Ministry Of Higher Education
Higher Institute of Engineering
6th of October City
Department of Basic Science

Prep. Year: Final Exam
Mathematics: (Calculus II)
Course Code, BAS 115
Date: 8 / 6 / 2009

الامتحان (5) أسئلة في صفحة واحدة و المطلوب الإجابة عن كل الأسئلة — الزمن: 3 ساعات

Marks

(1)(a) Find $A + B$ and $(BA)^{-1}$, if possible, where $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}$ (3)

(b) Using binomial theorem, expand $\dfrac{1}{\sqrt{1+4x}}$ (3)

(c) Test the series $\sum_{n=1}^{\infty} \dfrac{n}{3^n}$ (3)

(d) Determine the interval of convergence of the series $\sum_{n=1}^{\infty} \dfrac{1}{n+3}(x-1)^n$ (3)

(2)(a) Compute the sum $\sum_{r=1}^{20} r(3r+1)$ (3)

(b) Find u(x, y), v(x, y) and $u_{xx} + u_{yy}$ of the complex function $f(z) = e^{2\ln z}$ (3)

(c) Determine the extrema of the function $f(x,y) = x^2 - 2x + y + \dfrac{1}{y}$ (3)

(d) Verify Euler's theorem for the function $f(x,y) = x^4 + y^4$ (3)

(3)(a) Find the envelope of the curves: $y = \alpha x + \dfrac{1}{\alpha}$ (4)

(b) Find $\overline{U} + \overline{V}$ $\overline{U}.\overline{V}$, $\overline{U} x \overline{V}$ where $\overline{U} = 2i + 2j + k$ and $\overline{V} = 2i - 2j + k$ (4)

(c) If $\overline{U} = (xyz)i + (xz\sin y)j + (yz\cos x)k$. Find $\overline{U_x}$, $\overline{U_y}$, $\overline{U_z}$, $\nabla.\overline{U}$ and $\nabla x \overline{U}$ (4)

(4) Evaluate the following integrals: (12)

(a) $\int_0^1 \int_0^{2x} (2x+6y)\, dydx$ (b) $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dydx$

(c) $\int_{(0,0)}^{(1,1)} (x+y)\, dx + (x+2y)dy$, through the curve (i) $y = x^2$ (ii) $y = x$ (iii) $y^2 = x$

(5)(a) Find the integral $\iint_S dS$, where S is $x^2 + y^2 + z = 1,\ z \ge 0$ (4)

(b) Verify Green's theorem for the integral $\oint_C (x+y)\, dx + (2x+y)dy$,

where C is formed by: $x^2 + y^2 = 1$, $x + y = 1$ (8)

Good Luck *Dr. M. H. Eid*