Automatic Control

If you have a smart project, you can say "I'm an engineer"

Lecture 4

Staff boarder

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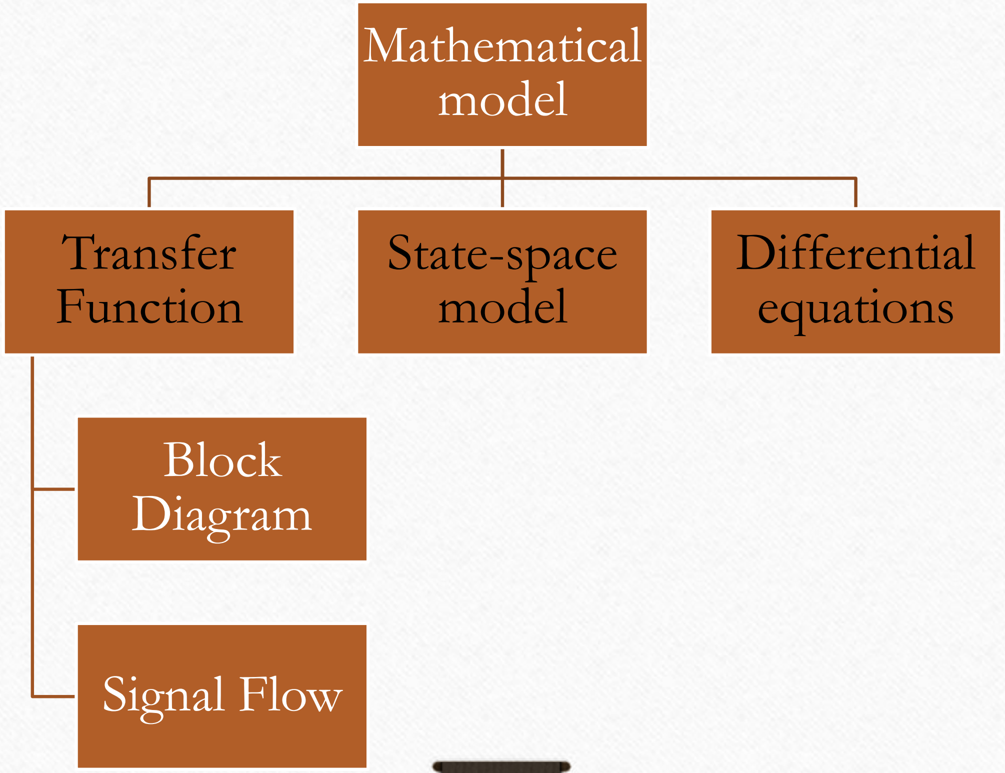
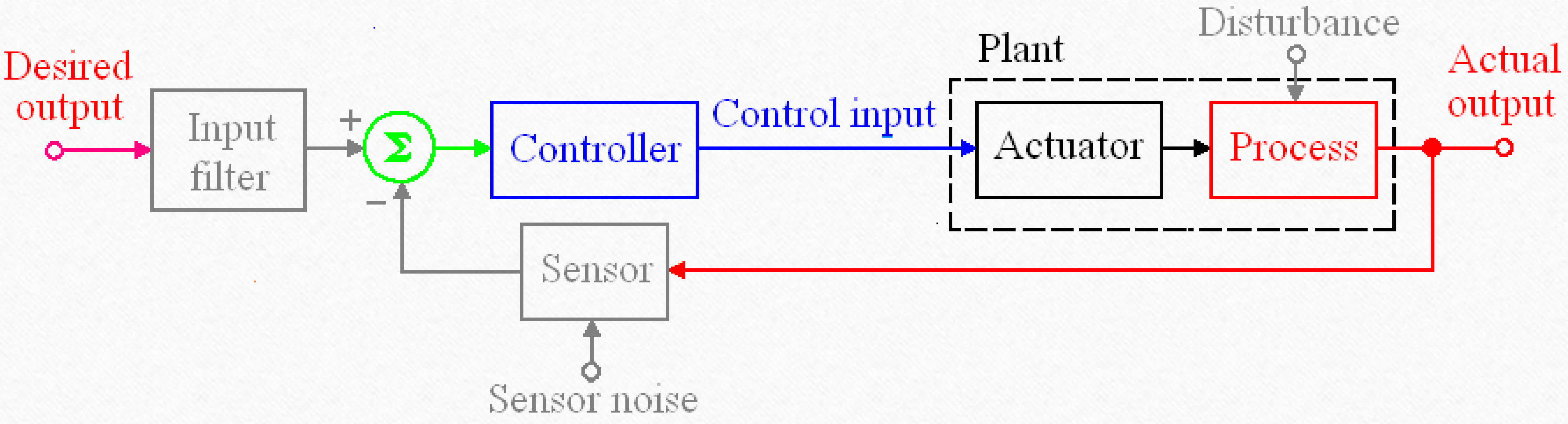
Automatic Control
MPE 424

- Lecture aims:
  - Understand the Block reduction techniques
  - Identify the transfer function

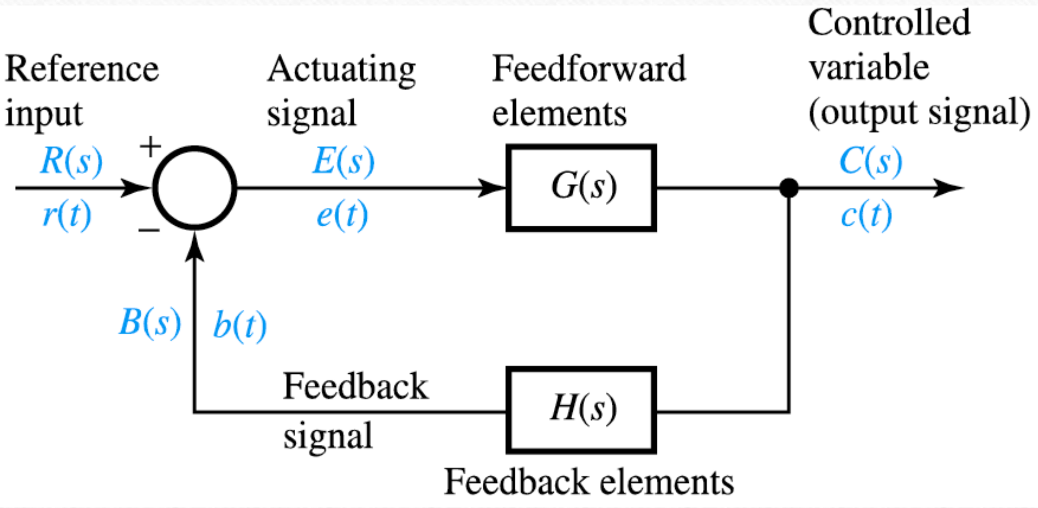
Mathematical Modeling



Component Block Diagram



Component Block Diagram

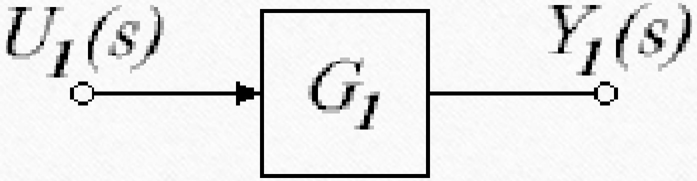


$R(s)$	Reference input
$C(s)$	Output signal (controlled variable)
$B(s)$	Feedback signal = $H(s)C(s)$
$E(s)$	Actuating signal (error) = $[R(s) - B(s)]$
$G(s)$	Forward path transfer function or open-loop transfer function = $C(s)/E(s)$
$M(s)$	Closed-loop transfer function = $C(s)/R(s) = G(s)/[1 + G(s)H(s)]$
$H(s)$	Feedback path transfer function
$G(s)H(s)$	Loop gain

$$\frac{E(s)}{R(s)} = \text{Error-response transfer function} \quad \frac{1}{1 + G(s)H(s)}$$

Component Block Diagram

- It represents the ***mathematical relationships*** between the elements of the system.



$$U_1(s)G_1(s) = Y_1(s)$$

- The ***transfer function*** of each component is placed ***in box***, and the ***input-output relationships*** between components are indicated by ***lines and arrows***.

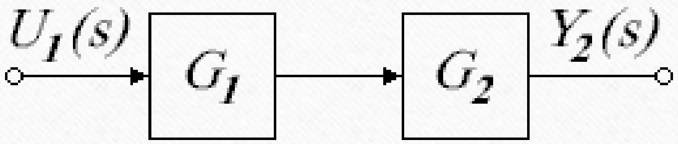
Component Block Diagram

- We can ***solve the equations by graphical simplification***, which is often easier and more informative than algebraic manipulation, even though the methods are in every way equivalent.

- The interconnections of blocks include summing points, where any number of signals may be added together.

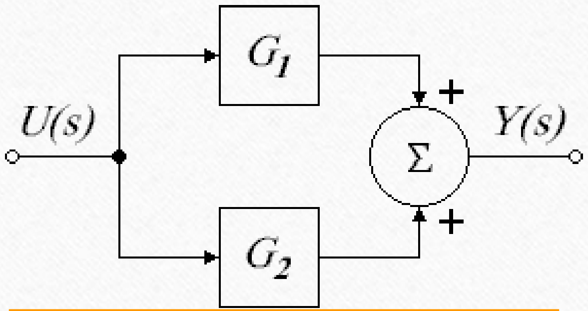
Block Diagram Reduction Technique

- Blocks in series:



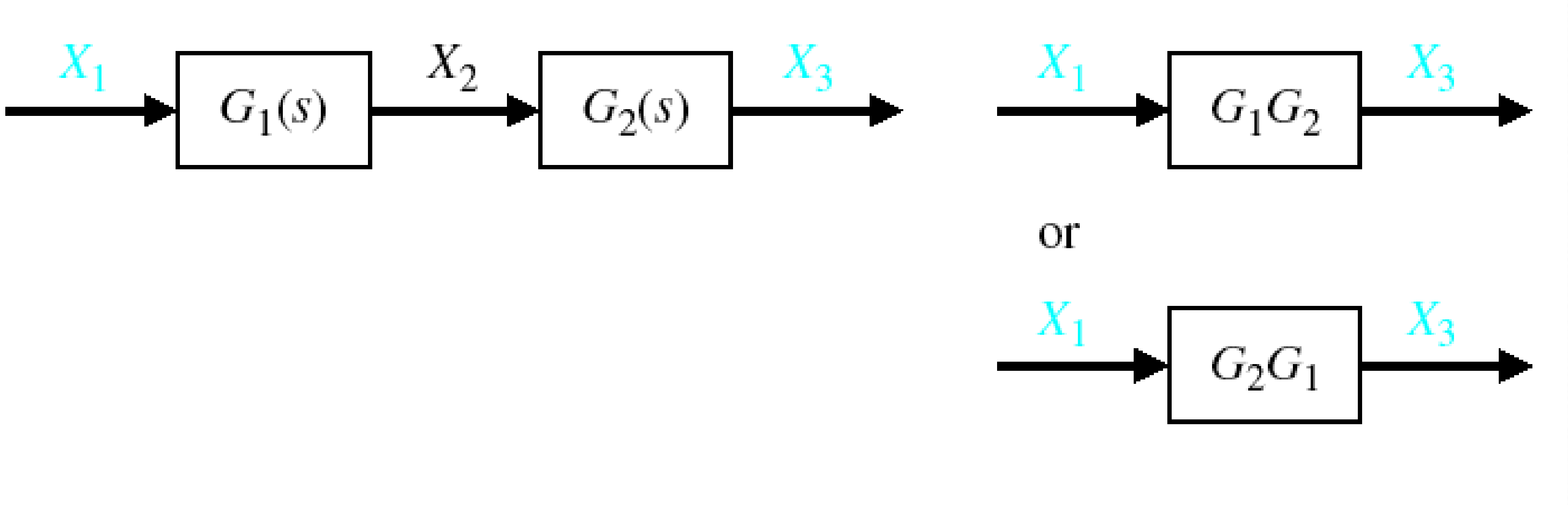
$$\frac{Y_2(s)}{U_1(s)} = G_1 G_2$$

- Blocks in parallel with their outputs added:



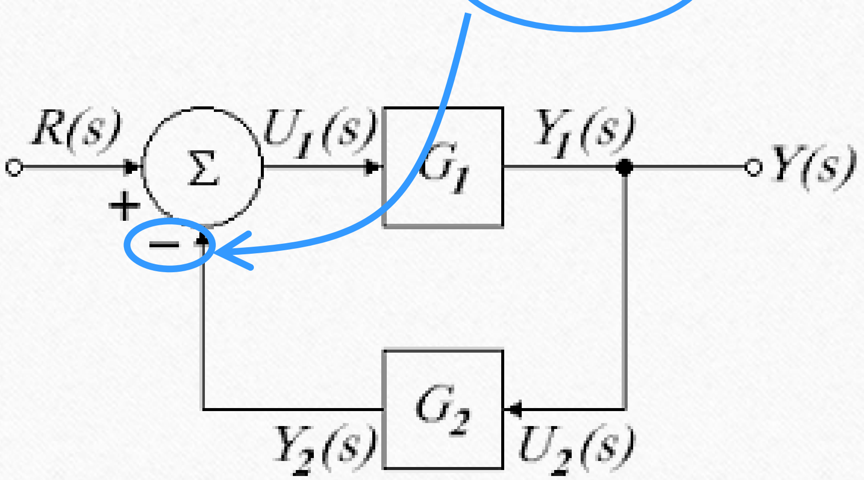
$$\frac{Y(s)}{U(s)} = G_1 + G_2$$

Combining blocks in cascade



Block Diagram Reduction Technique

- ***Single-loop negative feedback***



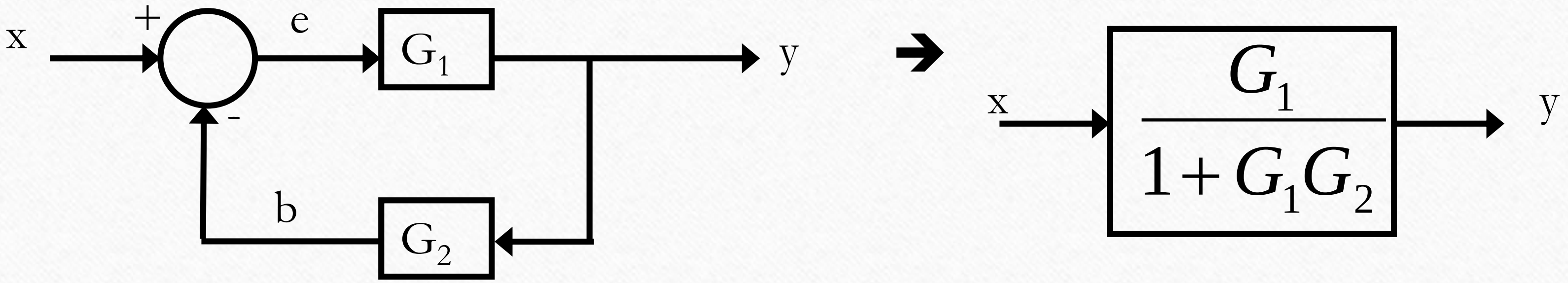
Two blocks are connected in a feedback arrangement so that each feeds into the other.

- **Transfer function**

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1G_2}$$

Block Diagram Reduction Technique

- Proof:



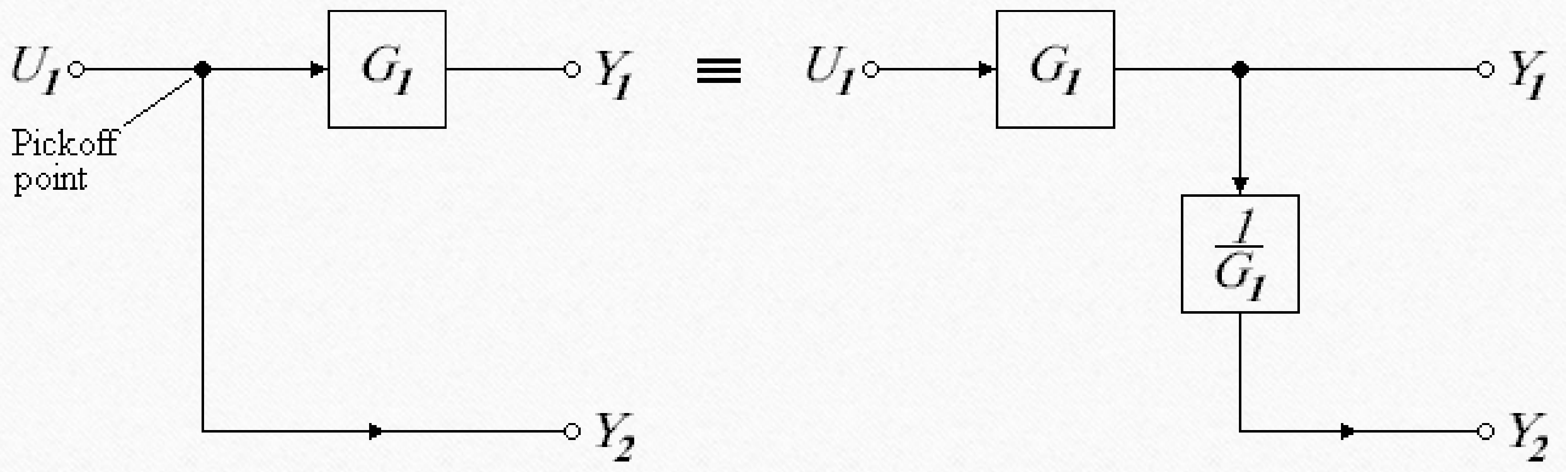
$$y = \frac{G_1}{1+G_1G_2}x$$

$$e = x - b, \quad b = G_2 y, \quad y = G_1 e \quad \Rightarrow \quad y = \frac{G_1}{1+G_1G_2}x$$

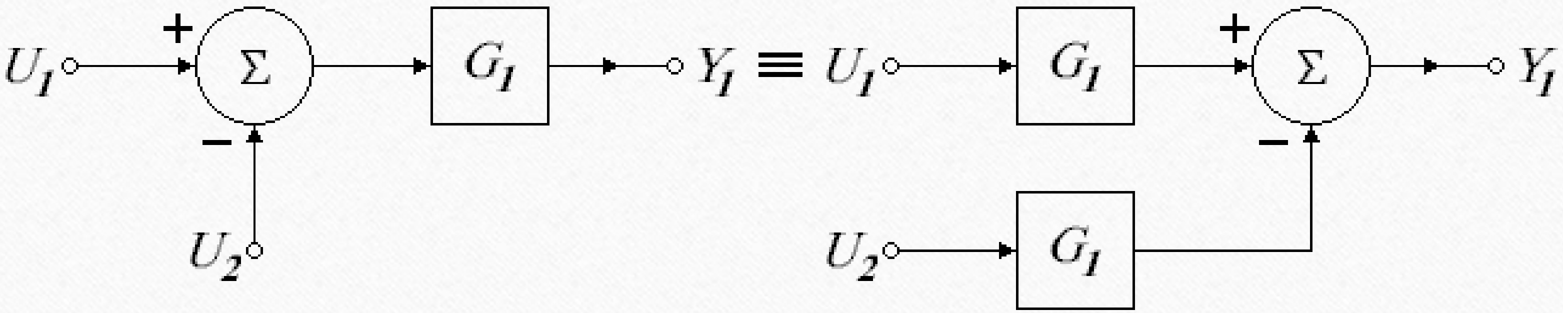
$$e = x - G_2G_1e$$

$$(1+G_1G_2)e = x \;\Rightarrow\; e = \frac{1}{1+G_1G_2}x$$

Block Diagram Reduction Technique



Block Diagram Reduction Technique



Block Diagram Reduction Technique

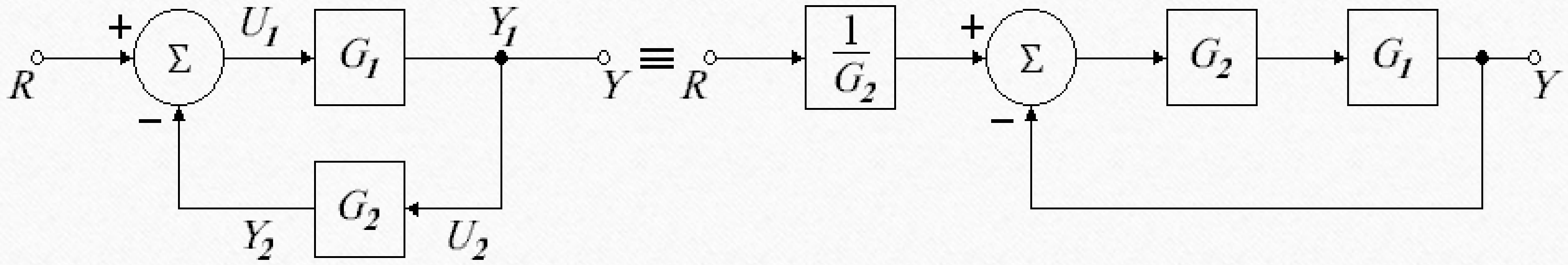
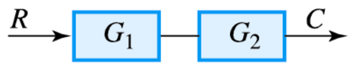
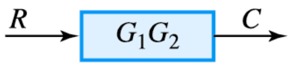
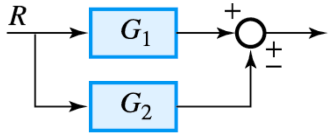
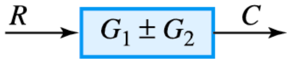
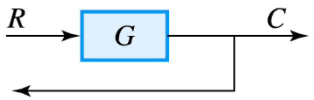
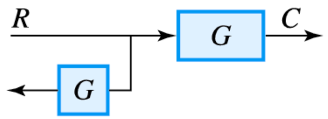
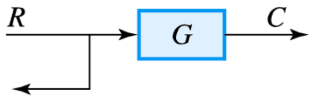
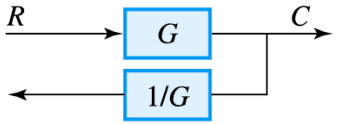
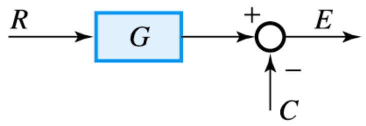
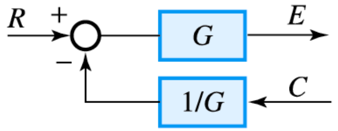
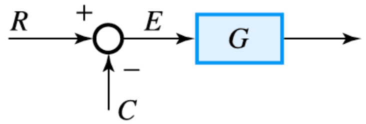
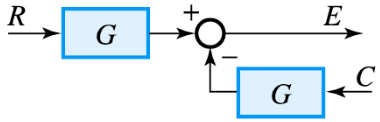
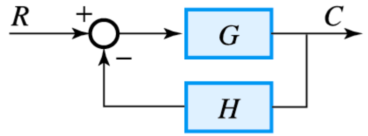
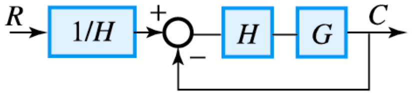
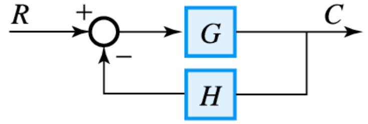
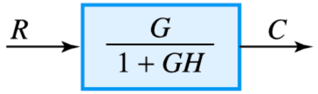


TABLE 3.4.1 Some of the Block Diagram Reduction Manipulations

Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pickoff point ahead of block	
	Shifting of pickoff point behind block	
	Shifting summing point ahead of block	
	Shifting summing point behind block	
	Removing H from feedback path	
	Eliminating feedback path	

Block Diagram Reduction Technique

Example

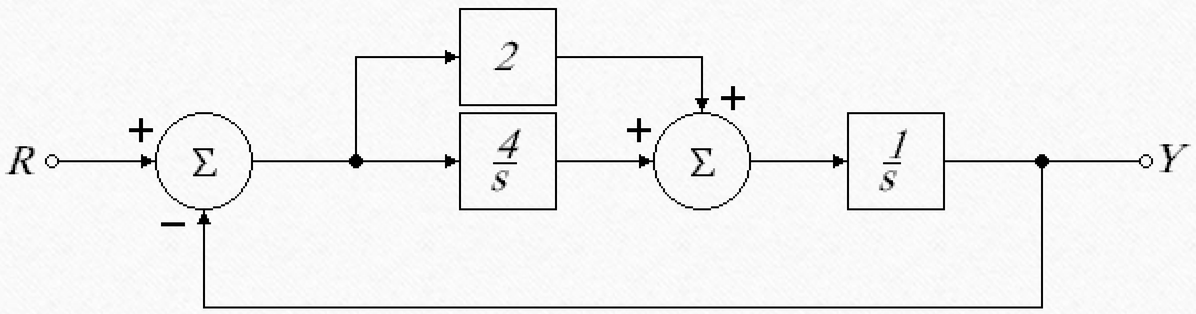


fig. (a)

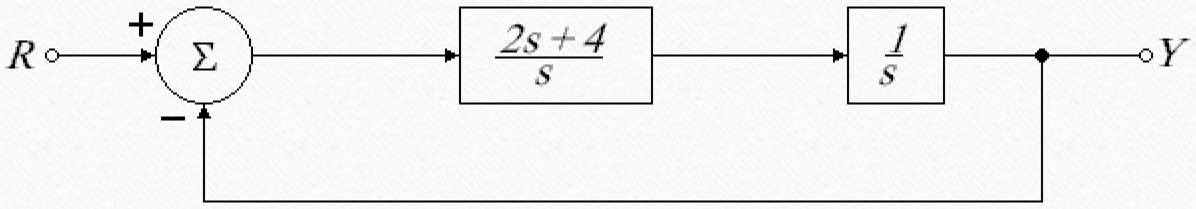


fig. (b)

$$T(s)=\frac{Y(s)}{R(s)}$$

$$T(s)=\frac{\dfrac{2s+4}{s^2}}{1+\dfrac{2s+4}{s^2}}$$

$$T(s)=\frac{2s+4}{s^2+2s+4}$$

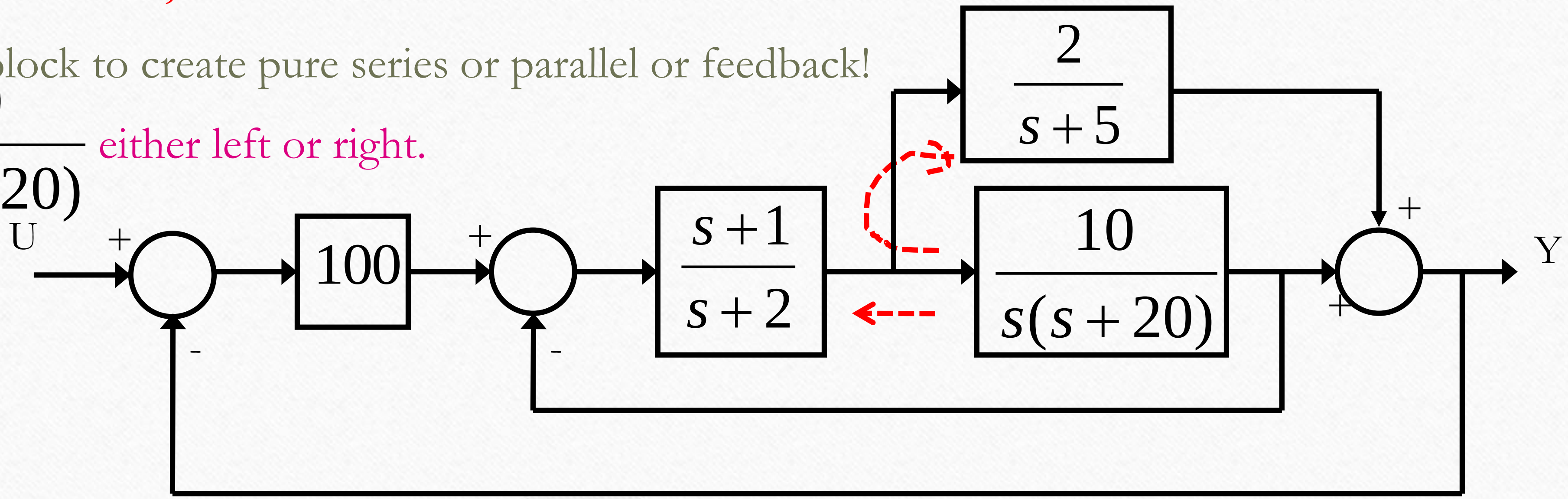
Block Diagram Reduction Technique

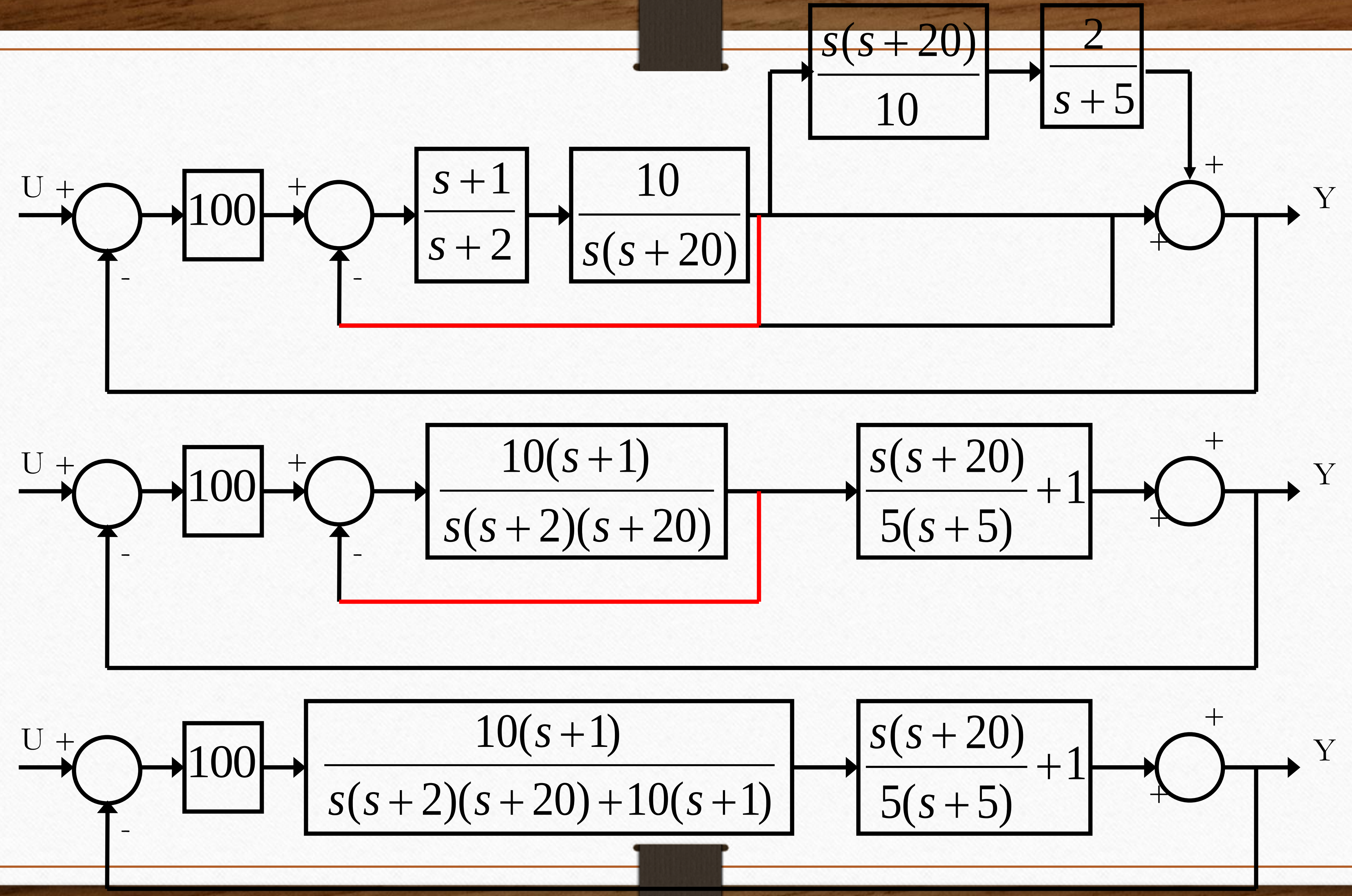
Example 2: Find TF from U to Y:

- No pure series/parallel/feedback
- Needs to move a block, but which one?

Key: move one block to create pure series or parallel or feedback!

So move $\frac{10}{s(s+20)}$ either left or right.

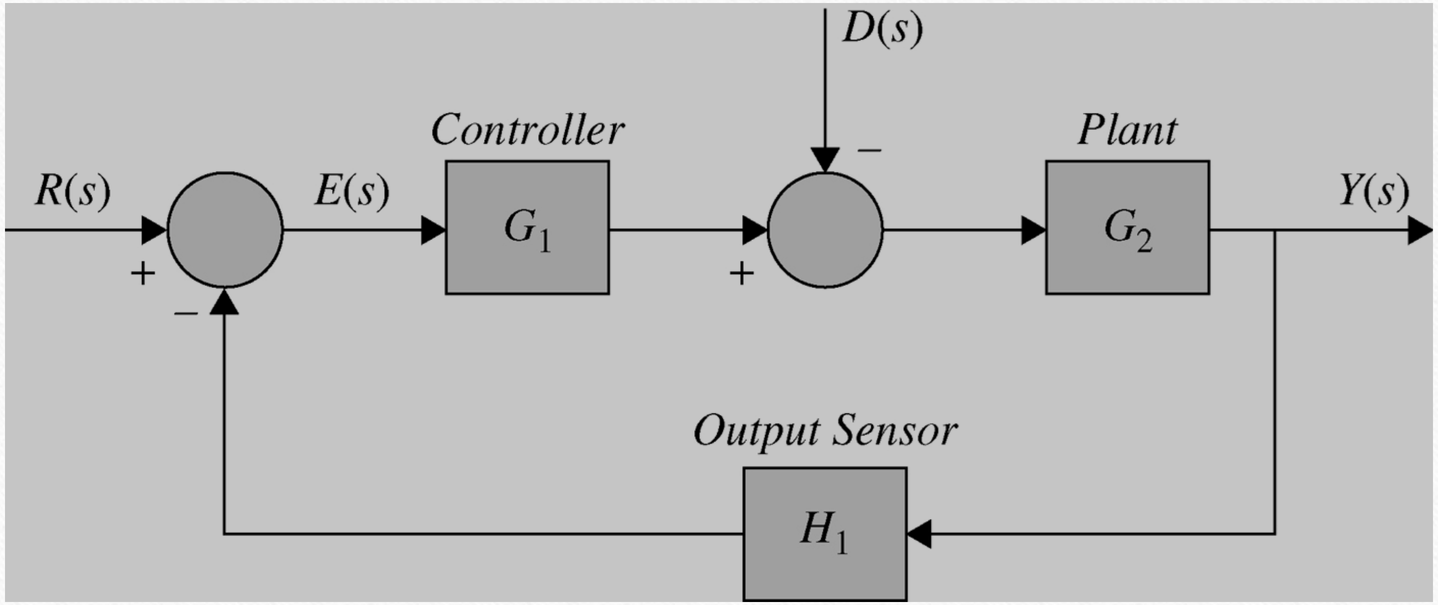




U
+
-
100
s+1 / s+2
10 / s(s+20)
s(s+20) / 10
2 / s+5
Y
10(s+1) / s(s+2)(s+20)
s(s+20) / 5(s+5) +1
10(s+1) / s(s+2)(s+20)+10(s+1)

Block Diagram Reduction Technique

Example



Can use superposition:

First set D=0, find Y due to R

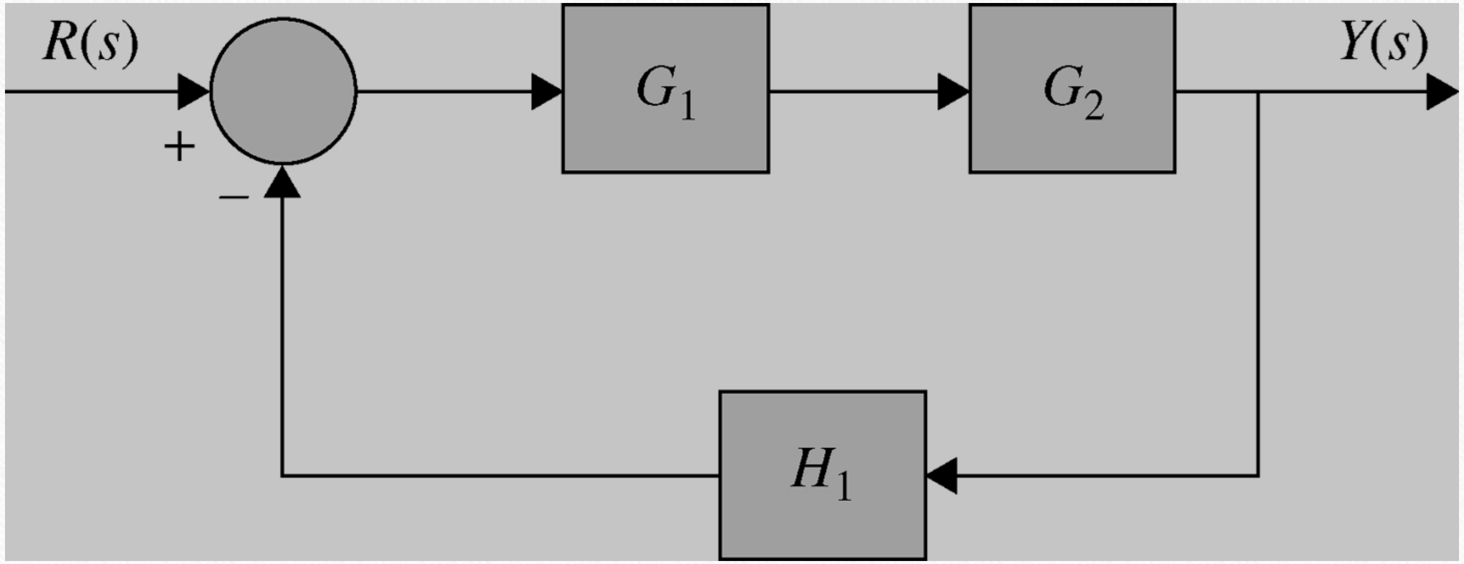
Then set R=0, find Y due to D

Finally, add the two component to get the overall Y

Block Diagram Reduction Technique

First set D=0, find Y due to R

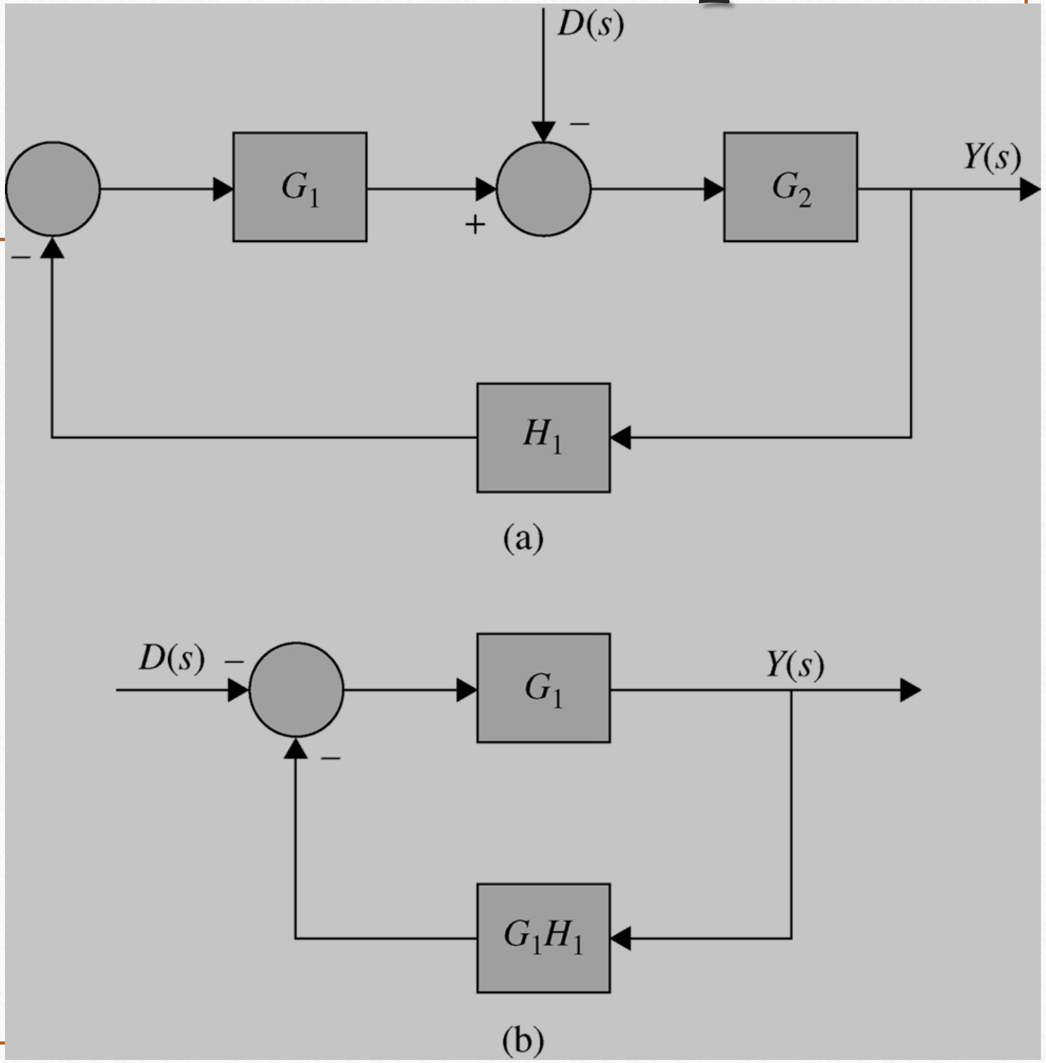
$$Y_1(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s)$$



Block Diagram Reduction Technique

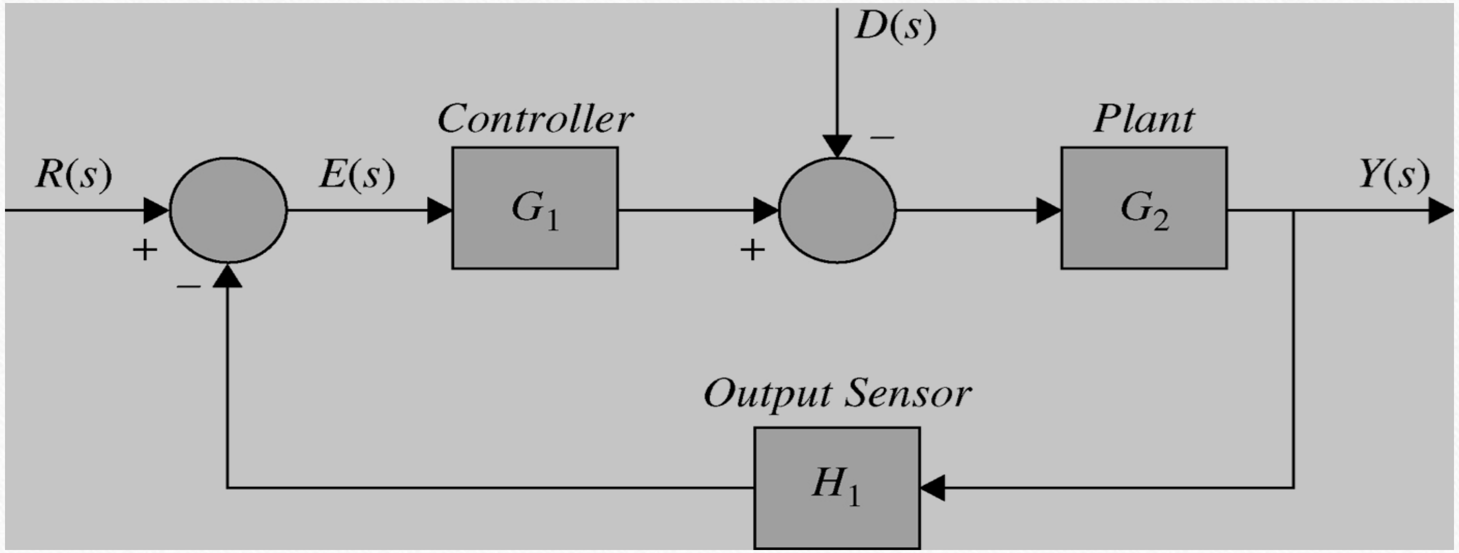
Then set R=0, find Y due to D

$$Y_2(s) = \frac{G_2}{1+G_1G_2H_1}\left(-D(s)\right)$$



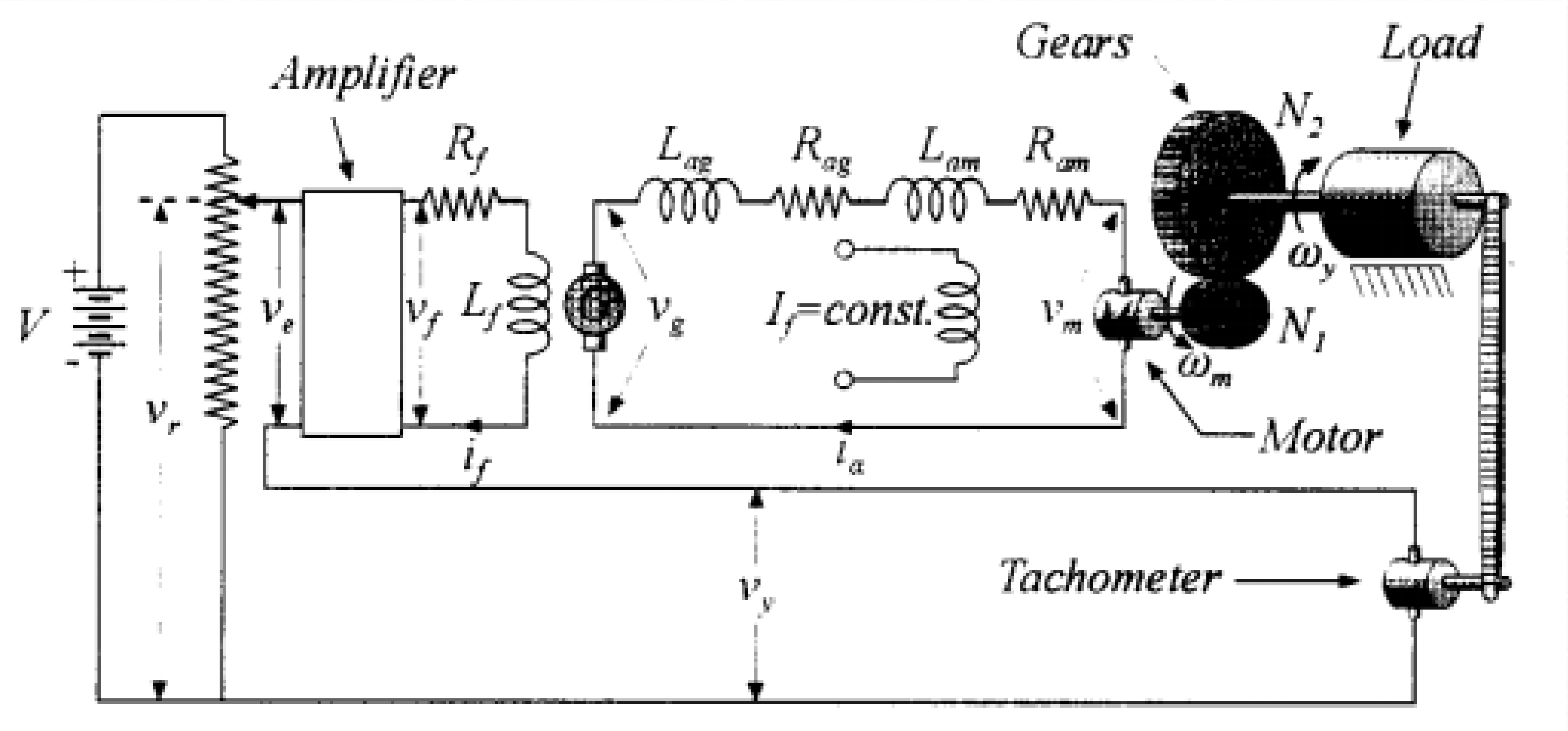
Block Diagram Reduction Technique

Finally, add the two component to get the overall Y



$$Y(s) = \frac{G_1G_2}{1+G_1G_2H_1}R(s) - \frac{G_2}{1+G_1G_2H_1}D(s)$$

Modeling of Motors



Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

The voltage v_g of the generator G is proportional to the current i_f, i.e.,

$$v_g = K_g i_f$$

The voltage v_m of the motor M is proportional to the angular velocity ω_m, i.e.,

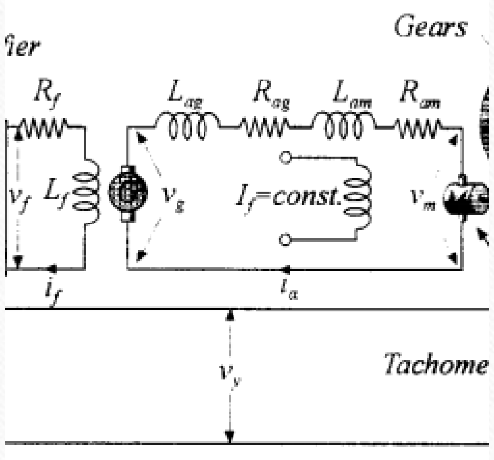
$$v_m = K_b \omega_m$$

The differential equation for the current i_a is

$$R_a i_a + L_a \frac{di_a}{dt} = v_g - v_m = K_g i_f - K_b \omega_m$$

The torque T_m of the motor is proportional to the current i_a

$$T_m = K_m i_a$$

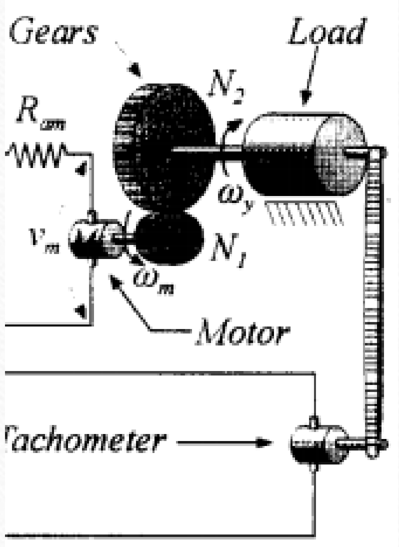


Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

The rotational motion of the rotor is described by

$$J_m^* \frac{d\omega_m}{dt} + B_m^* \omega_m = K_m i_a$$



where J_m*= J_m + $N^2 J_L$ and B_m* =B_m + N^2 B_L, where N = N_1/N_2. Here, J_m is the moment of inertia and B_m the viscosity coefficient of the motor: likewise, for J_L and B_L of the load.

where use was made of the relation

$$\omega_y = N\omega_m.$$

The tachometer equation

$$v_y = K_t \omega_y$$

the amplifier equation

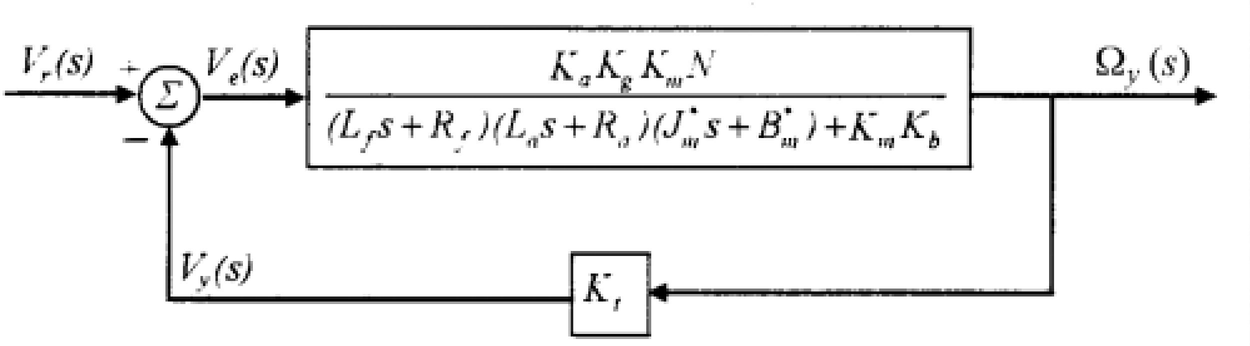
$$v_f = K_a v_e$$

Mathematical Modeling

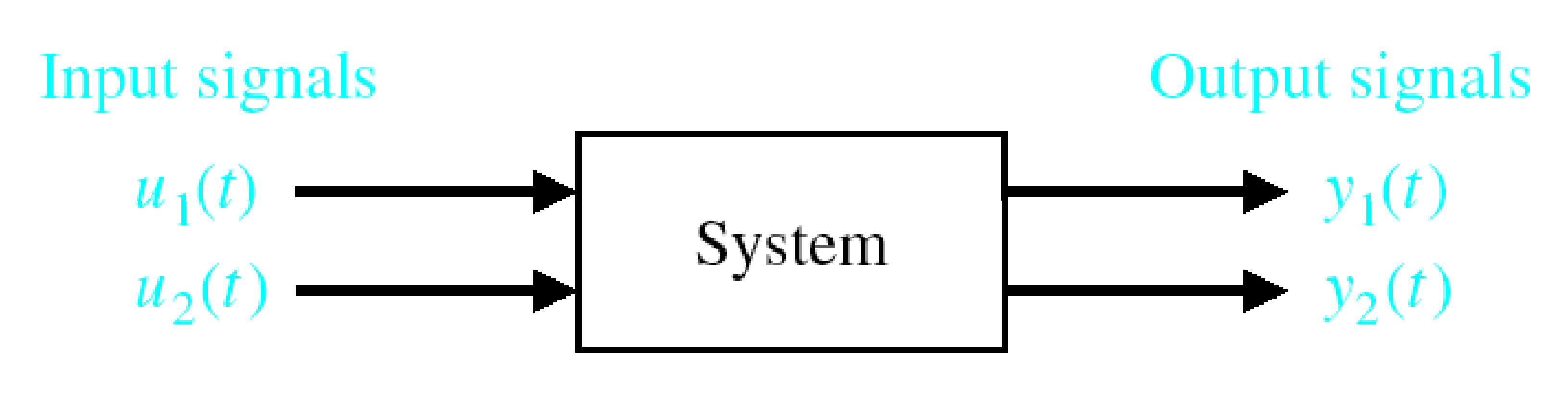
The mathematical model of the Ward–Leonard layout are as follows .

$$\frac{\Omega_y(s)}{V_f(s)} = \frac{K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$

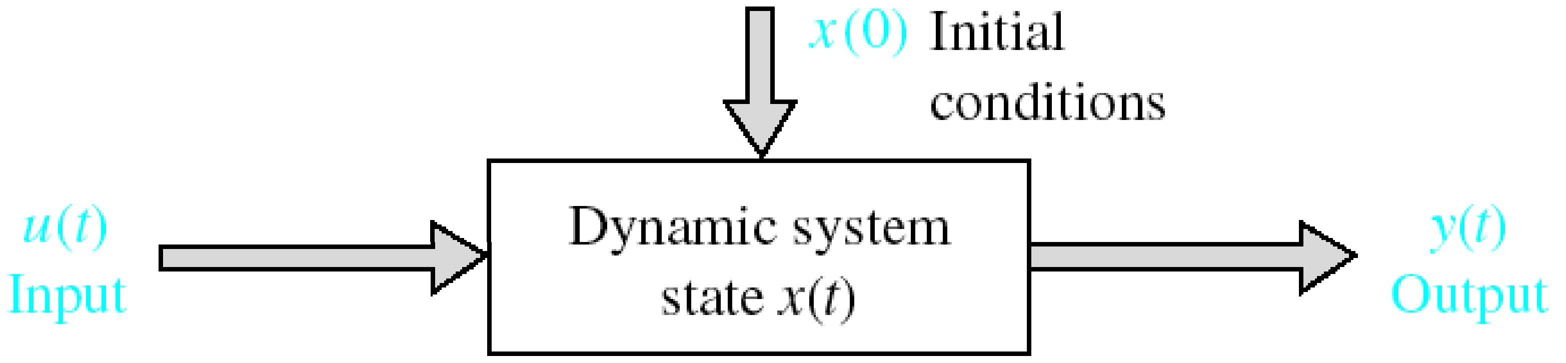
$$\frac{\Omega_y(s)}{v_e(s)} = \frac{K_a K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$



The state variables of a dynamic system



The general form of a dynamic system



State Space Equations

- **State equations** is a description which relates the following four elements: input, system, state variables, and output

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Matrix A has dimensions nxn and it is called the **system** matrix, having the general form

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Matrix B has dimensions nxm and it is called the **input** matrix, having the general form

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}$$

Matrix C has dimensions pxn and it is called the **output** matrix, having the general form

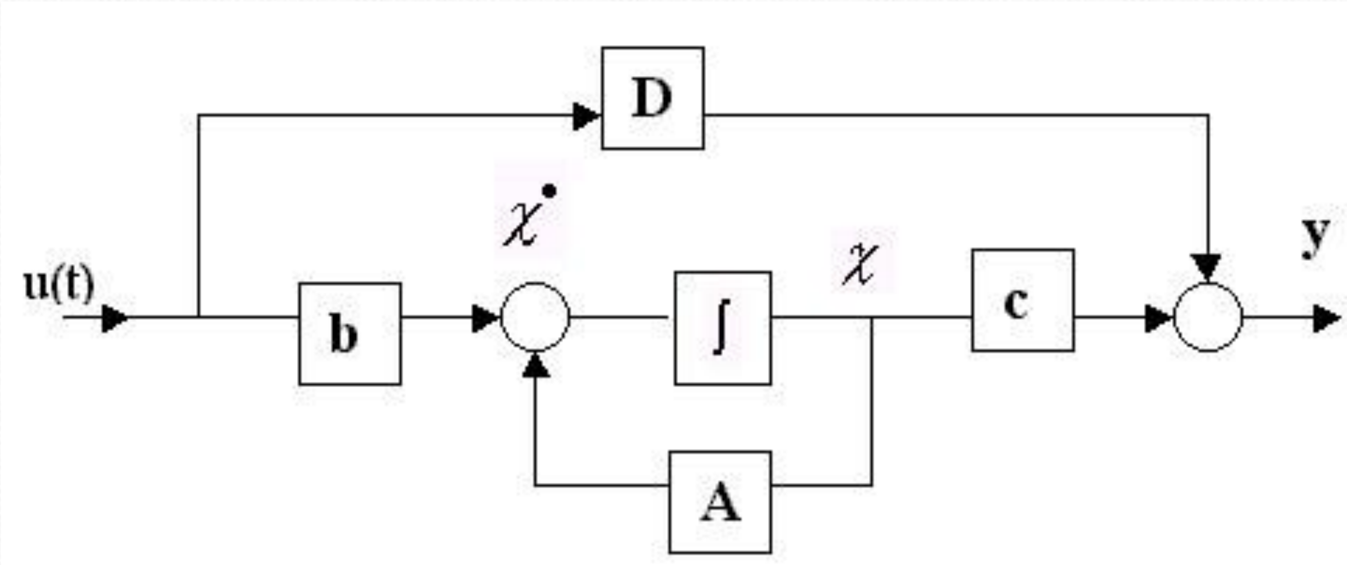
$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix}$$

Matrix D has dimensions pxm and it is called the **feedforward** matrix, having the general form

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix}$$

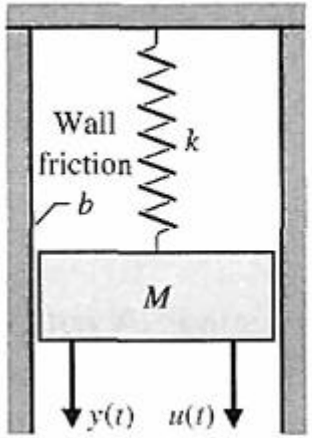
State Space Equations

$$SISO \Rightarrow \dot{X} = Ax(t) + Bu(t)$$
$$Y = Cx(t) + Du(t)$$



State Space representation

- **The general form of a dynamic system**



The concept of a set of state variables that represent a dynamic system can be illustrated in terms of the spring-mass-damper system. A set of state variables sufficient to describe this system includes the position and the velocity of the mass.

- We will define a set of state variables as *(x1, x2)*, where

$$x_1(t) = y(t) \quad \text{and} \quad x_2(t) = \frac{dy(t)}{dt}. \qquad \frac{dx_1}{dt} = x_2$$

To write Equation of motion in terms of the state variables, we substitute the state variables as already defined and obtain

$$M\frac{dx_2}{dt} + bx_2 + kx_1 = u(t)$$

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = u(t)$$

Therefore, we can write the equations that describe the behavior of the spring-mass damper system as the set of two first-order differential equations

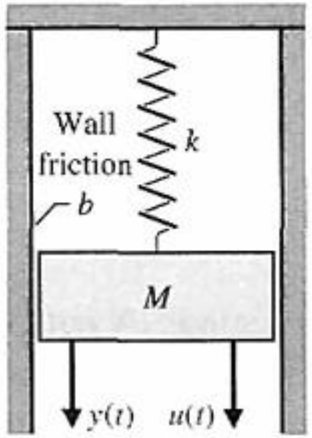
$$\frac{dx_2}{dt} = \frac{-b}{M}x_2 - \frac{k}{M}x_1 + \frac{1}{M}u$$

State Space representation

- **State space matrix**

$$\frac{dx_1}{dt} = x_2 \qquad \frac{dx_2}{dt} = \frac{-b}{M}x_2 - \frac{k}{M}x_1 + \frac{1}{M}u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \left[u_{(t)} \right]$$



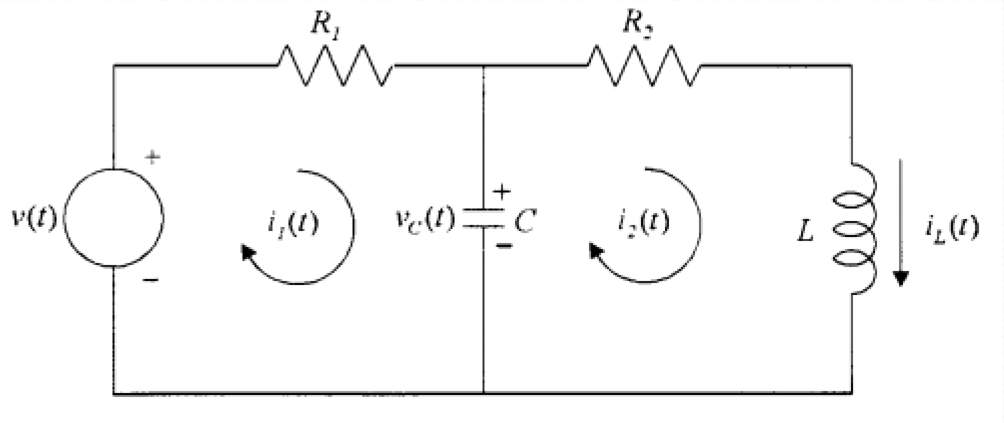
$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = u(t)$$

State Space representation

- Transfer from time domain to frequency domain:

$$R_1 i_1(t) + \frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t - \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = v(t)$$

$$\left[R_1 + \frac{1}{Cs}\right] I_1(s) - \frac{1}{Cs} I_2(s) = V(s)$$



$$-\frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t + R_2 i_2(t) + L\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = 0$$

$$-\frac{1}{Cs} I_1(s) + \left[R_2 + Ls + \frac{1}{Cs}\right] I_2(s) = 0$$

- Transfer function

$$\frac{I_2(s)}{V(s)} = \frac{Cs}{(R_1 Cs + 1)(LCs^2 + R_2 Cs + 1) - 1} = \frac{1}{R_1 LCs^2 + (R_1 R_2 C + L)s + R_1 + R_2}$$

State Space representation

$$\begin{cases} e(t) - R_1\, i_1(t) - L_1 \dfrac{di_1}{dt} - V_C(t) = \phi \\ V_C(t) - L_2 \dfrac{di_2}{dt} - R_2 i_2 = \phi \\ i_c = i_1 - i_2 = C \dfrac{dv_c}{dt} \end{cases}$$

$$x = (i_1 \;\; i_2 \;\; v_c)^T$$

$$X^{\bullet} = \begin{pmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-1}{L_1} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2} \\ \dfrac{1}{C} & \dfrac{-1}{C} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} \dfrac{1}{L_1} \\ 0 \\ 0 \end{pmatrix} e(t)$$

$$y(t) = (0 \quad R_2 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

State Space representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Laplace

$$\begin{cases} S\,x(s) - x(0) = Ax(s) + Bu(s) \\ Y(s) = C\,x(s) + Du(s) \end{cases}$$

$$X(S) = (SI - A)^{-1} Bu(s)$$
$$y(s) = (C(SI - A)^{-1} B + D)u(s)$$

Laplace

$$G(S) = C(SI - A)^{-1} B + D$$

$$G(S) = \frac{Q(S)}{|SI - A|}$$

State Space representation

$$\begin{cases}\dot{x}_1 = -5x_1 - x_2 + 2u \\ \dot{x}_2 = 3x_1 - x_2 + 5u\end{cases} \Rightarrow \dot{x} = \begin{pmatrix}-5 & -1 \\ 3 & -1\end{pmatrix} x + \begin{pmatrix}2 \\ 5\end{pmatrix} u$$

$$y = x_1 + 2x_2$$

$$y = (1 \quad 2)\, x$$

$$(SI - A) = \begin{pmatrix}2+5 & 1 \\ -3 & S+1\end{pmatrix}$$

$$(SI - A)^{-1} = \underbrace{\frac{1}{(S+5)(S+1)+3}}_{\Delta = (S+2)(S+4)} \begin{pmatrix}S+1 & -1 \\ 3 & S+5\end{pmatrix}$$

$$G(S) = [1 \quad 2]\frac{1}{\Delta}\begin{bmatrix}S+1 & -1 \\ 3 & S+5\end{bmatrix}\begin{bmatrix}2 \\ 5\end{bmatrix}$$

$$G(s) = \frac{12S + 59}{(S+2)(S+4)}$$

Model Examples

Thank you