(1)Find |A|, |BA|, if possible, where $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$

(2)Compute the nth sum of the series: $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots$

(3)Find u(x, y) and v(x, y) of the function $f(z) = \frac{1}{z+2}$ and find u_x, v_y

(4)Find the maximum (minimum) values of the function: $f(x, y) = x^2 + y^2 - 4x + 6y$

(5)Test the series (a) $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3 + 2}$

(1)Find A^{-1} where $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

(2)Compute the nth sum of the series: $1.3 + 2.5 + 3.7 + 4.9 + \ldots$

(3)Find u(x, y) and v(x, y) of the function $f(z) = \sqrt{z}$.

(4)Find the extrema of the function $f(x, y) = x^2 + 3y^2 + 2x - 18y$

(5)Test the series (a) $\sum_{n=1}^{\infty} \frac{2}{\ln(n+1)}$ (b) $\sum_{n=1}^{\infty} \frac{3^n}{(2n+1)^n}$

(1)Find $(B^tA)^{-1}$, if possible, where $A=\begin{bmatrix}1 & 2\\0 & 1\\2 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 1\\0 & 1\\2 & 1\end{bmatrix}$

(2)Using the binomial theorem, write the first four terms of $\frac{1}{\sqrt{4-8x}}$

(3)Find the envelope of the curves: $x^2+(y-2\alpha)^2=4$, α is parameter

(4)Find the maximum and minimum values of: $f(x,y)=x^2+y^3-4xy+4y$.

(5)(a)Test $\sum_{n=1}^{\infty}\frac{3^n}{n+3}$

(b)Determine the interval of convergence of $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{2^n}(x-1)^n$

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Higher Institute of Engineering
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Department of Basic Science

Prep. Year: Final Exam
Mathematics: (Calculus II)
Course Code, BAS 115
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الزمن: 3 ساعات — الامتحان مكون من (5) أسئلة فى صفحة واحدة و المطلوب الإجابة عن كل الأسئلة

(1)(a)Find |A| and $(AB)^{-1}$, if possible, where $A=\begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 1 \end{bmatrix}$

(b)Test the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)^n}$

(c)Determine the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n}{2^n} x^n$

(2)(a)Compute the sum of the series $\sum_{r=1}^{n} \frac{3}{(r+2)(r+3)}$

(b)Find u(x, y), v(x, y) and $u_{xx} + u_{yy}$ of the complex function f(z) = $3 + e^{2z}$.

(c)Determine the extrema of the function $f(x,y) = y^2 - 4y + x + \frac{1}{x}$

(3)(a)Show that the envelope of the curves: $(x-\alpha)^2 + (y-\alpha)^2 = 1$ is $(x-y)^2 = 2$

(b)If $f(x,y,z) = \frac{x}{y} + \sin\frac{y}{z}$. Find ∇f and $x.f_x + y.f_y + z.f_z$

(c)Find $\overline{U}.\overline{V}$, $\overline{U}x\overline{V}$ where $\overline{U} = i + 2j + 2k$ and $\overline{V} = i + 2j - 2k$.

(4)Evaluate the following integrals:

(a) $\int_0^1 \int_0^{2x} (2x + 6y)\, dydx$

(b) $\iint_D (xy)dxdy$, where D is the region inside the circle $x^2 + y^2 = 9$

(c) $\int_{(0,0)}^{(2,4)} (12xy)\, dx + (3x + 2y)dy$, through the curve $y = x^2$

(5)(a)If $\overline{U} = (xy + z^2)i + (xz + \sin y)j + (y + 2^z)k$. Find $\nabla.\overline{U}$ and $\nabla x\overline{U}$.

(b)Verify Green's theorem for the integral $\oint_C (x + 2y)\, dx + (3x - y)dy$,

where C is formed by: $x^2 + y^2 - 2x = 0$, $y = x^2$

Good Luck — *Dr. M. H. Eid*