

<p>Communications Eng. Department Level: 3rd Year Examiner: Dr. Mohamed Eid Time allowed: 3 hours</p>		<p>Semester: Autumn 2018 Final Exam Course: Mathematics IV Code: Math 301 Date: January 2, 2019</p>
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The Exam consists of one page Answer all questions No. of questions: 4 Total Mark: 55

Question 1

(a) Prove that : $B(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ 4

(b) Find the integrals : (i) $\int_1^\infty e^{-(x-1)^2} dx$ (ii) $\int_0^\infty \frac{1}{1+x^4} dx$ 6

Question 2

(a) Solve the PDE : (i) $2u_{xx} - 5u_{xy} + 2u_{yy} = e^{2x+3y}$ (ii) $9u_{xx} - 4u_{yy} = \cos(2x)$ 8

(b) Solve the wave equation: $u_{tt} - 16u_{xx} = 0, \quad 0 < x < 1$
 B. C. $u(0, t) = u(1, t) = 0$ and I. C. $u(x, 0) = x, \quad u_t(x, 0) = 1$ 4

(c) If $J_k(x)$ is the Bessel's function. Find $\int_0^2 x^3 J_0 dx$ where $J_0(2) = 0.2239$ and $J_1(2) = 0.5767$ 4

Question 3

(a) From the data: (1, 2), (2, 5), (3, 6), (4, 10), (5, 12) 6

Find the regression line $y = a + bx$, \bar{x} , \bar{y} , σ_x , σ_y and the correlation coefficient r .

(b) If x is random variable with pdf $f(x) = e^{-x}, x \geq 0$. Find the moment generating function $M_x(t)$ and from it, find m_0, m_1, m_2, V and σ . 6

(c) If x, y are random variables with pdf $f(x, y) = \frac{3}{2}x^2 + y, 0 < x, y < 1$. Find $cov(x, y)$. 4

Question 4

(a) Find the probabilities $P(x = 3), P(x \leq 5), P(x < 5), P(x > 4)$ from the data:

x_i	$x < 2$	2	3	4	5	6	$x > 6$
$f(x_i)$	0	0.1	0.2	0.3	0.1	0.3	0

(b) Four fair coins are tossed simultaneously and x is the number of appearing heads H . From the binomial distribution, find the probabilities : $P(x = 2), P(x < 3), P(x \geq 3)$ 3

(c) In a production of iron rods, if the diameter x is normally distributed with $\mu = 2$ and $\sigma = 0.008$. Find the percentage of non defectives with tolerance limits 2 ± 0.02 that is, find $P(1.98 \leq x \leq 2.02)$ where $\phi(2.5) = 0.9938$ 3

(d) From the Gamma distribution: $f(x) = \begin{cases} \frac{1}{\Gamma(n)} x^{n-1} e^{-x}, & x, n > 0 \\ 0, & \text{Otherwise} \end{cases}$ 4

Show that $\sigma = \sqrt{n}$. Also, find $P(x \leq 2)$ and $P(x > 2)$ when $n = 2$.

Model Answer

Answer of Question 1

(a) Proof: $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

-----4 Marks

(b)(i) Put $y = (x - 1)^2$, $dy = 2(x - 1)dx$ and $dx = \frac{1}{2\sqrt{y}} dy$

Then $I = \frac{1}{2} \int_0^\infty y^{-\frac{1}{2}} e^{-y} dy = \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{1}{2} \sqrt{\pi}$

(b)(ii) Put $y = x^4$, $dy = 4x^3 dx$ and $dx = \frac{1}{4} \frac{1}{y^{\frac{3}{4}}} dy$

Then $I = \frac{1}{4} \int_0^\infty \frac{y^{-\frac{3}{4}}}{1+y} dy = \frac{1}{4} B\left(\frac{1}{4}, \frac{3}{4}\right) = \frac{\pi}{4} \sqrt{2}$

-----6 Marks

Answer of Question 2

(a)(i) A.E $2k^2 - 5k + 2 = 0$, then $k = 2, \frac{1}{2}$ and $y_c = f(y + 2x) + g\left(y + \frac{1}{2}x\right)$

$$y_p = \frac{1}{2D^2 - 5DE + 2E^2} e^{2x+3y} = \frac{1}{8 - 30 + 18} e^{2x+3y} = -\frac{1}{4} e^{2x+3y}$$

(a)(ii) A.E $9k^2 - 4 = 0$, then $k = \pm \frac{2}{3}$ and $y_c = f\left(y + \frac{2}{3}x\right) + g\left(y - \frac{2}{3}x\right)$

$$y_p = \frac{1}{9D^2 - 4E^2} \cos 2x = \frac{1}{-36 - 0} \cos 2x = -\frac{1}{36} \cos 2x$$

-----8 Marks

(b) Wave equation $c = 4$, $L = 1$, $f(x) = x$, $g(x) = 1$. Then

$$A_n = \frac{2}{1} \int_0^1 x \cdot \sin n\pi x dx = 2 \left[\frac{-x}{n\pi} \cos n\pi x + \frac{1}{(n\pi)^2} \sin n\pi x \right] = \frac{2}{n\pi} (-1)^{n+1}$$

$$B_n = \frac{2}{4n\pi} \int_0^1 1 \cdot \sin n\pi x dx = \frac{-1}{2n\pi} \frac{\cos n\pi x}{n\pi} = \frac{-1}{2(n\pi)^2} [(-1)^n - 1] = \begin{cases} \frac{1}{(n\pi)^2}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

$$U(x, t) = \sum_{n=1}^{\infty} \frac{2}{n\pi} (-1)^{n+1} \sin n\pi x \cos 4n\pi t + \sum_{n=1}^{\infty} \frac{1}{\pi^2 (2n-1)^2} \sin(2n-1)\pi x \cdot \cos 4(2n-1)\pi t$$

-----4 Marks

(c) Integrate by parts: $u = x^2$, $dv = x J_0 dx$, then $du = 2x dx$, $v = x J_1$. Then

$$I = x^3 J_1 - 2 \int_0^2 x^2 J_1 dx = x^3 J_1 - 2x^2 J_2 = 8 J_1(2) - 8 J_2(2)$$

From the relation $J_{k-1} + J_{k+1} = \frac{2k}{x} J_k$, putting $k = 1$, $x = 2$, we get

$$J_2(2) = J_{k+1} = J_1(2) - J_0(2)$$

Then $I = 8 J_0(2) = 8(0.2239) = 1.7912$

-----4 Marks

Answer of Question 3

(a) By regular calculator, the regression line is $y = a + bx = -0.5 + 2.5x$, $\bar{x} = 3$, $\bar{y} = 7$,
 $\sigma_x = 1.414$, $\sigma_y = 3.578$ and the correlation coefficient $r = 0.988$

-----6 Marks

$$(b) M_x(t) = \int_{-\infty}^{\infty} e^{xt} f(x) dx = \int_0^{\infty} e^{xt} e^{-x} dx = \int_0^{\infty} e^{xt} e^{-(1-t)x} dx = \frac{-1}{1-t} e^{-(1-t)x} = \frac{1}{1-t}$$

By the binomial theorem, we get

$$\begin{aligned} \frac{1}{1-t} &= 1 + t + t^2 + t^3 + \dots \\ &= 1 + t + 2 \frac{t^2}{2!} + 6 \frac{t^3}{3!} + \dots \end{aligned}$$

Then $m_0 = 1$, $m_1 = 1$, $m_2 = 2$, $m_3 = 6$, $V = 2 - 1 = 1$ and $\sigma = 1$

-----6 Marks

$$(c) E(x) = \int_0^1 \int_0^1 x f(x,y) dx dy = \int_0^1 \int_0^1 x \left(\frac{3}{2} x^2 + y \right) dx dy = \frac{5}{8}$$

$$E(y) = \int_0^1 \int_0^1 y f(x,y) dx dy = \int_0^1 \int_0^1 y \left(\frac{3}{2} x^2 + y \right) dx dy = \frac{7}{12}$$

$$E(xy) = \int_0^1 \int_0^1 xy f(x,y) dx dy = \int_0^1 \int_0^1 xy \left(\frac{3}{2} x^2 + y \right) dx dy = \frac{17}{48}$$

Then $cov(x,y) = E(xy) - E(x)E(y) = -\frac{1}{96}$

-----4 Marks

Answer of Question 4

(a) $P(x = 3) = 0.2$, $P(x \leq 5) = 0.7$, $P(x < 5) = 0.6$, $P(x > 4) = 0.4$

-----3 Marks

(b) $n = 4$, $P(x = 2) = \binom{4}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$$P(x < 3) = f(0) + f(1) + f(2) = \binom{4}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + \binom{4}{1} \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 + \binom{4}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} = \frac{11}{16}$$

$$P(x \geq 3) = 1 - P(x < 3) = \frac{5}{16}$$

-----3 Marks

(c) Since $P(a \leq x \leq b) = F(b) - F(a) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$

$$\begin{aligned} \text{Then } P(1.98 \leq x \leq 2.02) &= \Phi\left(\frac{2.02-2}{0.008}\right) - \Phi\left(\frac{1.98-2}{0.008}\right) \\ &= \Phi(2.5) - \Phi(-2.5) = \Phi(2.5) - [1 - \Phi(2.5)] \\ &= 0.9938 - [1 - 0.9938] = 0.9876 = 98.76 \% \end{aligned}$$

-----3 Marks

(d) $E(x) = \mu = \frac{1}{\Gamma(n)} \int_0^{\infty} x \cdot x^{n-1} e^{-x} dx = \frac{\Gamma(n+1)}{\Gamma(n)} = n$

$$E(x^2) = \frac{1}{\Gamma(n)} \int_0^{\infty} x^2 \cdot x^{n-1} e^{-x} dx = \frac{\Gamma(n+2)}{\Gamma(n)} = n(n+1)$$

$$V = E(x^2) - (\mu)^2 = n(n+1) - n^2 = n \quad \text{and} \quad \sigma = \sqrt{n}$$

When $n = 2$, then $P(x \leq 2) = \frac{1}{\Gamma(2)} \int_0^2 x e^{-x} dx = 0.59$

$$P(x > 2) = 1 - P(x \leq 2) = 0.41$$

-----4 Marks

Dr. Mohamed Eid