MICROWAVE FILTERS
What is a Microwave filter?

- Circuit that controls the frequency response at a certain point in a microwave system
- Provides perfect transmission of signal for frequencies in a certain passband region
- Infinite attenuation for frequencies in the stopband region
FILTER DESIGN METHODS

Filter Design Methods

Two types of commonly used design methods:
- Image Parameter Method
- Insertion Loss Method

• Image parameter method yields a usable filter

Filters designed using the image parameter method consist of a cascade of simpler two port filter sections to provide the desired cutoff frequencies and attenuation characteristics but do not allow the specification of a particular frequency response over the complete operating range. Thus, although the procedure is relatively simple, the design of filters by the image parameter method often must be iterated many times to achieve the desired results.
Filter Design by The Insertion Loss Method

The perfect filter would have zero insertion loss in the pass-band, infinite attenuation in the stop-band, and a linear phase response in the pass-band.

**Characterization by Power Loss Ratio**

\[
P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{\text{inc}}}{P_{\text{load}}} = \frac{1}{1 - |\Gamma(\omega)|^2}
\]

If both load and source are matched.

\[
P_{LR} = \frac{1}{|S_{12}|^2}
\]

The insertion loss (IL) is dB is

\[
IL = 10 \log P_{LR}
\]
Filter Design by The Insertion Loss Method

Characterization by Power Loss Ratio

\[
\Gamma(\omega) = \frac{Z(\omega) - Z_o}{Z(\omega) + Z_o} = \frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)}
\]

\[
\Gamma(-\omega) = \frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)} = \Gamma^*(\omega)
\]

\[
|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2
\]

(Even functions of \(\omega\))

\[
|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}
\]

Where \(M\) and \(N\) are real polynomials in \(\omega^2\).

\[
P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2} \rightarrow P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}
\]

The magnitude of the voltage gain
Filter Design by The Insertion Loss Method

Proof

\[
\Gamma(\omega) = \frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)} \quad \Gamma^*(\omega) = \frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)}
\]

\[
|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega)
\]

\[
\Gamma(\omega)\Gamma^*(\omega) = \left(\frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)}\right)\left(\frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)}\right)
\]

\[
\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) + Z_o)^2 + X^2(\omega)}
\]

\[
\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) + Z_o)^2 + X^2(\omega) - (R(\omega) - Z_o)^2 + (R(\omega) - Z_o)^2}
\]

\[
\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) - Z_o)^2 + X^2(\omega) + 4R(\omega)Z_o}
\]
Filter Design by The Insertion Loss Method

\[ P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)} \]

- Maximally Flat
  \[ P_{LR} = 1 + k^2 \left( \frac{\omega}{\omega_c} \right)^{2N} \]

- Equal Ripple
  \[ P_{LR} = 1 + k^2 T_N^2 \left( \frac{\omega}{\omega_c} \right) \]

- Linear Phase
  \[ \phi(\omega) = A\omega \left[ 1 + p \left( \frac{\omega}{\omega_c} \right)^{2N} \right] \]
Filter Design by The Insertion Loss Method

Practical filter response:

Maximally flat:
- also called the binomial or Butterworth response,
- is optimum in the sense that it provides the flattest possible passband response for a given filter complexity.

\[
P_{LR} = 1 + k^2 \left( \frac{\omega}{\omega_c} \right)^N
\]

Equal ripple also known as Chebyshev.
- sharper cutoff

\[
P_{LR} = 1 + k^2 T_N^2(\omega)
\]

\(\omega\) – frequency of filter
\(\omega_c\) – cutoff frequency of filter
\(N\) – order of filter
Filter Design by The Insertion Loss Method

• The insertion loss method (ILM) allows a systematic way to design and synthesize a filter with various frequency response.

• Design usually begins by designing a normalized low-pass prototype (LPP). The LPP is a low-pass filter with source and load resistance of $1\, \Omega$ and cutoff frequency of 1 Radian/s.

• Impedance transformation and frequency scaling are then applied to denormalize the LPP and synthesize different type of filters with different cutoff frequencies.

The process of filter design by the insertion loss method.

![Diagram of filter design process]
Low pass filter prototype, $N = 2$
\[ P_{LR} = 1 + k^2 \left( \frac{\omega}{\omega_c} \right)^{2N} \]

for \( N = 2 \), \( P_{LR} = 1 + \omega^4 \).

\[ P_{LR} = \frac{1}{1 - |\Gamma|^2} \qquad \Gamma = \frac{Z_{in} - 1}{Z_{in} + 1} \]

\[ P_{LR} = 1 + \frac{1}{4R} \left[ (1 - R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4 \right] \]

\[ L = C = \sqrt{2}. \quad = 1.4142 \]
Low Pass Filter Prototype – Ladder Circuit

For an arbitrary number of elements $N$. 

Ladder circuit for low pass filter prototypes and their element definitions. (a) begin with shunt element. (b) begin with series element.
\[ g_0 = \text{generator resistance, generator conductance.} \]

\[ g_k = \text{inductance for series inductors, capacitance for shunt capacitors.} \]

\[ (k=1 \text{ to } N) \]

\[ g_{N+1} = \text{load resistance if } g_N \text{ is a shunt capacitor, load conductance if } g_N \text{ is a series inductor.} \]
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<th>g_1</th>
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Element values for maximally flat LPF prototypes
Low Pass Filter Prototype – Maximally Flat

To determine the order (size) of the filter, it is usually determined by the specification of the insertion loss at some frequency in the stopband of the filter.

Attenuation versus normalized frequency for maximally flat filter prototypes.
Low Pass Filter Prototype – Equal Ripple

For an equal ripple low pass filter with a cutoff frequency $\omega_c = \ldots$

The power loss ratio is:

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$
## Element Values for Equal-Ripple LPF Prototype

\((g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10, 0.5\text{dB ripple})\)

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<th>N</th>
<th>(g_1)</th>
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Element values for equal ripple LPF prototypes (0.5 dB ripple level)
### The Insertion Loss Method

#### Element Values for Equal-Ripple LPF Prototype

\( g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10, \ 3.0 \text{dB ripple} \)

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Element values for equal ripple LPF prototypes (3.0 dB ripple level).
Attenuation versus normalized frequency for equal-ripple filter prototypes. (0.5 dB ripple level)
THE INSERTION LOSS METHOD

Attenuation versus normalized frequency for equal-ripple filter prototypes (3.0 dB ripple level)
Scaling and TRANSFORMATIONS
Scaling
Example 1: LPF

\[ L = C = \sqrt{2} = 1.4142 \]

for \( N = 2 \).

\[
\begin{array}{cccc}
N & g_1 & g_2 & g_3 & g_4 \\
1 & 2.0000 & 1.0000 & & \\
2 & 1.4142 & 1.4142 & 1.0000 & \\
3 & 1.0000 & 2.0000 & 1.0000 & 1.0000 \\
\end{array}
\]
Low Pass Filter Prototype – Impedance Scaling

\[ L' = R_0 L \]

\[ C' = \frac{C}{R_0} \]

\[ R'_s = R_0 \]

\[ R'_L = R_0 R_L \]
Frequency scaling for the low pass filter:

\[ \omega \leftarrow \frac{\omega}{\omega_c} \]

The new element values of the prototype filter:

\[ jX_k = j \frac{\omega}{\omega_c} L_k = j \omega L'_k \]

\[ jB_k = j \frac{\omega}{\omega_c} C_k = j \omega C'_k \]
The new element values are given by:

\[ L_k' = \frac{L_k}{\omega_c} \]

\[ C_k' = \frac{C_k}{\omega_c} \]
finally

The new element values are given by:

\[
L'_k = \frac{R_0 L_k}{\omega_c}
\]

\[
C'_k = \frac{C_k}{R_0 \omega_c}
\]
Example 1: LPF

\[ L = C = \sqrt{2}. \quad = 1.4142 \]

for \( N = 2 \).

\begin{array}{ccccc}
N & g_1 & g_2 & g_3 & g_4 \\
1 & 2.0000 & 1.0000 & \ & \ \\
2 & 1.4142 & 1.4142 & 1.0000 & \ \\
3 & 1.0000 & 2.0000 & 1.0000 & 1.0000 \\
\end{array}
TRANSFORMATIONS
Low-pass to high-pass transformation

\[ jX_k = -j \frac{\omega_c}{\omega} L_k = \frac{1}{j \omega C'_k}, \]

\[ jB_k = -j \frac{\omega_c}{\omega} C_k = \frac{1}{j \omega L'_k}, \]

\[ C'_k = \frac{1}{\omega_c L_k}, \]

\[ L'_k = \frac{1}{\omega_c C_k}. \]

\[ \omega \leftarrow -\frac{\omega_c}{\omega} \]

\[ C'_k = \frac{1}{R_0 \omega_c L_k}, \]

\[ L'_k = \frac{R_0}{\omega_c C_k}. \]
Low pass to high pass transformation

The frequency substitution:
\[
\omega \leftarrow - \frac{\omega_c}{\omega}
\]

The new component values are given by:
\[
C'_k = \frac{1}{R_0 \omega_c L_k}
\]
\[
L'_k = \frac{R_0}{\omega_c C_k}
\]
Low pass to Bandpass transformation

\[
\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{\Delta} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)
\]

Where,

\[
\Delta = \frac{\omega_2 - \omega_1}{\omega_0}
\]

The center frequency is:

\[
\omega_0 = \sqrt{\omega_1 \omega_2}
\]
The series inductor, $L_k$, is transformed to a series $LC$ circuit with element values:

$$L'_k = \frac{L_k}{\Delta \omega_0}$$

$$C'_k = \frac{\Delta}{\omega_0 L_k}$$

The shunt capacitor, $C_k$, is transformed to a shunt $LC$ circuit with element values:

$$L'_k = \frac{\Delta}{\omega_0 C_k}$$

$$C'_k = \frac{C_k}{\Delta \omega_0}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$
Low pass to Bandstop transformation

\[ \omega \leftarrow -\Delta \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} \]

Where,

\[ \Delta = \frac{\omega_2 - \omega_1}{\omega_0} \]

The center frequency is:

\[ \omega_0 = \sqrt{\omega_1 \omega_2} \]
The series inductor, $L_k$, is transformed to a parallel $LC$ circuit with element values:

\[
L_k' = \frac{\Delta L_k}{\omega_0}
\]

\[
C_k' = \frac{1}{\omega_0 \Delta L_k}
\]

The shunt capacitor, $C_k$, is transformed to a series $LC$ circuit with element values:

\[
L_k' = \frac{1}{\omega_0 \Delta C_k}
\]

\[
C_k' = \frac{\Delta C_k}{\omega_0}
\]
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<thead>
<tr>
<th>Low-pass</th>
<th>High-pass</th>
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<th>Bandstop</th>
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<td><img src="image1" alt="Low-pass L" /></td>
<td><img src="image2" alt="High-pass 1/ω_c L" /></td>
<td><img src="image3" alt="Bandpass L/ω_0 Δ" /></td>
<td><img src="image4" alt="Bandstop 1/ω_0 LΔ" /></td>
</tr>
<tr>
<td><img src="image1" alt="Low-pass C" /></td>
<td><img src="image2" alt="High-pass 1/ω_c C" /></td>
<td><img src="image3" alt="Bandpass Δ/ω_0 C" /></td>
<td><img src="image4" alt="Bandstop 1/ω_0 CΔ" /></td>
</tr>
</tbody>
</table>

**Formulas:**
- Low-pass: $L$
- High-pass: $\frac{1}{\omega_c L}$
- Bandpass: $\frac{L}{\omega_0 \Delta}$, $\frac{\Delta}{\omega_0 L}$
- Bandstop: $\frac{L\Delta}{\omega_0}$, $\frac{1}{\omega_0 L\Delta}$
Example 2:
Design a maximally flat low pass filter with a cutoff freq of 2 GHz, impedance of 50 Ω, and at least 15 dB insertion loss at 3 GHz. Compute and compare with an equal-ripple (3.0 dB ripple) having the same order.
Solution:
First find the order of the maximally flat filter to satisfy the insertion loss specification at 3 GHz.

We can find the normalized freq by using:

$$\left| \frac{\omega}{\omega_c} \right| - 1 = \left| \frac{3}{2} \right| - 1 = 0.5$$
Element Values for Maximally-Flat LPF Prototype

\((g_0 = 1, \omega_c = 1, N = 1 to 10)\)

<table>
<thead>
<tr>
<th>N</th>
<th>(g_1)</th>
<th>(g_2)</th>
<th>(g_3)</th>
<th>(g_4)</th>
<th>(g_5)</th>
<th>(g_6)</th>
<th>(g_7)</th>
<th>(g_8)</th>
<th>(g_9)</th>
<th>(g_{10})</th>
<th>(g_{11})</th>
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<td>0.9080</td>
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<td>1.4142</td>
<td>0.9080</td>
<td>0.3129</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

\[
g_1 = 0.618 \\
g_2 = 1.618 \\
g_3 = 2.0 \\
g_4 = 1.618 \\
g_5 = 0.618
\]
The ladder diagram of the LPF prototype to be used is as follow:

\[ L' = R_0 L \]
\[ C' = \frac{C}{R_0} \]
\[ R_s' = R_0 \]
\[ R_L' = R_0 R_L \]
\[ C_1 = \frac{g_1}{R_0 \omega_c} \]
\[ L_2 = \frac{R_0 g_2}{\omega_c} \]
\[ C_3 = \frac{g_3}{R_0 \omega_c} \]
\[ L_4 = \frac{R_0 g_4}{\omega_c} \]
\[ C_5 = \frac{g_5}{R_0 \omega_c} \]
LPF prototype for maximally flat filter

\[ C_1 = \frac{g_1}{R_0 \omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pF} \]

\[ L_2 = \frac{R_g}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nH} \]

\[ C_3 = \frac{g_3}{R_0 \omega_c} = \frac{2.00}{(50)(2\pi \times 2 \times 10^9)} = 3.183 \text{ pF} \]

\[ L_4 = \frac{R_g}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nH} \]

\[ C_5 = \frac{g_5}{R_0 \omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pF} \]
LPF prototype for **equal** ripple filter:

\[
C_1 = \frac{g_1}{R_0\omega_c} = \frac{3.4817}{(50)(2\pi \times 2 \times 10^9)} = 5.541 \text{ pF}
\]

\[
g_1 = 3.4817
\]

\[
L_2 = \frac{R_0 g_2}{\omega_c} = \frac{50 \times 0.7618}{(2\pi \times 2 \times 10^9)} = 3.031 \text{ nH}
\]

\[
g_2 = 0.7618
\]

\[
C_3 = \frac{g_3}{R_0\omega_c} = \frac{4.5381}{(50)(2\pi \times 2 \times 10^9)} = 7.223 \text{ pF}
\]

\[
g_3 = 4.5381
\]

\[
L_4 = \frac{R_0 g_4}{\omega_c} = \frac{50 \times 0.7618}{(2\pi \times 2 \times 10^9)} = 3.031 \text{ nH}
\]

\[
g_4 = 0.7618
\]

\[
C_5 = \frac{g_5}{R_0\omega_c} = \frac{3.4817}{(50)(2\pi \times 2 \times 10^9)} = 5.541 \text{ pF}
\]

\[
g_5 = 3.4817
\]
Example:
Design a **band pass** filter having a 0.5 dB equal-ripple response, with $N = 3$. The center frequency is 1 GHz, the bandwidth is 10%, and the impedance is 50 Ω.
Solution: The low pass filter (LPF) prototype ladder diagram is shown as follow:

\[ g_1 = 1.5963 = L_1 \]
\[ g_2 = 1.0967 = C_2 \]
\[ g_3 = 1.5963 = L_3 \]
\[ g_4 = 1.000 = R_L \]
Transforming the LPF prototype to the **BPF** prototype

\[ \Delta = 0.1 \quad N = 3 \quad \omega = 1 \text{ GHz} \]
\[L_1' = \frac{L_1Z_0}{\omega_0\Delta} = \frac{1.5963 \times 50}{(2\pi \times 1 \times 10^9 \times 0.1)} = 127.0\text{nH}\]

\[C_1' = \frac{\Delta}{Z_0\omega_0L_1} = \frac{0.1}{(50)(2\pi \times 2 \times 10^9 \times 1.5963)} = 0.199\text{pF}\]

\[L_2' = \frac{\Delta Z_0}{\omega_0C_2} = \frac{0.1 \times 50}{(2\pi \times 1 \times 10^9 \times 1.0967)} = 0.726\text{nH}\]

\[C_2' = \frac{C_2}{\omega_0\Delta Z_0} = \frac{1.0967}{(2\pi \times 1 \times 10^9)(0.1)(50)} = 34.91\text{pF}\]
\[ L'_3 = \frac{L_3 Z_0}{\omega_0 \Delta} = \frac{1.5963 \times 50}{(2\pi \times 1 \times 10^9 \times 0.1)} = 127.0 \text{nH} \]

\[ C'_3 = \frac{\Delta}{Z_0 \omega_0 L_3} = \frac{0.1}{(50)(2\pi \times 2 \times 10^9 \times 1.5963)} = 0.199 \text{pF} \]
Thank you