Assessment of Uncertainty in Reactive Power Compensation Analysis of Distribution Systems

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Abstract— **Rigorous assessment of uncertainties associated with capacitor installation in distribution systems is the aim of the present paper. Interval mathematics provides a powerful tool for modeling uncertainties. To account for such uncertainties, a heuristic method coupled with interval mathematics is developed with the aim of maximizing a cost saving function. The effects of uncertain inputs within the proposed model are examined for various assumed levels of overall uncertainties. To assess the relative contribution of each uncertain input, an interval sensitivity analysis is carried out. While catering for uncertainties, the proposed method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. This should enable utilities to make informed decisions regarding installing capacitors for reactive power compensation in their distribution systems. A procedure is devised in order to produce sharp bounds of the interval outcomes. Successful implementation of the proposed method is described using a nine- bus radial distribution system.**

Index Terms— **reactive power compensation, uncertainty, distribution systems, interval mathematics.**

I. INTRODUCTION

ENERGY management through reactive power compensation on distribution systems has, recently, emerged as a topic of current research interest [1]-[4]. Reactive power flow in a distribution system produces losses and results in increased rating for the system components. Shunt capacitors are usually installed to reduce these power losses, increase the released thermal capacities of the lines and transformers and improve the system voltage profile.
However, the data employed in the reactive power compensation analysis is usually derived from many sources with varying degrees of accuracy. Accounting for such uncertainties is necessary to produce realistic results which utilities can employ to make informed decisions regarding reactive power compensation in their distribution systems.

Uncertainties can be looked upon as a condition in which the possibility of errors exists as a result of having less than total information about the surrounding environment. They are beyond the utility's foreknowledge or control. In a distribution system, the reactive load is always varying and it is not a realistic proposition to determine capacitor sizes and locations based on an average of the reactive loads as even this number is subject to change as the load varies. In addition, many of the reactive power compensation techniques involve the optimization of a cost function which require parameters such as the cost of the capacitors, the cost of energy and the cost of the peak power savings to which only an estimation (single-point) without exact certainty can be obtained [3]. Consequently, the validity of the results generated are questionable.

Interval mathematics provides a powerful tool for the implementation and extension of the "unknown but bounded" concept [5]-[7]. Using interval analysis, there is no need for many simulation runs as the total variation of the solution considers the simultaneous variations of all inputs in a single run. In this form of mathematics, interval numbers are used instead of single point numbers.

This paper presents an interval method coupled with a heuristic technique for maximizing the cost saving; by placing optimal capacitors at proper locations in interval format. Uncertainties in the parameters are integrated into the analysis, as interval numbers, to allocate, sequentially, the capacitors according to the upper limit of the maximum interval saving outcome. Once locations are identified, the standard capacitor size, at a selected location, is determined through the optimization of the cost saving function. A comprehensive uncertainty level analysis is presented. The relative significance of each uncertain input is established through an interval sensitivity analysis. The method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. To overcome the difficulty of conservative bounds, a procedure is devised in order to produce sharp bounds of the interval outcomes and consequently enhances the decision making process. The proposed method is tested on a nine-bus distribution system and encouraging results are reported.

II. THE GOVERNING EQUATIONS

In order to account for uncertainties associated with the capacitors sizing and location problem, the maximum cost saving analysis is followed [4]. The input parameters' uncertainties, in interval format, are integrated into the governing equations as follows:

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$$P = \sum_{i=1}^{n} I_i^2 R_i \tag{1}$$

where P is the total active power loss for a distribution system with n branches, I_i and R_i are the current magnitude and resistance, respectively of branch i. The branch current can be obtained from the load flow solution. This current has two components; active (I_a) and reactive (I_r). Thus, the system losses can be written as

$$P = \sum_{i=1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{n} I_{ri}^2 R_i \tag{2}$$

If a capacitor of current I_{ck} is placed at a node k, the system losses are

$$P = \sum_{i=1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{k} (I_{ri} + I_{ck})^2 R_i + \sum_{i=k+1}^{n} I_{ri}^2 R_i \tag{3}$$

Subtracting (3) from (2), the loss reduction ΔP_k is

$$\Delta P_k = -2 I_{ck} \sum_{i=1}^{k} I_{ri} R_i - I_{ck}^2 \sum_{i=1}^{k} R_i \tag{4}$$

Assuming there is no significant change in the node voltage after setting the capacitor and using the cost function equation, the cost reduction can be defined as

$$\Delta S = K_p \Delta P + K_e \Delta E - K_{ck} Q_{ck} \tag{5}$$

where K_p is the annual cost in \$/KW and K_{ck} is the annual cost in \$/KVAr for the capacitor placed at node k both represented in interval format. K_e is the interval annual cost of KWh losses in \$/KWh with the energy losses, defined over a time period T, using (4) as

$$\Delta E_k = -2 L_f T I_{ck} \sum_{i=1}^{k} I_{ri} R_i - T I_{ck}^2 \sum_{i=1}^{k} R_i \tag{6}$$

where L_f is the interval load factor. Q_{ck} is the capacitor size at node k and equals

$$Q_{ck} = I_{ck} V_k \tag{7}$$

Substituting for (4), (6-7) in (5), we get

$$\Delta S = -2 K_p^{'} I_{ck} \sum_{i=1}^{k} I_{ri} R_i - K_p^{"} I_{ck}^2 \sum_{i=1}^{k} R_i \tag{8}$$

where

$$K_p^{'} = K_p + K_e T L_f + \frac{K_{ck} V_k}{2 \sum_{i=1}^{k} I_{ri} R_i} \tag{9}$$

$$K_p^{"} = K_p + K_e T$$

The value of I_{ck} that maximizes the cost reduction is obtained by

$$\frac{\partial \Delta S}{\partial I_{ck}} = 0 \tag{10}$$

From (9), we get the interval I_{ck} as

$$I_{ck} = -\frac{K_p^{'}}{K_p^{"}} \frac{\sum_{i=1}^{k} I_{ri} R_i}{\sum_{i=1}^{k} R_i} \tag{11}$$

Substituting from (11) into (8) and (4), we obtain the interval maximum net saving and the corresponding interval loss reduction as follows:

$$\Delta S_{\max} = \frac{(K_p^{'})^2}{K_p^{"}} \frac{(\sum_{i=1}^{k} I_{ri} R_i)^2}{\sum_{i=1}^{k} R_i} \tag{12}$$

$$\Delta P_{\Delta S \max} = (\frac{K_p^{'}}{K_p^{"}})(2 - \frac{K_p^{'}}{K_p^{"}}) \frac{(\sum_{i=1}^{k} I_{ri} R_i)^2}{\sum_{i=1}^{k} R_i} \tag{13}$$

Using (7) and (11-13), respectively, we can calculate the size of the capacitor used at a certain node k that maximizes the total system cost reduction and we can compute the maximum cost reduction as well as the corresponding loss reduction, all in interval format.

III. INTERVAL MATHEMATICS

Interval mathematics provides a useful tool in determining the effects of uncertainty in parameters used in a computation. In this form of mathematics, interval numbers are used instead of ordinary single point numbers. An interval number is defined as an ordered pair of real numbers representing the lower and upper bounds of the parameter range [6], [7]. An interval number can then be formally defined as follows; [a, b], where $a \le b$. In the special case where the upper and lower bounds of an interval number are equal, the interval is referred to as a point or a degenerate interval and interval mathematics is reduced to ordinary single point arithmetic.

Given two interval numbers, [a, b] and [c, d], the rules for interval addition, subtraction, multiplication, and division are as follows:

$$\begin{aligned} &[a,b]+[c,d]=[a+c,b+d] \\ &[a,b]-[c,d]=[a-d,b-c] \\ &[a,b]*[c,d]=[\min(ac,ad,bc,bd),\max(ac,ad,bc,bd)] \\ &[a,b]/[c,d]=[a,b]*[1/d,1/c],\text{where} 0\notin[c,d] \end{aligned} \tag{14}$$

Implementing interval analysis techniques confronts some obstacles because its algebraic structure is unlike that of common single point arithmetic. Accordingly, interval computations may produce wide bounds.

Given a set of interval input parameters, the bounds of the resulting interval computations may depend on the calculation procedure as well as the input parameters. Therefore, an effort has to be made to reduce the width of the resulting interval bounds. Normally, the approach to producing better bounds has been to rearrange the expression to reduce the appearance of the interval parameters [6], [7].

IV UNCERTAINTIES IN THE CAPACITOR PLACEMENT ROBLEM

Inspection of equations (7-13) reveals that it is likely that values for K_p, K_e, K_{ck} and L_f can not be obtained with absolute certainty. For instance, K_p and K_e , the costs for the peak power and energy losses respectively can be calculated in many ways but it is probably known that there is an upper and lower bound for these costs which can be attributed with more certainty than a single- point value for each cost [3]. Likewise, for the reactive load factor L_f a range of values can also be determined. Thus by using interval mathematics, the uncertainties associated with the capacitor allocation technique could be more effectively understood if these input parameters were treated as interval numbers whose ranges contain the uncertainties in those parameters. The resulting computations, carried out entirely in interval form, would then literally carry the uncertainties associated with the data through the analysis. Likewise, the final outcome in interval form would contain all possible solutions due to the variations in input parameters.

V. ALGORITHM

The implementation of the proposed optimal capacitor sizing and placement technique in interval mathematics is performed in the Matlab® environment. The steps of the algorithm are summarized as follows:

1) Run the load flow program for the original uncompensated feeder to calculate the voltages and currents at each bus using the Gauss-Seidel method.

2) Assume an initial value for the single point estimate capacitor cost K_{ck} as the average cost for all available standards for the studied feeder.

3) Let the input parameters (K_p, K_e, K_{ck} and L_f) as an interval numbers with a realistic tolerance of ±5% of their single point estimates.

4) Select a bus and apply (7), (11-13) to compute the interval capacitor size, the interval current capacitor, the interval maximum saving and the corresponding interval loss reduction respectively. Repeat this step for all buses in the system, except the source bus.

5) Identify the candidate bus that has the highest interval cost saving (defined here as the bus with the highest upper bound in the interval cost saving) provided that the evaluated interval loss reduction and interval capacitor size are positive.

6) Once a bus is identified as a candidate bus, determine all the standard capacitor sizes lying within the interval capacitor size at this bus. In case no standard size lies within the interval, then the one nearest to the interval is selected (i.e. the closest standard size to both the lower upper bounds of the interval). These procedure are applied at any one candidate bus selected.

7) Perform the load flow calculations, for every single standard capacitor selected earlier, to ensure that no voltage violation takes place. If there is a voltage violation for one or more standard capacitor sizes, eliminate them from further consideration. If all the capacitor sizes result in voltage violation, then go to step 5 to select the next candidate bus.

8) If there is no voltage violation, set the standard capacitor size, among the series of standard sizes in this interval, that provides the highest cost saving at this bus and take the corresponding exact capacitor cost value K_{ck}.

9) Repeat steps 4-8 to get the next capacitor bus and hence the sequence of buses to be compensated until it is found that there is no significant cost saving can be achieved by further capacitor placement.

The above algorithm can be looked upon as consisting of two nested loops. The first is a global one that loops over candidate buses to determine the interval capacitor values at all buses and the corresponding interval standard sizes. While the second is local as it searches for the optimal standard capacitor size, within an interval, at a specific candidate bus.

VI. SIMULATION RESULTS

To illustrate the numerical algorithm presented above, a test system whose load and feeder data given in [8], is investigated.

The radial distribution feeder has 9 load buses and its rated substation voltage is 23kV. The estimated values for K_p, K_e and L_f are $168/kW, $0.3/kWh, and 0.5 respectively [3],[9]. Commercially available capacitor sizes with $/kVAr are used in the analysis. As the maximum capacitor size Q^C_{max} should not exceed the reactive load (i.e. 4186kVAr), this results in 27 possible capacitor sizes with their corresponding cost/kVAr and their values may be derived assuming a life expectancy of ten years (placement, maintenance, and running costs are neglected) [9].

Applying load flow solution on this feeder, before compensation, the cost function and the total power losses are $ 131,675 and 783.8 kW respectively. The maximum and minimum bus voltage magnitudes are 0.9929 and 0.8375 p.u., where the voltage of the substation (bus number 0) is assumed to be 1 p.u., thus we have generally $0.8375 \leq V_i \leq 1$ p.u. The following sections describe the compensation procedure for the test feeder; with the input parameters K_p, K_e, K_{ck} and L_f all assumed to be interval numbers. The computations are carried out using the Intlab toolbox [10].

i) Base Case (5% tolerance)

To demonstrate the application of the proposed algorithm, equations (7), (11-13) are employed to obtain the required outcomes. A tolerance of ±5% is assumed in all parameters.

Table I

Optimal sizes of singly located capacitors, cost savings and losses reduction

Bus no.	Q_c (kVAr)	ΔS ($)	ΔP (kW)
1	[2018.1, 3057.1]	[170.16, 353.27]	[3.4883, 6.0349]
2	[2247.2, 3270.7]	[237.57, 455.33]	[4.1538, 6.937]
3	[3568.2, 4437]	[4049, 5664.6]	[27.184, 40.741]
4	[3415.8, 4215.2]	[6860.9, 9452.8]	[42.76, 63.979]
5	[2309, 2838.4]	[7548, 10320]	[45.223, 67.63]
6	[1997, 2453.4]	[7236.8, 9882.3]	[43.086, 64.431]
7	[1561.9, 1917.2]	[6729.8, 9173.7]	[39.697, 59.358]
8	[1100.6, 1349.5]	[6213.1, 8450.7]	[36.22, 54.156]

9	[836.88, 1025.5]	[5558.4, 7552.1]	[32.218, 48.171]

Table I shows the optimal size of a single located capacitor (Q_c), the maximum cost saving (ΔS) and corresponding loss reduction (ΔP), for all buses, as interval outcomes. It is noticed that the bus that provides the highest upper bound in the cost saving is bus 5 (10,320$) which corresponds to the interval capacitor size [2309, 2838.4] KVAr. This identifies bus 5 as the first bus to be compensated. There are 3 standard sizes which fall within this range (i.e. 2400, 2550, 2700 KVAr). Computing the cost saving for each of the 3 standard sizes (provided no voltage violation occurs), it was found that the size of 2700KVAr provides the highest cost saving. The single- point estimates for the capacitor size, maximum cost saving and corresponding loss reduction at bus 5 were also computed and found to be 2560.4KVar, 8839.6$, and 55.849kW respectively. It is clear that the estimated values of the outcomes are within the lower and upper bounds of the corresponding interval results. When the compensation procedure was continued, it was found that the next node to be compensated is node 9 with 450KVAr followed by node 4 with 900KVAr.

The proposed technique produces a total cost saving and a total loss reduction of [9015.52, 12358.7] $ and [54.74, 81.92] kW respectively. The estimated values of maximum cost saving and loss reduction are 10875 $ and 69.65 kW respectively by setting capacitor sizes of 2550, 450, and 900 KVAr at bus 5, 9, and 4 respectively. Again, it is noted that the single-point values are within the lower and upper bounds of the interval outcomes. There was an improvement in voltage profile of about 5% in the obtained minimum voltage.

The proposed interval technique furnishes utilities with alternatives of using any available standard capacitor size, lying within the interval capacitor size outcome, together with the associated cost saving. The maximum cost saving, achieved by the selection of any of these standard sizes, would *certainly* have a lower limit which corresponds to the lower bound of the interval outcome. Prior knowledge of such information could be of significance in utility planning.

ii) Assessment of the uncertainty level

In order to assess the uncertainties associated with the various input parameters K_p, K_e, K_{ck} and L_f , the level of uncertainty of all parameters has been taken to vary by 10% in case and 15% in another. Table II shows that the compensated nodes remain the same in all cases. However, the number of available standard capacitor sizes, for node 4, has changed to be 750, 900, and again 900KVAr for tolerances of 5, 10, 15% respectively while those at node 5 and node 9 remain in all cases the same at 2700 and 450 KVAr, respectively. It is observed that the interval bounds of ΔP and ΔS for the higher tolerances contains those of lower tolerances, e.g., the interval outcome of ΔS for a 5% uncertainty is contained within the intervals of the 10% and 15% levels. It is also noted that the radius of the outcome increases proportional to the increase of the uncertainty level. Results showed that while the single point estimate for ΔS overestimates the interval outcome (represented here by the interval midpoint) for the lower tolerance of 5%, the opposite is true for the higher tolerance levels of 10 and 15%. As the interval outcome width increases (e.g. for 10% and 15% tolerances), the number of standard capacitor sizes available, for use by utilities, increases at all compensated nodes. For example, the number of standard capacitor sizes at node 5, for 10% tolerance, becomes 7 instead of 3 at 5% tolerance.

Table II

Results for different uncertainty levels

Tol.	Node No.	Qc kVAr	No. of st. cap.	Q_{Cst} kVAr	ΔP (kW)	ΔS ($)
10%	5	[2080,3147]	7	2700	[35.6,80.8]	[6419.3,12020]
	4	[370,560]	1	450	[5.9,13.5]	[1063.1,1994.3]
	9	[541,905]	3	900	[1.5,3.7]	[173.3,397.6]
	Total				**[42.9,97.95]**	**[7655.7,14412]**
15%	5	[1871,3493]	11	2700	[26,96]	[5432,13979]
	4	[332,621]	2	450	[4.4,15.9]	[899,2320]
	9	[469,1023]	3	900	[1.1,4.4]	[137,480]
	Total				**[31.5,116.3]**	**[6468,16779]**

iii) Sensitivity analysis of the input parameters

By using interval analysis, there is no need for many simulation runs as the total variation of the solution considers the simultaneous variations of all inputs in a single run. In order to evaluate the relative influence of each input parameter K_p, K_e, K_{ck} and L_f , an interval sensitivity analysis has been carried out. The quantities of interest are the interval maximum cost saving ΔS and the corresponding interval loss reduction ΔP. Table III shows the total interval outcomes for ΔP and ΔS when every input parameter is assumed to vary alone with tolerances of 5%, 10% and 15%. It was found that the compensated nodes have not changed for all cases. Close inspection of Table III, reveals that K_p is the most influencing parameter on the interval outcomes followed by K_{ck}. The radius of the interval ΔS , when varying K_p alone, is 1479.6, 3293.3. and 5022.3 for tolerance of 5, 10, and 15% respectively. As for varying K_{ck} alone, these values are 41.5, 82.5, and 124 respectively. The other two parameters K_e and L_f have almost a negligible effect on ΔS for the same tolerances. The above results point out to the importance of accurate determination of these parameters as the confidence in the computed interval outcomes depends mainly on the confidence in the input parameters and not on computational procedures. The standard capacitor sizes at the compensated nodes when K_p alone is varied, are the same as those when all the uncertainties are included.

Table III

Results for Sensitivity analysis of the input parameters

par.	Tol.	5%	10%	15%
K_p	ΔP(kW)	[54.99,81.61]	[43.39,97.29]	[33.08,114.59]
	ΔS($)	[9353,12312]	[7720,14307]	[6559,16603]
K_{ck}	ΔP(kW)	[69.38,69.92]	[69.12,70.18]	68.86,70.44]
	ΔS($)	[10834,10917]	[10793,10958]	[10752,11000]
K_e	ΔP(kW)	[69.63,69.66]	[69.61,69.69]	[69.59,69.7]
	ΔS($)	[10873,10877]	[10871,10879]	[10869,10881]
L_f	ΔP(kW)	[69.64,69.65]	[69.64,69.66]	[69.63,69.67]
	ΔS($)	[10874,10876]	[10873,10877]	[10872,10878]

iv) Proposed technique for sharp bounds

In view of the fact that the algebraic structure of interval mathematics is unlike that of common single point arithmetic, interval computations may, sometimes, produce conservative bounds [6], [7]. In order to produce better bounds (i.e. sharp bounds) of the interval outcomes the term K_p'/K_p'', appearing in the governing equations, is proposed to be of the following form:

$$\frac{K_p'}{K_p''} = 1 + \frac{K_e T (L_f - 1) + \dfrac{K_{ck} V_k}{2 \sum_{i=1}^{k} I_{ri} R_i}}{K_p + K_e T} \tag{15}$$

Equation (15) is then used to modify (7) and (11-13). It is expected with such modification to get sharp bounds of the interval outcomes as the appearance of the interval input parameter K_p has been reduced [6].

Table IV shows the results of the modified algorithm and also its significance. For instance, at bus number 5, the earlier radius (half the interval width) of the capacitor size interval was 264.7. This value had led to the possible use of 3 standard capacitor sizes falling within that range. The corresponding radius of the interval numbers of the cost saving, and loss saving were 1385.8 and 11.2, respectively. With the modified algorithm, the radius of the interval number of the capacitor size is reduced to 8.2467. This value would lead to the use of a single standard capacitor size within that range. The corresponding radius of the interval numbers of the cost saving, and loss saving after modification become 497.96 and 0.34904, respectively.

Table IV

Optimal sizes of singly located capacitors, cost savings and losses reduction using the modified technique

Bus no.	Q_c (kVAr)	ΔS ($)	ΔP (kW)
1	[2230.1, 2766.3]	[188.03, 319.67]	[4.0079, 5.3224]
2	[2483.2, 2959.6]	[262.52, 412.03]	[4.7958, 6.0912]
3	[3943.4, 4014.8]	[4474.7, 5125.6]	[33.039, 34.153]
4	[3775, 3814.1]	[7582.3, 8553.3]	[52.295, 53.332]
5	[2551.8, 2568.3]	[8341.7, 9337.6]	[55.5, 56.198]
6	[2207, 2219.9]	[7997.8, 8941.8]	[52.908, 53.512]
7	[1726.2, 1734.8]	[7437.5, 8300.7]	[48.787, 49.262]
8	[1216.4, 1221.1]	[6866.5, 7646.5]	[44.563, 44.901]
9	[924.88, 927.95]	[6142.9, 6833.4]	[39.66, 39.919]

Using the above results, a first standard capacitor size of 2550 KVAr is placed at bus 5. When the procedure is repeated, a second interval outcomes of [479.8, 483] KVAr, [1552.9, 1738.6] $, and [10.34, 10.47] kW, respectively, are achieved at bus 9. This will lead to a standard size of 450KVAr at this bus. Final cost saving and loss reduction of [352.61, 427.84] $, and [3.26, 3.53] kW are achieved by placing a third interval capacitor of [828.9, 868.7] KVAr at bus 4, leading to a standard capacitor size of 900 KVAr. The technique provides a total cost saving and total loss reduction of [10247.25, 11504.01] $ and [69.09, 70.19] kW respectively, when the above 3 standard capacitor sizes are installed. After compensation, the maximum and minimum bus voltage magnitudes are found to be 0.9961 and 0.88196 p.u. These results show that the widths of the interval outcomes of the maximum cost saving and loss reduction have been reduced and their corresponding estimated values, still fall within the modified interval outcomes.

VII CONCLUSIONS

The capacitor sizing and placement problem is modeled using a combined heuristic and interval mathematics method. Use of interval mathematics enables the integration of the effects of parameters' uncertainties into the analysis and eliminates the need for many simulation runs. The effects of uncertain inputs within the proposed model are examined for various overall uncertainty levels. The relative contribution of each uncertain input is assessed through an interval sensitivity analysis. While catering for uncertainties, the method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. This enhances their ability to make informed decisions regarding installing capacitors for reactive power compensation in their distribution systems. A procedure is devised in order to produce sharp bounds of the interval outcomes. Successful implementation of the method is described using a nine- bus test distribution system.

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