Three – dimensional Flow over a Stretching Surface with chemical reaction and suction/injection

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Abstract: The effects of suction/injection and chemical reaction on mass transfer characteristics over a stretching surface subjected to three dimensional flows are studied. The governing boundary layer equations are transformed to ordinary differential equations containing suction/injection parameter, stretching ratio parameter, concentration parameter, chemical reaction parameter, and Schmidt number. These equations are solved through applying symbolic computation to Runge-Kutta technique in order to solve three point boundary value problem. The Concentration profiles are computed and discussed in details for various values of the different parameters.

Key words : chemical reaction, stretching surface, suction / injection, three dimensional flows.

Introduction

The study of the boundary layer flow of fluid over the stretching surface has important applications in several industrial manufacturing processes. Such as hot rolling, wire drawing, metal and polymer extrusion, crystal growing, continuous casting, glass fiber and paper production, drawing of plastic film, ect. During these processes there could be some chemical reactions. Das, Deka , et al. [1] have studied the effect of chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux. Anderson et al. [2] have studied the diffusion of chemical reactive species from a linear stretching sheet. Muthucumaraswamy et al. [3] have studied the effect of homogenous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Muthucumaraswamy et al. [4] have investigated the effect of chemical reaction on the unsteady flow past an impulsively started semi-infinite vertical plate which is subjected to uniform heat flux. Muthucumaraswamy et al. [5] have analyzed the effect of chemical reaction on the unsteady flow past an impulsively started vertical plate which is subjected to uniform mass flux and in the presence of heat transfer. Muthucumaraswamy [6] studied the effects of suction on heat and mass transfer along a moving vertical surface in the presence of chemical reaction. Muthucumaraswamy et al. [7] have studied the effect of first order chemical reaction on unsteady flow past an uniformly accelerated isothermal infinite vertical plate in the presence of heat and mass transfer. EL-Arabawy [8] studied the effects of suction/injection and chemical reaction on mass transfer over a stretching surface. Loganathan et al. [9] have studied the effect of first order chemical reaction on flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation. Rajeswari et al. [10] have studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical porous surface with heat source in the presence of suction. Kandasamy et al. [11] have studied the effect of variable viscosity, heat and mass transfer on nonlinear mixed convection flow over a porous wedge with chemical reaction in the presence of heat radiation. Mahmoud [12] studied the effect of slip velocity at the wall on flow and mass transfer of an electrically conduction visco-elastic fluid past a stretching sheet embedded in a porous medium in the presence of chemical reaction and concentration dependent viscosity. All these studies considered the two dimensional flow problem, by developing the problem to three dimensional flow we found a good lists of references which discussed this problem. Nazar et. al. [13] studied the effects of viscoelastic fluid on the velocity profiles of three dimensional flow over a stretching surface. Takhar et. al. [14] studied the effects of heat transfer on three dimensional MHD boundary layer flow through A stretching surface. El-dabe et. al. [15] studied the effects of heat generation / absorption and chemical reaction on three dimensional viscoelastic fluid through a stretching surface. The purpose of this work is to study the effects of chemical reaction and suction/injection on mass transfer over a stretching surface.

Formulation of the problem

Consider a steady, laminar, incompressible, and viscous fluid on a continuous stretching surface with chemical reaction and suction/injection. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to the other chemical species, the concentration of species far from the surface, C∞ very small [16]. The x- , y- axes are run along the plan of a continuous surface , and the z-axis is perpendicular to it as shown in fig (1). The conservation equations for the steady three dimensional flow are

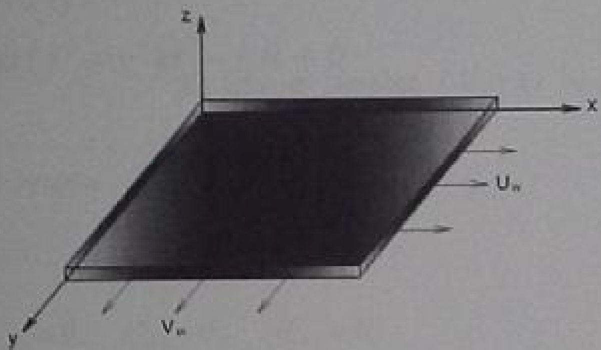

Fig (1) physical model and coordinate system.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\upsilon\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\upsilon\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}-k_1 C \tag{4}$$

Subjected to The Boundary Conditions

$$u=U_w,\quad v=V_w\quad,\quad C=C_w\quad,\quad at\ z=0$$

$$u=0,\quad v=0,\quad \frac{\partial u}{\partial z}=0,\quad \frac{\partial v}{\partial z}=0\quad,\qquad C=C_\infty\quad as\ z\to\infty \tag{5}$$

Where u, v and w are velocity components in the x, y and z directions, respectively , t is the time, υ is the viscosity, C is the concentration of the fluid, D is the effective diffusion coefficient, and $k1$ is the reaction rate coeffecient.

We assume that the stretching velocities Uw , Vw and the fluid Concentration Cw are of the form

$$U_w = ax, \quad V_w = by, \quad C_w = C_\infty + d_1 x^{\alpha} = C_\infty + d_2 y^{\alpha} \tag{6}$$

Where a and b are constant and called stretching rate, $d1$, $d2$ are constant and α is the concentration parameter.

We now introduce the following dimensionless functions f , g and θ , and the similarity variables η

$$\eta = \sqrt{\frac{a}{\upsilon}} z, \quad u = (ax) f'_{(\eta)}, \quad v = (ay) g'_{(\eta)}, \quad w = -\sqrt{a\upsilon}\,(f + g) \tag{7}$$

Where prime denotes the differntiation with respect to η , using (7) the mass conservation equation (1) is indentically satisfies, and substituting into eqs. (2-4) we obtain

(8)

$$f''' + (f + g) f'' - f'^2 = 0 \tag{9}$$

$$g''' + (f + g) g'' - g'^2 = 0 \tag{10}$$

$$\theta'' + Sc\left[(f + g)\theta' - \alpha(f' + g')\theta\right] - L\theta = 0$$

where (Sc = υ/D) is the schmidt number, (L = k1 Sc / a) is th chemical reaction parameter

The boundary condition (5) become

$$f_{(0)} + g_{(0)} = \beta, \quad f'_{(0)} = 1, \quad g'_{(0)} = \zeta, \quad \theta_{(0)} = 1$$

$$f'_{(\infty)} = 0, \quad f''_{(\infty)} = 0, \quad g'_{(\infty)} = 0, \quad g''_{(\infty)} = 0, \quad \theta_{(\infty)} = 0 \tag{11}$$

where (β) is the suction/injection parameter (β > 0) corresponds to suction, (β < 0) corresponds to injection and (β = 0) corresponds to impermeable surface.

where ($\zeta = b/a$) is the stretching ratio parameter , and when ($\zeta = 0$) the problem reduces to the two-dimensional case [$g = 0$], and when ($\zeta = 1$) the problem reduces to the axisymmetric flow [$f = g$]

Numerical Solutions and Results

We first convert the Equations (9-12) into a system of linear equations of first order , by using

$$S_1 = f, \quad S_2 = f', \quad S_3 = f'', \quad S_4 = g, \quad S_5 = g', \quad S_6 = g'', \quad S_7 = \theta, \quad S_8 = \theta'$$

$$\begin{aligned}
S_1' &= S_2 \\
S_2' &= S_3 \\
S_3' &= S_2^{\,2} - (S_1 + S_4) S_3 \\
S_4' &= S_5 \\
S_5' &= S_6 \\
S_6' &= S_5^{\,2} - (S_1 + S_4) S_6 \\
S_7' &= S_8 \\
S_8' &= -Sc\left[(S_1 + S_4) S_8 - \alpha (S_2 + S_5) S_7\right] + L S_7
\end{aligned} \tag{12}$$

Subjected to The initial Conditions

$$S_{1(0)}+S_{4(0)}=\beta,\;\; S_{2(0)}=1,\;\; S_{3(0)}=m,\;\; S_{4(0)}=0,\;\; S_{5(0)}=\zeta,\;\; S_{6(0)}=n,\;\; S_{7(0)}=1,\;\; S_{8(0)}=\ell \qquad (13)$$

where m, n and ℓ are unknown to be determined as a part of the numerical solution.

Using Mathematica, we define a function F[m_, n_, ℓ_] that gives the solution of the system of D.E's , the values of m, n, ℓ are determined upon solving the equations S2(ηmax) =0 , S5(ηmax) =0 , and S7(ηmax) =0. once we obtain the values of m, n and ℓ. we use Mathematica to solve the system of D.E's numerically. consequently , only one integration pass is enough to solve the problem instead of an iteration technique like shooting method .
The computations have been carried out for a various values of the stretching parameter (ζ) , concentration parameter (α), suction/injection parameter (β), chemical reaction parameter (L), and the schmidt number (Sc).

To validate the numerical method used in this study , the steady flow (A = 0) case and (β = 0) was considered in table (1) and the results for $-f''(0)$ and $-g''(0)$ are compared with the numerical solution which reported in Nazar [13].

Table (1) : Values of $-f''(0)$, $-g''(0)$ for a various values of (ζ)

	ζ	Nazar [13]	present results
$-f''(0)$	0.00	1.0013	1.0000
	0.50	1.0935	1.0931
	1.00	1.1738	1.1737
$-g''(0)$	0.00	0.0000	0.0000
	0.50	0.4656	0.4652
	1.00	1.1738	1.1737

Also the concentration surface gradient values for a various values of concentration parameter (α) for suction/injection case was considered in table (2) and the results are compared with the exact solution which reported in Al-Arabawy [8].

Table (2) : Values of $-\theta'(0)$ for a various values of (α) at Sc=0.7, ζ =0 , L =0.8

β	α	L	Al-Arabawy [8]	present results
1	-2	0.00	1.04672	1.04672
	-1	0.20	1.25353	1.25353
	1	0.80	1.62421	1.62421
	2	1.20	1.79220	1.79220
-1	-2	0.00	0.06266	0.06266
	-1	0.20	0.42367	0.42367
	1	0.80	0.95568	0.95568
	2	1.20	1.16929	1.16929

Discussions

The influence of the suction/injection parameter (β), concentration parameter (α), stretching parameter (ζ), chemical reaction parameter (L), and the schmidt number (Sc) on the dimensionless concentration are shown in figures (1-5).

Figure (1) shows the effect of the suction/injection parameter (β) on the concentration. We observe that the increases of the suction/injection parameter decrease the concentration for any case of concentration parameter. Also it is clear that the concentration in the case of negative value of the concentration parameter (α = - ve) is high compared to the case of positive value of the concentration parameter near the surface.

Figure (2) shows the effect of the concentration parameter (α) on the concentration. We observe that the increases of the concentration parameter decrease the concentration. Also it is clear that the concentration in the case of injection is high compared to suction.
Also the concentration in the case of injection increases to a certain point then decay to zero especially in the case of negative value of concentration parameter.

Figure (3) shows the effect of stretching parameter (ζ) on the concentration. We observe that the increases of the stretching parameter decrease the concentration for both suction and injection case which mean that the concentration in the case of 2D-problem ($\zeta=0$) is high compared to 3D-problem ($\zeta \neq 0$).

Figure (4) shows the effect chemical reaction parameter (L) on the concentration. We observe that the increases of the chemical reaction parameter decrease the concentration. Also it is clear that the concentration in the case of injection is high compared to suction.

Figure (5) shows the effect of different chemical species on the concentration profiles, where Diffusing chemical species of most interest in air has Schmidt number in the range from 0.1 to 10 We observe that the concentration decreases with the increases of Schmidt number. Also it is clear that the concentration boundary layer decreases with increase of Schmidt number.

Conclusion

Numerical solution has been obtained for the effects of the suction/injection parameter (β), concentration parameter (α), stretching parameter (ζ), chemical reaction parameter (L), and the schmidt number (Sc) on the dimensionless concentration over a steady stretching surface. The following results are obtained:

- The concentration decreases with the increase of suction/injection parameter (β), concentration parameter (α), stretching parameter (ζ), chemical reaction parameter (L), and the schmidt number (Sc).
- The values of $-f''_{(0)}$ and $-g''_{(0)}$ increase with increase of suction/injection parameter (β), stretching parameter (ζ) which leading to decreasing of surface shear stress with increasing of these parameter.
- The concentration gradient at the surface $-\theta'_{(0)}$ increases with decrease of suction/injection parameter (β)), chemical reaction parameter (L) and concentration parameter (α).
- The concentration gradient at the surface $-\theta'_{(0)}$ in the case of 3D- problem is high compared to 2D-problem.

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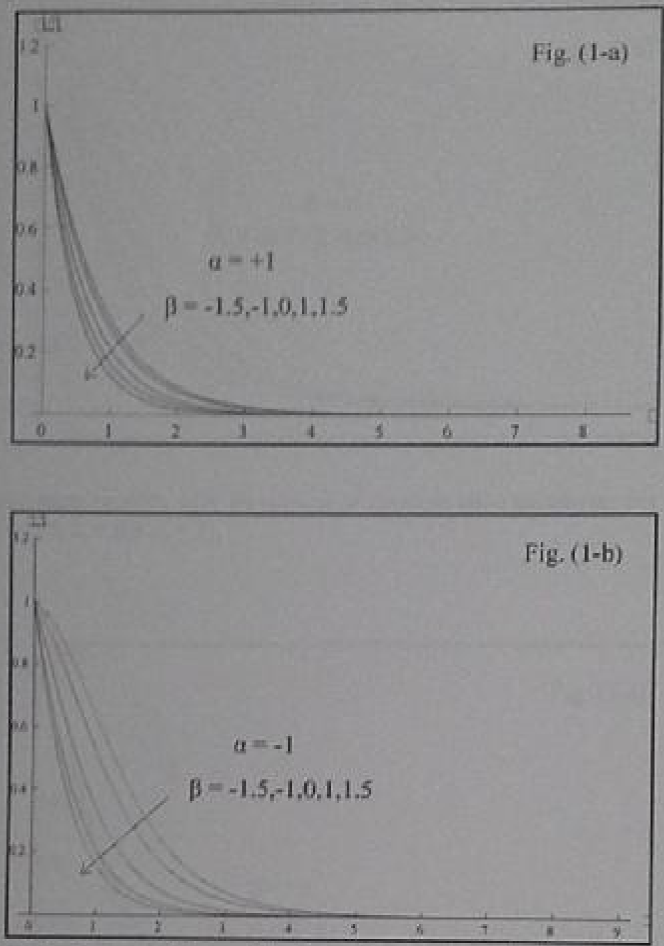
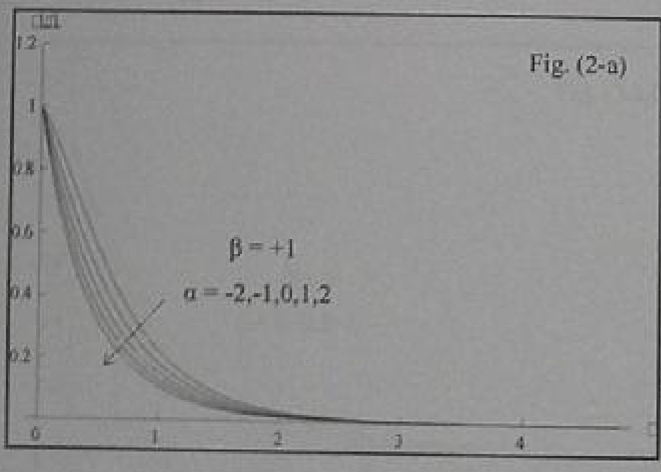


Fig (1): The Concentration profiles with increasing of suction/injection parameter (β) and concentration parameter (α) at (Sc = 0.7, L = 0.8, ζ = 1).



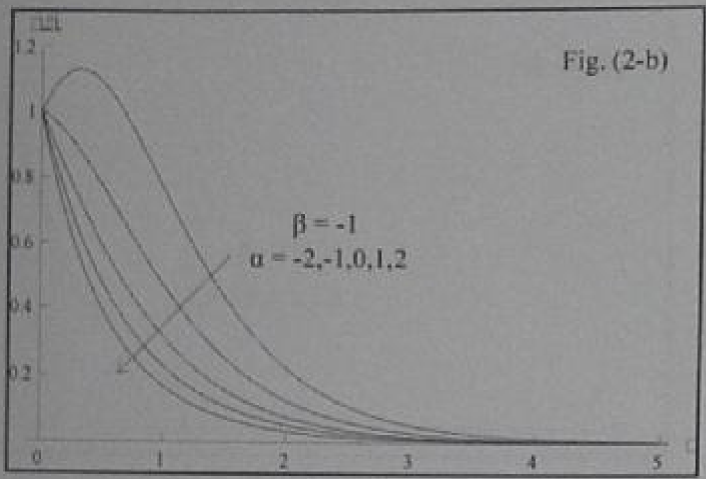
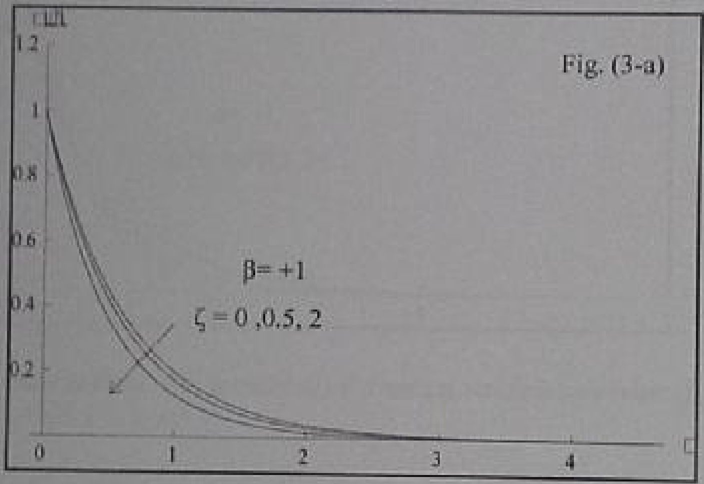


Fig (2): The Concentration profiles with increasing of concentration parameter (α) for suction (a) and injection (b) at ($Sc = 0.7$, $L = 0.8$, $\zeta = 2$).



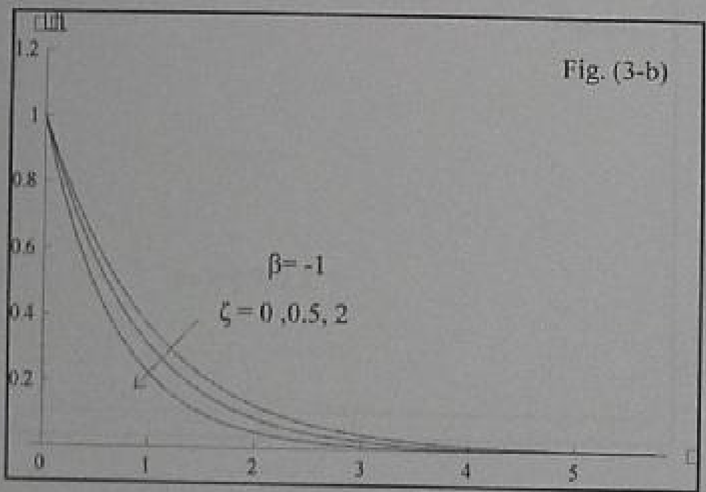
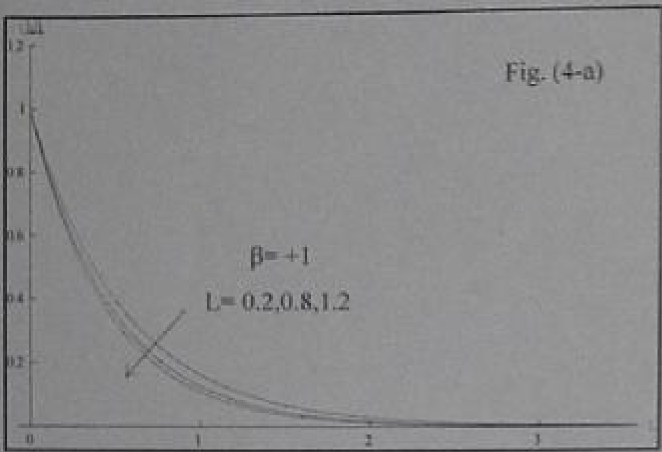


Fig (3): The Concentration profiles with increasing of stretching parameter (ζ) for suction (a) and injection (b) at ($Sc = 0.7$, $L = 0.8$, $\alpha = 1$).

$+ S_c (f\theta'$



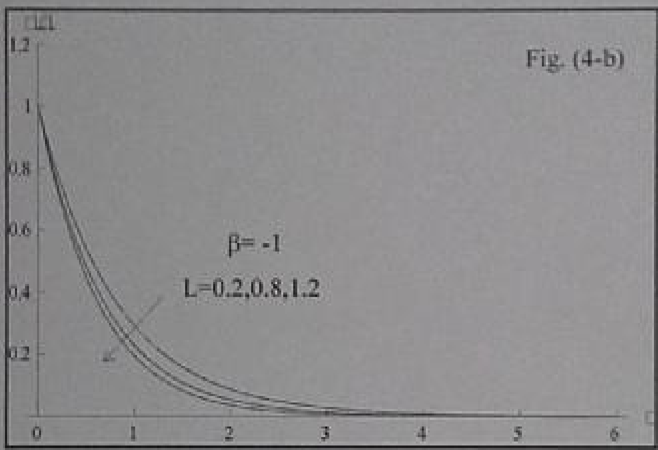


Fig (4): The Concentration profiles with increasing of chemical reaction parameter (L) for suction (a) and injection (b) at (Sc = 0.7, ζ = 2, α = 1).

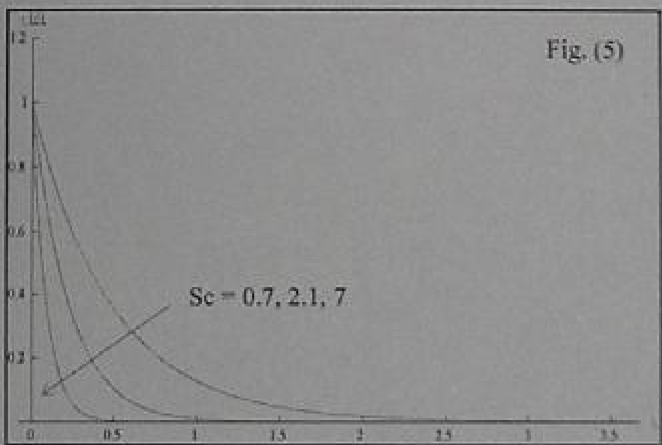


Fig (5): The Concentration profiles with increasing of Schmidt number Sc at (L = 0.8, β=1, α = 1, ζ =2).