Linear Array Antenna

Smart antennas for wireless communication
(Gross F.B. - 2005 - McGraw-Hill)(Autosaved)

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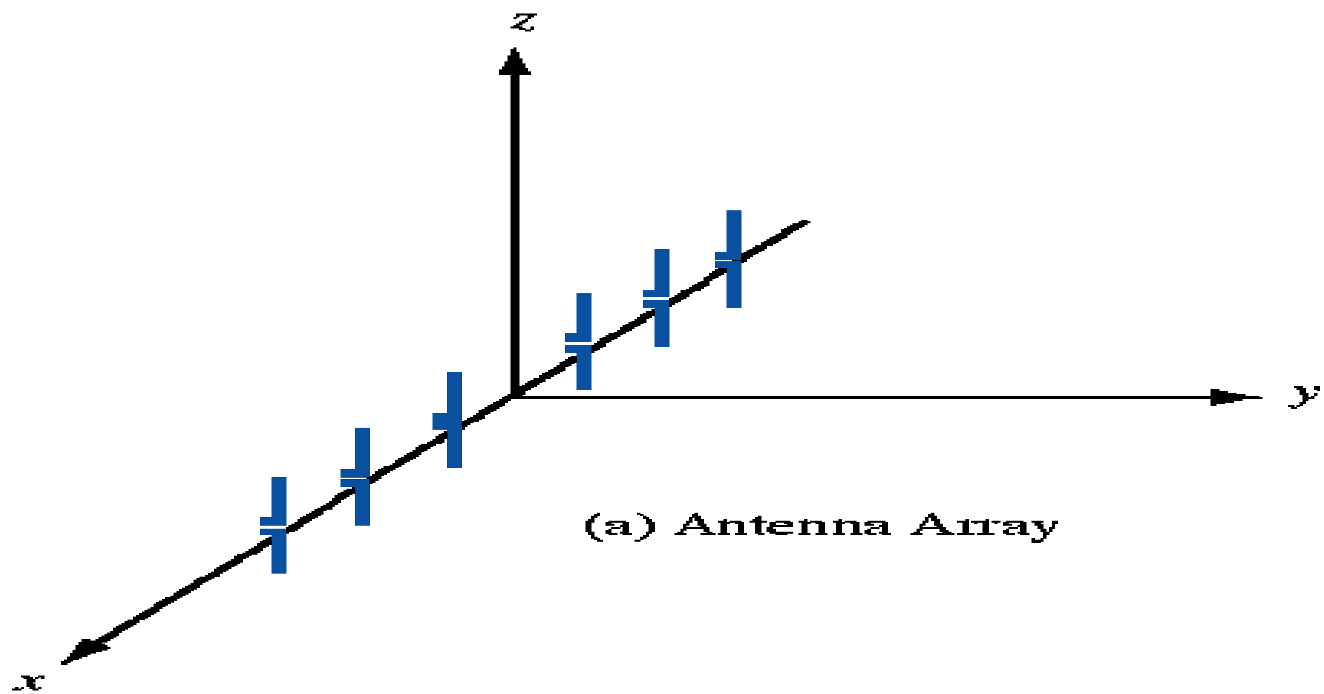
Linear Array Antenna

- The simplest array geometry is the linear array.
- All elements are aligned along a straight line.
- The minimum length linear array is the 2-element array.



(a) Antenna Array

Linear Arrays of n isotropic point sources of equal amplitude and spacing.

- General Case.
- Broadside arrays.
- Ordinary End-fire Arrays.
- Phased arrays.

Analyze is the two-element array

- two vertically polarized infinitesimal dipoles aligned along the *y axis and separated by a distance d.*
- The field point is located at a distance *r from the origin such that r >>d.*
- We can therefore assume that the distance vectors $\bar{r}_1$, $\bar{r}$, *and* $\bar{r}_2$ *are all approximately parallel to* each other.

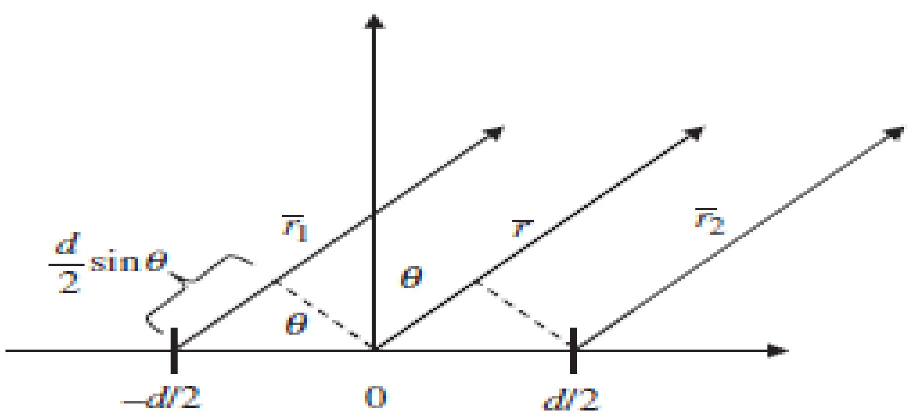


Figure 4.1 Two infinitesimal dipoles.

$r_1 \approx r + d\,2\sin\vartheta$ & $r_2 \approx r - d2\sin\vartheta$

assuming that $r_1 \approx r_2 \approx r$ *in the denominator, we can* now find the total electric field.

$$E_\theta = \frac{jk\eta I_0 e^{-j\frac{\delta}{2}} L\sin\theta}{4\pi r_1} e^{-jkr_1} + \frac{jk\eta I_0 e^{j\frac{\delta}{2}} L\sin\theta}{4\pi r_2} e^{-jkr_2}$$

$$= \frac{jk\eta I_0 L\sin\theta}{4\pi r} e^{-jkr} \left[e^{-j\frac{(kd\sin\theta+\delta)}{2}} + e^{j\frac{(kd\sin\theta+\delta)}{2}} \right]$$

where *δ = electrical phase difference between the two adjacent* elements

L = dipole length

ϑ = angle as measured from the z axis in spherical coordinates

d = element spacing

- the element factor is the far field equation for one dipole and the array factor is the pattern function associated with the array geometry.
- The distant field from an array of identical elements can always be broken down into the product of the element factor (EF) and the array factor (AF).
- the far field pattern of any array of antennas is always given

  by (EF) × (AF).

$$E_\theta = \underbrace{\frac{jk\eta I_0 L e^{-jkr}}{4\pi r} \sin\theta}_{\text{Element factor}} \cdot \underbrace{\left(2\cos\left(\frac{(kd\sin\theta + \delta)}{2}\right)\right)}_{\text{Array factor}}$$

- The AF is dependent on
- ❖ the geometric arrangement of the array elements
- ❖ the spacing of the elements "d"
- ❖ the electrical phase of each element "δ"

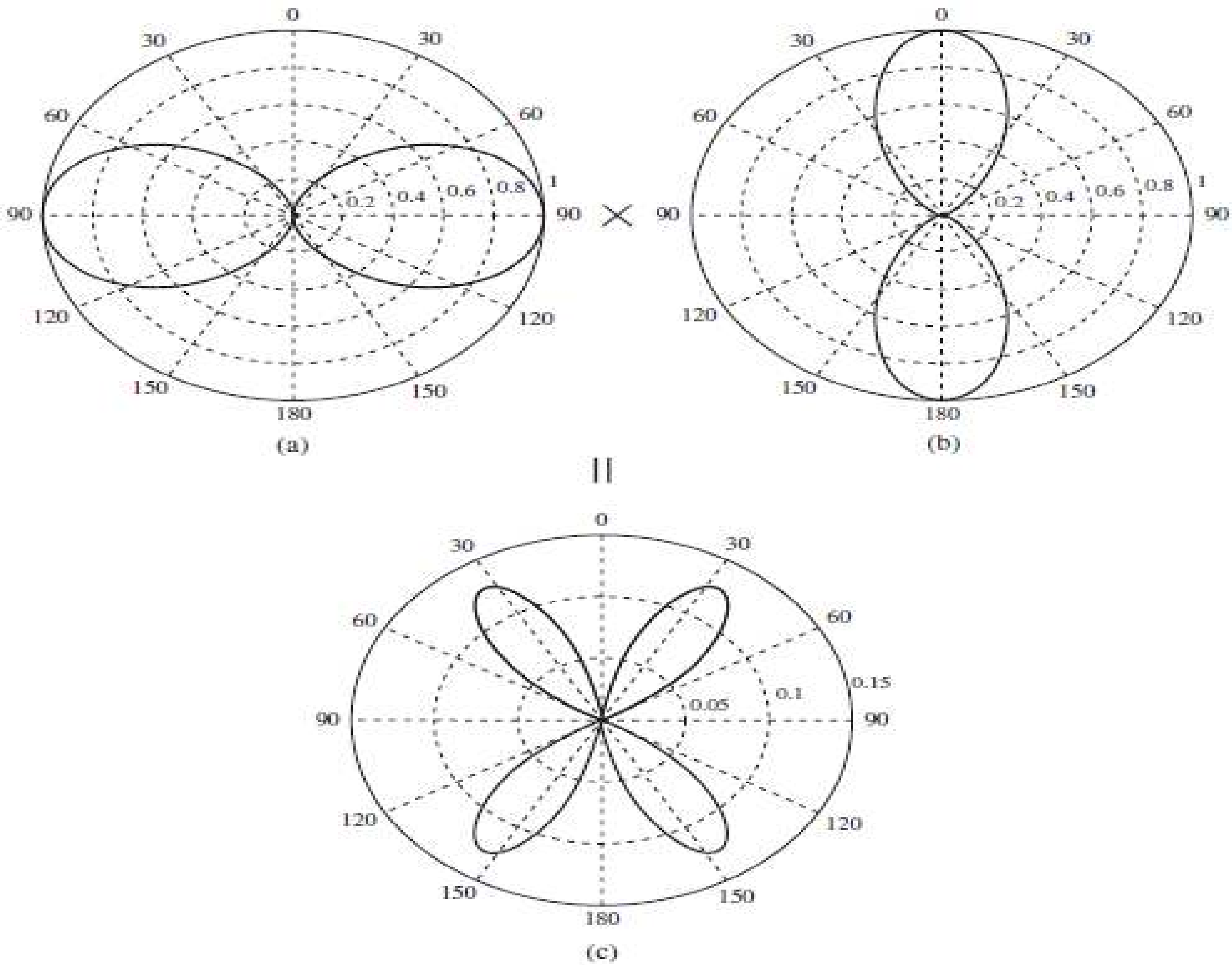


Figure 4.2 (*a*) Dipole pattern, (*b*) Array factor pattern, (*c*) Total pattern.

Linear Arrays of n isotropic point sources of equal amplitude and spacing

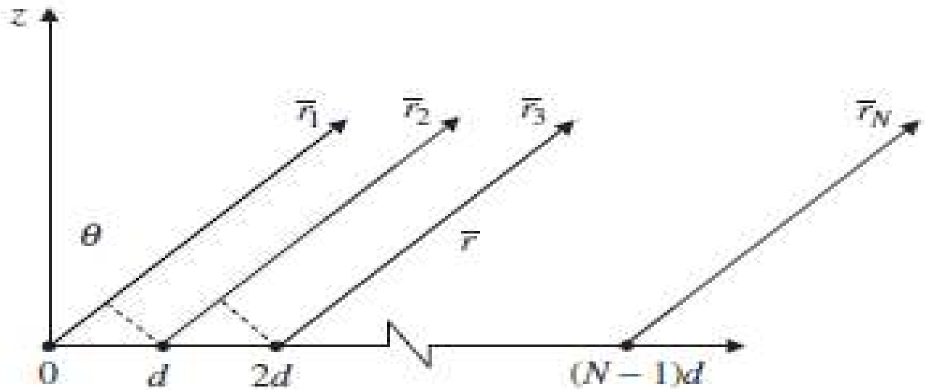


Figure 4.3 N-element linear array.

Assuming far field conditions such that $r \gg d$, we can derive the array factor as follows:

$$AF = 1 + e^{j(kd\sin\theta+\delta)} + e^{j2(kd\sin\theta+\delta)} + \cdots + e^{j(N-1)(kd\sin\theta+\delta)} \tag{4.6}$$

where δ is the phase shift from element to element.

This series can more concisely be expressed by

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\sin\theta+\delta)} = \sum_{n=1}^{N} e^{j(n-1)\psi} \tag{4.7}$$

where $\psi = kd\sin\theta + \delta$.

It should be noted that if the array is aligned along the z-axis then $\psi = kd\cos\theta + \delta$.

We may simplify the expression in Eq. (4.6) by multiplying both sides by $e^{j\psi}$ such that

$$e^{j\psi}\mathrm{AF} = e^{j\psi} + e^{j2\psi} + \cdots + e^{jN\psi} \tag{4.10}$$

Subtracting Eq. (4.6) from Eq. (4.10) yields

$$(e^{j\psi} - 1)\mathrm{AF} = (e^{jN\psi} - 1) \tag{4.11}$$

The array factor can now be rewritten.

$$\begin{aligned}\mathrm{AF} &= \frac{(e^{jN\psi} - 1)}{(e^{j\psi} - 1)} = \frac{e^{j\frac{N}{2}\psi}(e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi})}{e^{j\frac{\psi}{2}}(e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}})} \\ &= e^{j\frac{(N-1)}{2}\psi}\frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{\psi}{2})}\end{aligned} \tag{4.12}$$

The $e^{j\frac{(N-1)}{2}\psi}$ term accounts for the fact that the physical center of the array is located at $(N-1)d/2$. This array center produces a phase shift of $(N-1)\psi/2$ in the array factor. If the array is centered about the origin, the physical center is at 0 and Eq. (4.12) can be simplified to become

$$\mathrm{AF} = \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{\psi}{2})} \tag{4.13}$$

The maximum value of AF is when the argument $\psi = 0$. In that case $\mathrm{AF} = N$. This is intuitively obvious since an array of N elements should have a gain of N over a single element. We may normalize the AF to be reexpressed as

$$\mathrm{AF}_n = \frac{1}{N}\frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{\psi}{2})} \tag{4.14}$$

The maximum value of AF is when the argument *ψ = 0*

$$\mathrm{AF} = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}$$

$\psi = 0$ The function is indeterminate

$$\frac{\frac{d}{d\psi}\sin(n\psi/2)}{\frac{d}{d\psi}\sin(\psi/2)} = \frac{\frac{n}{2}}{\frac{1}{2}}\frac{\cos(n\psi/2)}{\cos(\psi/2)}$$

$\psi = 0$ Limit

An array of *N elements should* have a gain of *N over a single element.*

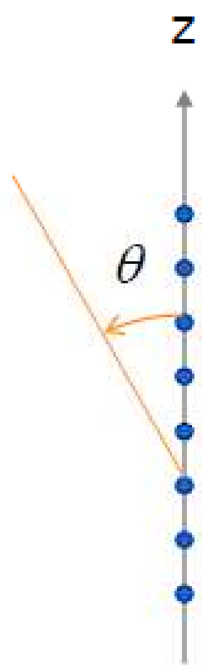
$$\mathbf{AF}_n = \frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}$$

The maximum will occur when $\psi = 0$

Broadside Array

Assume a linear array of n isotropic sources of the same amplitude and phase $\delta = 0$

$$\psi = \frac{2\pi d}{\lambda}\cos\theta + \delta = d_r \cos\theta + \delta \quad \text{———} \quad \psi = d_r \cos\theta$$

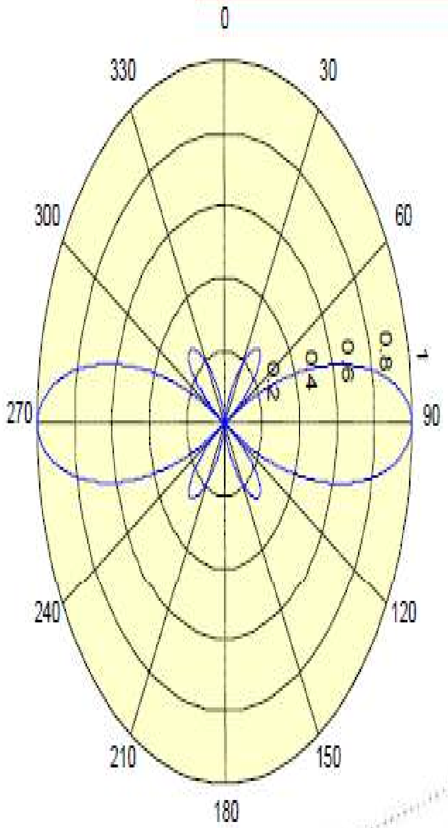


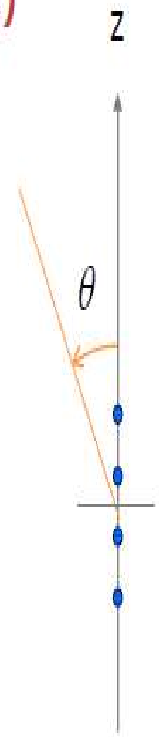
Maximum AF$_{max}$

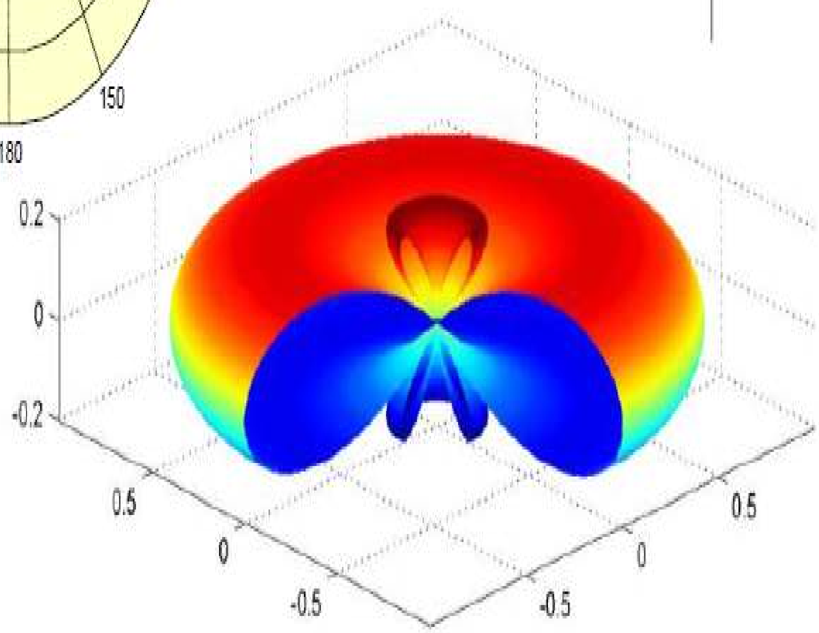
$$\psi = 0 \qquad \theta = \frac{(2k+1)\pi}{2}$$

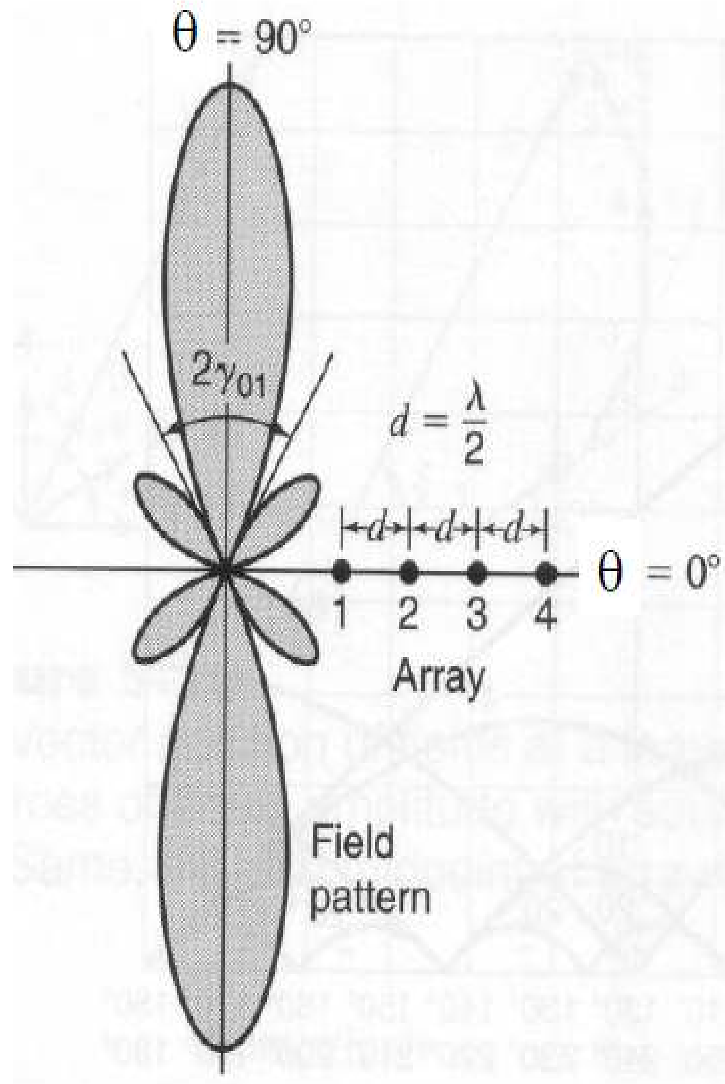
$$k = 0,1,2,3,\cdots \qquad \theta = \frac{\pi}{2} \;\&\; \frac{3\pi}{2}$$

Case 1. Broadside Array (Sources in Phase)







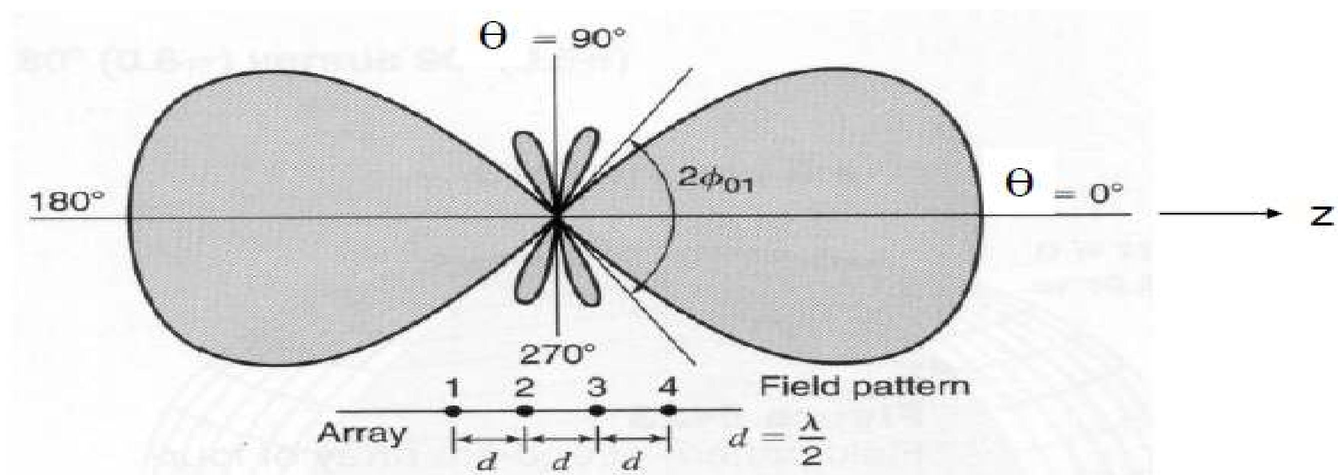


Case 2. Ordinary End-fire Array

Maximum $\theta = 0$

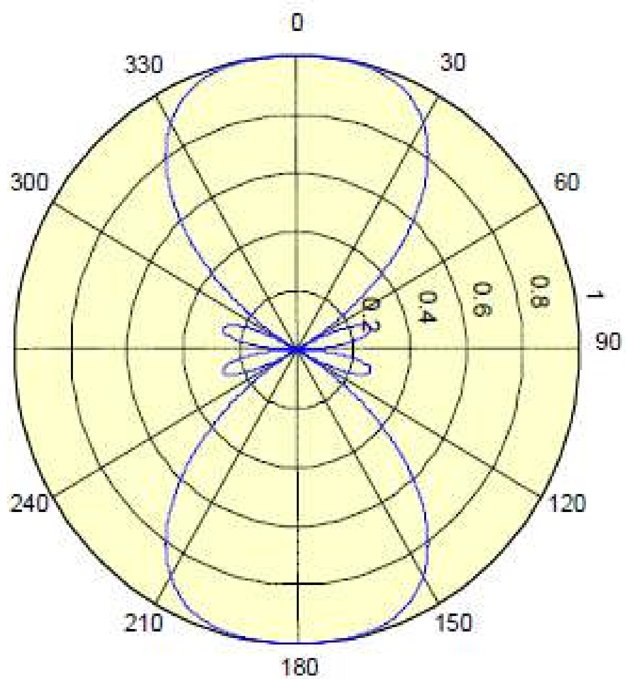
$$\psi = \frac{2\pi d}{\lambda} co\,\theta + \delta = d_r \cos\theta + \delta = d_r + \delta = 0 \longrightarrow \boxed{\delta = -d_r}$$

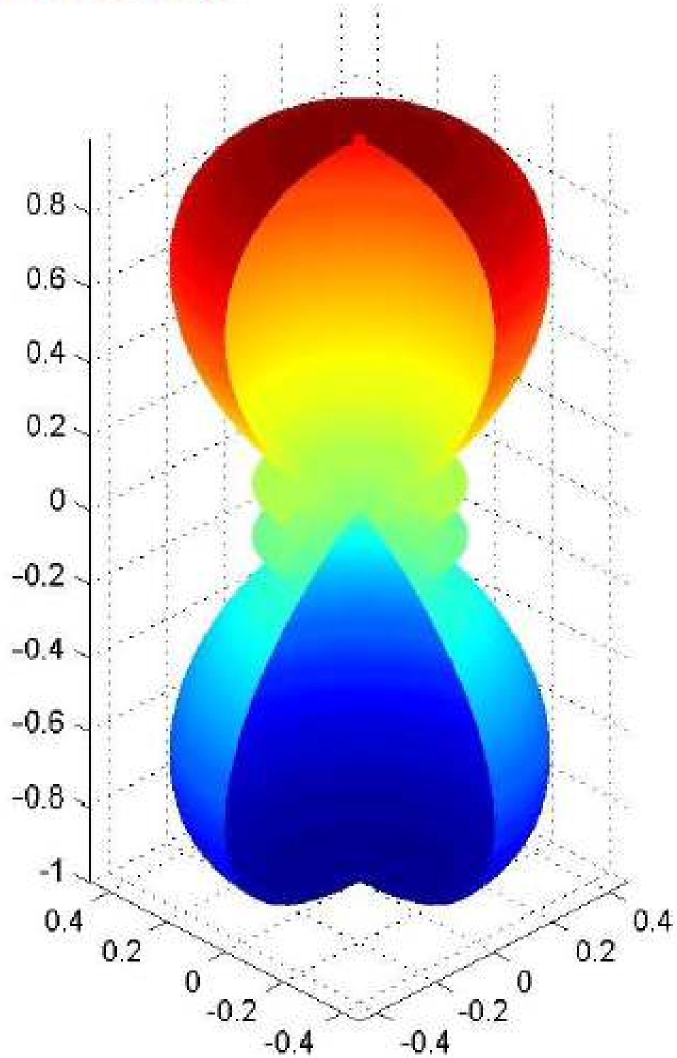
Case 2. Ordinary End-fire Array

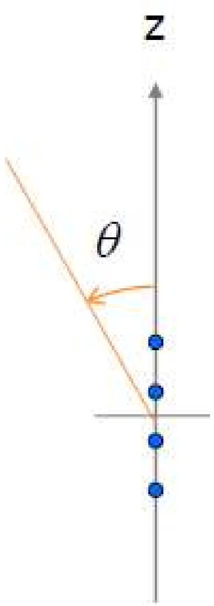


Radiation Pattern of End-Fire Array

Case 2. Ordinary End-fire Array



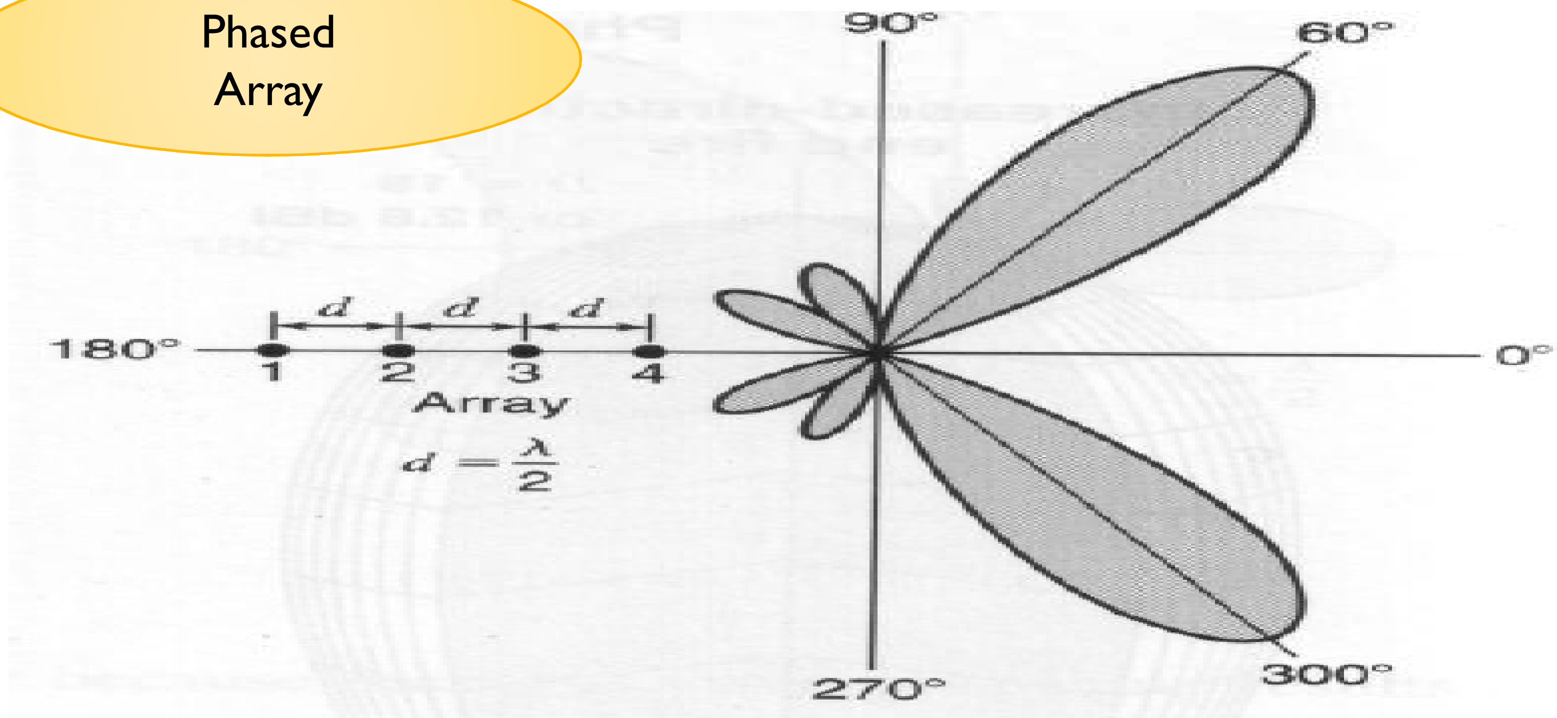




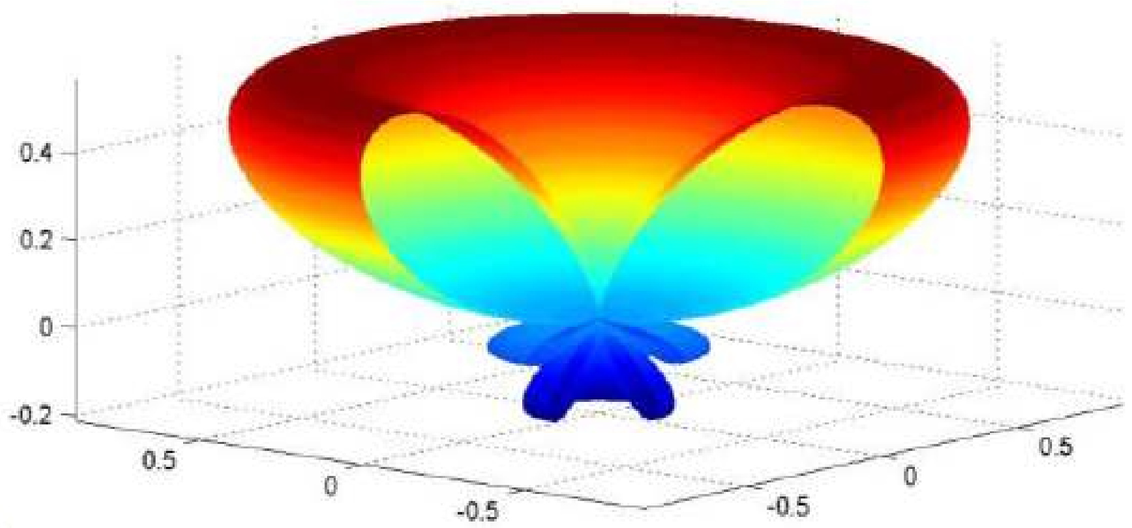
Case3 : Scanning Array "Phased Array"

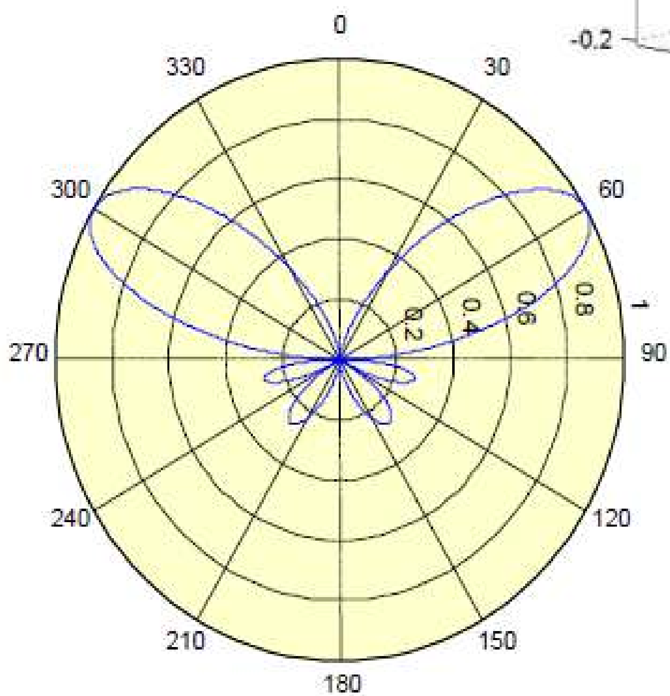
- **Array with maximum field in an arbitrary direction.**

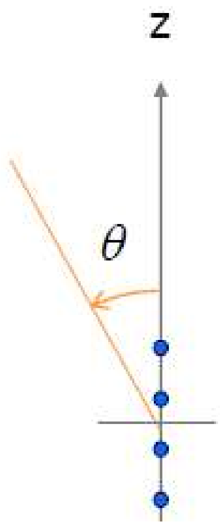
Phased Array



Radiation Pattern of Phased Array







Radiation Pattern Drawing

- *Ψ = k d cos(ϑ)+ δ ; k=2∏/λ*
- *To determine ϑ of Main lobe Ψ=0*
- *To determine ϑ of Nulls*

$$n\frac{\psi}{2} = \pm k\pi$$

To determine Side lobes

$$n\frac{\psi}{2} = \pm\frac{(2k+1)\pi}{2}$$

Calculation

- Broadside antenna array $\delta=0$

So $\Psi = k\, d \cos(\vartheta)$

- End-fire array $\delta= -kd$ So $\Psi = kd \cos(\vartheta) - kd$
- *Phased array (Scanning array)* ϑ *is given*

So determine δ