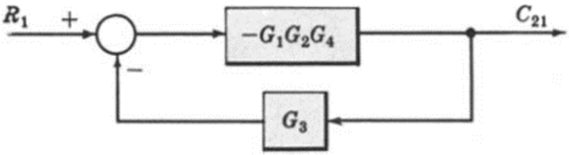


$$C_{22} = \frac{G_4 R_2}{1 - G_1 G_2 G_3 G_4}$$

Superposition of Multiple Inputs:

Example:

(5) Put $R_2 = 0$:



$$C_{21} = -\frac{G_1 G_2 G_4 R_1}{1 - G_1 G_2 G_3 G_4}$$

So, the total output $C_2 = C_{21} + C_{22}$

End of Lecture

Best Wishes

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