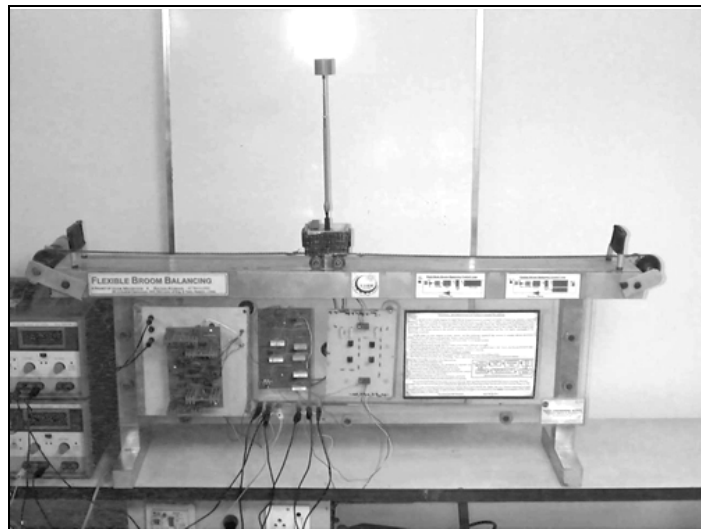# Inverted Pendulum

Analysis, Design and Implementation

Reference:

The work included in this document has been carried out in the **Instrumentation and Control Lab** at the Institute of Industrial Electronics Engineering, Karachi, Pakistan.

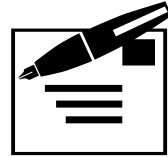# CONTENTS

WHAT'S INSIDE THIS REPORT
(CONTENTS IN DETAIL)

The "INVERTED PENDULUM, ANALYSIS, DESIGN AND IMPLEMENTATION" is a collection of MATLAB functions and scripts, and SIMULINK models, useful for analyzing Inverted Pendulum System and designing Control System for it.

This collection is developed by:

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http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=3087&objectType=FILE

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http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=1762&objectType=FILE

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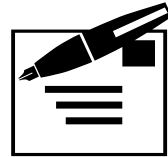Karachi - 75300, Pakistan.

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PREFACE

PREFACE

The "INVERTED PENDULUM, ANALYSIS, DESIGN AND IMPLEMENTATION" is a collection of MATLAB functions and scripts, and SIMULINK models, useful for analyzing Inverted Pendulum System and designing Control System for it.

This report & MATLAB-files collection are developed as a part of practical assignment on Control System Analysis, Design & Development practical problem. The assigned problem of INVERTED PENDULUM is a part of Lab Work of **Control System – III** Course at the INSTITUTE OF INDUSTRIAL ELECTRONICS ENGINEERING (IIEE), KARACHI, PAKISTAN.

The Inverted Pendulum is one of the most important classical problems of Control Engineering. Broom Balancing (Inverted Pendulum on a cart) is a well known example of nonlinear, unstable control problem. This problem becomes further complicated when a flexible broom, in place of a rigid broom, is employed. Degree of complexity and difficulty in its control increases with its flexibility. This problem has been a research interest of control engineers.

Control of Inverted Pendulum is a Control Engineering project based on the FLIGHT SIMULATION OF ROCKET OR MISSILE DURING THE INITIAL STAGES OF FLIGHT. The **AIM OF THIS STUDY** is to stabilize the Inverted Pendulum such that the position of the carriage on the track is controlled quickly and accurately so that the pendulum is always erected in its inverted position during such movements.

This practical exercise is a presentation of the analysis and practical implementation of the results of the solutions presented in the papers, "***Robust Controller for Nonlinear & Unstable System*: Inverted Pendulum**" [3] and "***Flexible Broom Balancing***" [4], in which this complex problem was analyzed and a simple yet effective solution was presented. The details of these papers can be looked in the BIBLIOGRAPHY section.

CONTENT OVERVIEW
This report comprises of EIGHT (8) major sections.

- ✓ **Section 1** introduces the classical control problem of Inverted Pendulum, and provides the details of the problem from the control engineering aspects. It also puts light on the possible applications of this problem.
- ✓ **Section 2** explores the mathematical model of the Inverted Pendulum System.
- ✓ **Section 3** provides the details of the analysis of the uncompensated system. The analysis includes the pole-zero map, impulse response and step response of the uncompensated open-loop system and root locus of the uncompensated system.
- ✓ **Section 4** explores the possible ways of designing the required control system for the Inverted Pendulum System.
- ✓ **Section 5** explains how the control system can be designed using Root-Locus techniques. The designing is done in MATLAB, using the SISO Design Tool. A brief primer to SISO Design Tool is also included in the report in this section.
- ✓ **Section 6** provides the details of the analysis of the compensated system. The analysis includes the pole-zero map of the PID compensated open-loop system and root locus, impulse response and step response of the PID compensated closed-loop system. In the conclusion of the Compensation Analysis section, it has been overviewed that how much of the compensation goals have been achieved.
- ✓ **Section 7** details the different ways of practically implementing the designed PID controller. It shows different circuit configurations for achieving the required transfer function.

- ✓ **Section 8** includes the practical results obtained recorded using a 8051-based Data Acquisition Card. Then the obtained experimental data is filtered to remove the unwanted high frequency components introduced due to sampling.
- ✓ **Appendix** contains the MATLAB m-files, used for analyzing uncompensated Inverted Pendulum System, designing Control System for it, and then analyzing the PID compensated system.

ACKNOWLEDGEMENTS

KHALIL SULTAN
July 25, 2003
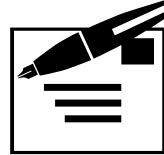
Remember when you were a child and you tried to balance a broom-stick on your index finger or the palm of your hand? You had to constantly adjust the position of your hand to keep the object upright. An INVERTED PENDULUM does basically the same thing. However, it is limited in that it only moves in one dimension, while your hand could move up, down, sideways, etc.

Just like the broom-stick, an Inverted Pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required. The Inverted Pendulum is essential in the evaluating and comparing of various control theories.

The *inverted pendulum* (IP) is among the most difficult systems to control in the field of control engineering. Due to its importance in the field of control engineering, it has been a task of choice to be assigned to Control Engineering students to analyze its model and propose a linear compensator according to the PID control law. Being an unstable system, Inverted Pendulum is very common control problem being assigned to a student of Control System Engineering (from Bachelor to Postgraduate level), to control it's dynamics.

The reasons for selecting the IP as the system are:

- It is the most easily available system (in most academia) for laboratory usage.
- It is a nonlinear system, which can be treated to be linear, without much error, for quite a wide range of variation.
- Provides a good practice for prospective control engineers.

The various stages of the work for accomplishing the task of controlling the Inverted Pendulum are as follows:

- Modeling the IP and linearizing the model for the operating range.
- Analyzing the uncompensated closed loop response with the help of a root locus plot.
- Designing the PID controller and simulating it in MATLAB for proper tuning and verification.
- Analyzing the compensated closed loop response of the system.
- Implementing the controller on the physical IP model.

APPLICATIONS OF INVERTED PENDULUM

Among the some considerable applications of *inverted pendulum* (IP) are:

✓ SIMULATION OF DYNAMICS OF A ROBOTIC ARM

The Inverted Pendulum problem resembles the control systems that exist in robotic arms. The dynamics of Inverted Pendulum simulates the dynamics of robotic arm in the condition when the center of pressure lies below the centre of gravity for the arm so that the system is also unstable. Robotic arm behaves very much like **Inverted Pendulum** under this condition.

✓ MODEL OF A HUMAN STANDING STILL

The ability to maintain stability while standing straight is of great importance for the daily activities of people. The central nervous system (CNS) registers the pose and changes in the pose of the human body, and activates muscles in order to maintain balance.

The *inverted pendulum* is widely accepted as an adequate model of a human standing still (quiet standing).

An inverted pendulum (assuming no attached springs) is unstable, and it is hence obvious that feedback of the state of the pendulum is needed to stabilize the pendulum.

Two models for the CNS feedback control are generally considered:
    → Time invariant, linear feedback control;
    → Linear feedback outside a threshold. No sensory feedback within the threshold.

There are certain passive mechanisms, such as stiffness in muscles and supportive tissue, which may be modeled as a spring and damper.
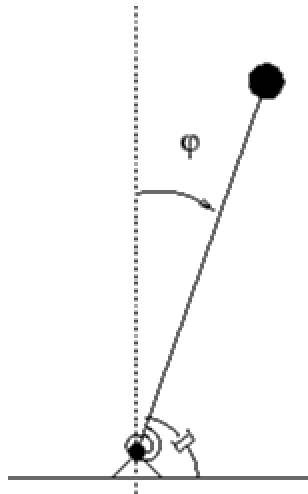
INVERTED PENDULUM WITH
PASSIVE MECHANISMS MODELED
AS A SPRING AND DAMPER

The spring and damper leads to a negative feedback loop that could be enough to stabilize the pendulum, if the spring is stiff enough.

More details about this application of Inverted Pendulum System can be looked at [15].
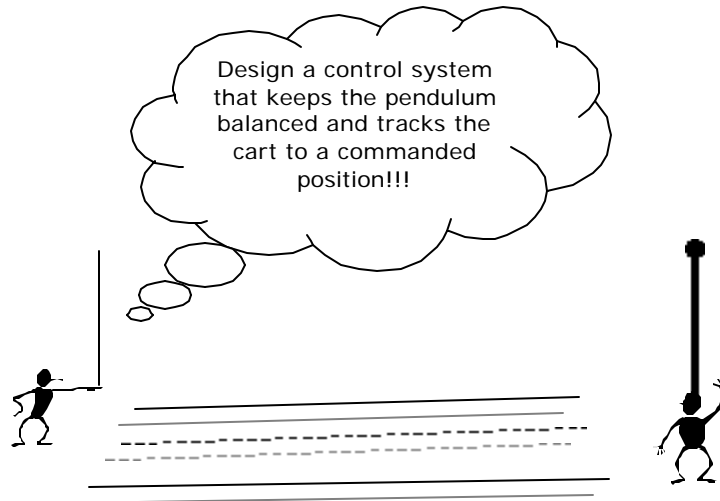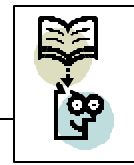
It is virtually impossible to balance a pendulum in the inverted position without applying some external force to the system. The Carriage Balanced Inverted Pendulum (CBIP) system, shown below, allows this control force to be applied to the pendulum carriage. This CBIP provides the control force to the carriage by means of a DC servo-motor through a belt drive system. The outputs from the CBIP rig can be carriage position, carriage velocity, pendulum angle and pendulum angular velocity (only pendulum angle in our case). The pendulum angle is fed back to an Analog Controller which controls the servo-motor, ensuring consistent and continuous traction. The **AIM OF THE STUDY** is to stabilize the pendulum such that the position of the carriage on the track is controlled quickly and accurately and that the pendulum is always maintained tightly in its inverted position during such movements.

The problem involves a cart, able to move backwards and forwards, and a pendulum, hinged to the cart at the bottom of its length such that the pendulum can move in the same plane as the cart, shown below. That is, the pendulum mounted on the cart is free to fall along the cart's axis of motion. The system is to be controlled so that the pendulum remains balanced and upright, and is resistant to a step disturbance.
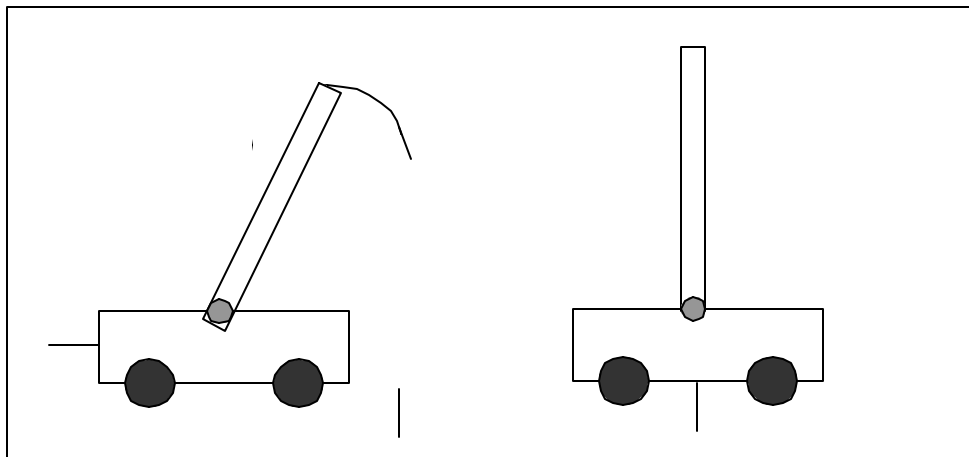
This problem involves **A SIMPLE COUPLED SYSTEM**. If the pendulum starts off-centre, it will begin to fall. The pendulum is coupled to the cart, and the cart will start to move in the opposite direction, just as moving the cart would cause the pendulum to become off centre. As change to one of parts of the system results in change to the other part, this

is a more complicated control system than it appears at first glance. For this reason, this problem is often used as a demonstration of fuzzy control.

The inverted pendulum cart runs along a track and is pulled by a belt connected to an electric motor. A potentiometer measures the cart position from its rotation and another potentiometer measures the angle of the pendulum.

If the output is the angle of the pendulum relative to the vertical axis (in upright position), we realize that the system is unstable, since the pendulum will fall down if we release it with a small angle. To stabilize the system, i.e., to keep the pendulum in upright position, a feedback control system must be used.
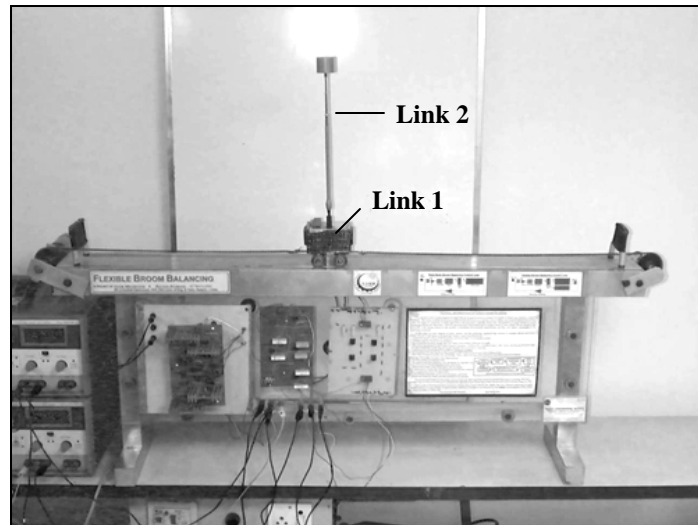
The inverted pendulum is an excellent test bed for linear control theory. In this classic inverted pendulum control experiment, we seek a feedback control law that balances Link 2 in its unstable, inverted position. The first link undergoes linear translation, our first link rotates.

The above photo shows the mechanism in its stable, pendant position.

So briefly, the Inverted Pendulum system is made up of a cart and a pendulum. The goal of the controller is to move the cart to its commanded position without causing the pendulum to tip over. In open loop this system is unstable.

THE TASK ASSIGNED IS: to **analyze, design & develop** a control loop for the given inverted pendulum (with servomechanism).

Following is the over all block-diagram for the feedback control system: [NEXT PAGE]

q_R → A → (⊝) e → PID CONTROLLER → E_A → Cart Pulley & Servo-mechanis → F → INVERTED PENDULUM → (⊕) → q_C

E_F (feedback, −)

U₁ (DISTURBANCE IN FORCE ON CART)

U₂ (DISTURBANCE IN POSITION OF BROOM)

SERVOMECHANISM
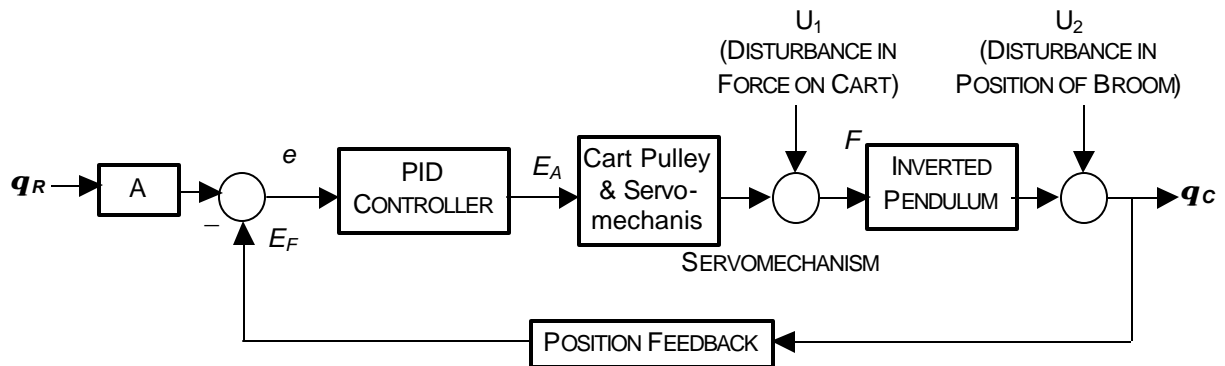
POSITION FEEDBACK

Our implementation contains only feedback from the pendulum angle (that is, only one out of the four states is used for feedback, the other states being carriage position, carriage velocity and pendulum angular velocity). The implementation may be enhanced by incorporation of cart-position control loop.

In this problem, the pendulum is first positioned upright manually, that is, in a position of unstable equilibrium, or it is given some initial displacement (position). The controller is then switched in to balance the pendulum and to maintain this balance in the presence of disturbances. A simple disturbance may be a light tap on the balanced pendulum. A complex disturbance may be gusts of wind (use a fan!).

This setup can be used to study the control of open loop unstable system. It is a demonstration of the stabilizing benefits of feedback control. A range of control techniques ranging from the simple phase advance compensator to neural net controllers can be applied.

SECTION 2

MATHEMATICAL WORK
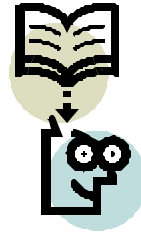
An **inverted pendulum** is a classic control problem. The process is non linear and unstable with one input signal and several output signals. The aim is to balance a pendulum vertically on a motor driven wagon.

The following figure shows an inverted pendulum. The aim is to move the wagon along the x direction to a desired point without the pendulum falling. The wagon is driven by a DC motor, which is controlled by a controller (analog in our implementation). The wagons x position (not in our case) and the pend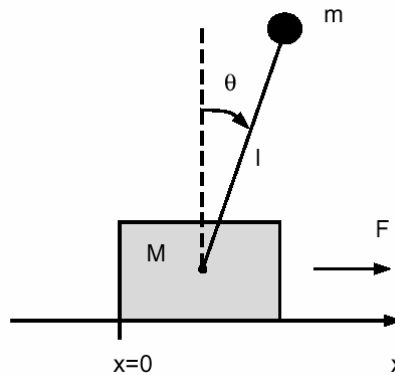ulum angle è are measured and supplied to the control system. A disturbance force, $F_{DISTURBANCE}$, can be applied on top of the pendulum.

A mathematical model of the system has been developed, giving the angle of the pendulum resulting from a force applied to the base.

Setup Description

The inverted pendulum is mounted on a moving *cart*. A servomotor is controlling the translation motion of the cart, through a belt/pulley mechanism. That is, the cart is coupled with a servo dc-motor through pulley and belt mechanism. The motor is derived by *servo* electronics, which also contains *controller-circuits*. A *rotary-potentiometer* is used to *feedback* the angular motion of the pendulum to servo electronics to generate *actuating-signal*.

Controller circuits process the error signal, which then drives the cart through the servomotor and driving pulley/belt mechanism. To-or/and-fro motion of the cart applies moments on the inverted pendulum and thus it keeps the pendulum *upright*.

Inverted Pendulum System Equations

The Free Body Diagram of the system is used to obtain the equations of motion. Below are the two Free Body Diagrams of the system.
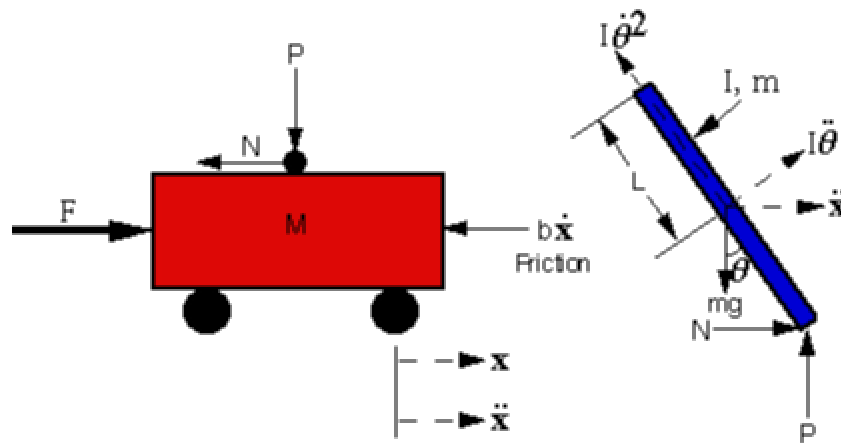
Summing the forces in the Free Body Diagram of the cart in the horizontal direction, you get the following equation of motion:

[1] $\qquad M\ddot{x} + b\dot{x} + N = F$

Note that you could also sum the forces in the vertical direction, but no useful information would be gained. The sum of forces in the vertical direction is not considered because there is no motion in this direction and we consider that the reaction force of the earth balances all the vertical forces.

The force exerted in the horizontal direction due to the moment on the pendulum is determined as follows:

$$t = r \times F = I\ddot{q}$$

$$F = \frac{I\ddot{q}}{r}$$

$$= \frac{ml^2 \ddot{q}}{l}$$

$$= ml\ddot{q}$$

Component of this force in the direction of N is $ml\ddot{q}\cos q$.

The component of the centripetal force acting along the horizontal axis is as follows:

$$F = \frac{I\dot{q}^2}{r}$$

$$= \frac{ml^2 \dot{q}^2}{l}$$

$$= ml\dot{q}^2$$

Component of this force in the direction of N is $ml\dot{q}^2 \sin q$.

Summing the forces in the Free Body Diagram of the pendulum in the horizontal direction, you can get an equation for N:

[2] $$N = m\ddot{x} + ml\ddot{q}\cos q - ml\dot{q}^2 \sin q$$

If you substitute this equation [2] into the first equation [1], you get the first equation of motion for this system:

[3] $$(M + m)\ddot{x} + b\dot{x} + ml\ddot{q}\cos q - ml\dot{q}^2 \sin q = F$$

To get the second equation of motion, sum the forces perpendicular to the pendulum. This axis is chosen to simplify mathematical complexity. Solving the system along this axis ends up saving you a lot of algebra. Just as the previous equation is obtained, the vertical components of those forces are considered here to get the following equation:

[4] $$P\sin q + N\cos q - mg\sin q = ml\ddot{q} + m\ddot{x}\cos q$$

To get rid of the P and N terms in the equation above, sum the moments around the centroid of the pendulum to get the following equation:

[5] $$-Pl\sin q - Nl\cos q = I\ddot{q}$$

Combining these last two equations, you get the second dynamic equation:

[6] $$(I + ml^2)\ddot{q} + mgl\sin q = -ml\ddot{x}\cos q$$

The set of equations completely defining the dynamics of the inverted pendulum are:

[3] $$(M + m)\ddot{x} + b\dot{x} + ml\ddot{q}\cos q - ml\dot{q}^2 \sin q = F$$

[6] $$(I + ml^2)\ddot{q} + mgl\sin q = -ml\ddot{x}\cos q$$

These two equations are non-linear and need to be linearized for the operating range. Since the pendulum is being stabilized at an unstable equilibrium position, which is 'Pi' radians from the stable equilibrium position, this set of equations should be linearized about theta = Pi. Assume that theta = Pi + ø, (where ø represents a small angle from the vertical upward direction).
Therefore, cos (theta) = -1, sin (theta) = -ø, and (d(theta)/dt)^2 = 0.

After linearization the two equations of motion become (where u represents the input):

[7] $$(M + m)\ddot{x} + b\dot{x} - ml\ddot{f} = u$$

[8] $$(I + ml^2)\ddot{f} - mglf = ml\ddot{x}$$

To obtain the transfer function of the linearized system equations analytically, we must first take the Laplace transform of the system equations. The Laplace transforms are:

$$(M + m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s)$$

$$(I + ml^2)\Phi(s)s^2 - mgl\Phi(s) = mlX(s)s^2$$

When finding the transfer function, initial conditions are assumed to be zero. The transfer function relates the variation from desired position [Output] to the force on the cart [Input].

Since we will be looking at the angle Phi as the output of interest, solve the first equation for X(s),

$$X(s) = [\frac{(I + ml^2)}{ml} - \frac{g}{s^2}]\Phi(s)$$

Then, substituting into the second equation will yield:

$$(M + m)[\frac{(I + ml^2)}{ml} + \frac{g}{s}]\Phi(s).s^2 + b[\frac{(I + ml^2)}{ml} + \frac{g}{s}]\Phi(s).s - ml\Phi(s).s^2 = U(s)$$

Re-arranging, the transfer function is:

$$\frac{\Phi(s)}{U(s)} = \frac{\dfrac{ml}{q} \cdot s^2}{s^4 + \dfrac{b(I + ml^2)}{q} \cdot s^3 - \dfrac{mgl(M + m)}{q} \cdot s^2 - \dfrac{bmgl}{q}.s}$$

where,
$$q = (M + m)(I + ml^2) - (ml)^2.$$

From the transfer function above it can be seen that there is both a pole and a zero at the origin. These can be canceled and the transfer function becomes:

$$\frac{\Phi(s)}{U(s)} = \frac{\dfrac{ml}{q} \cdot s}{s^3 + \dfrac{b(I + ml^2)}{q} \cdot s^2 - \dfrac{mgl(M + m)}{q} \cdot s - \dfrac{bmgl}{q}}$$

The transfer function can thus be simplified as:

$$\frac{\Phi(s)}{U(s)} = \frac{ml \cdot s}{q \cdot s^3 + b(I + ml^2) \cdot s^2 - mgl(M + m) \cdot s - bmgl}$$
$$\text{where} \quad q = (M + m)(I + ml^2) - (ml)^2.$$

If we NEGLECT THE FRICTION in the system, that is, we take the coefficient of friction **b=0**, then

$$\frac{\Phi(s)}{U(s)} = \frac{K_p}{s^2/A_p^2 - 1}$$

$$\text{where} \quad K_p = \frac{1}{(M + m)g}, \text{and} \quad A_p = \pm\sqrt{\frac{(M + m)mgl}{(M + m)(I + ml^2) - (ml)^2}}$$

Thus, the LINEARIZED APPROXIMATION TRANSFER FUNCTION for the IP has been obtained. In time domain, the transfer function can be stated as:

$$\frac{\Phi(t)}{u(t)} = \frac{Kp}{D^2/Ap^2 - 1}$$

Actuation Mechanism

The actuation mechanism consists of a **movable cart** (on rail), driven by a **DC motor** via **a pulley and belt**. So the overall transfer function of the actuation mechanism will depend upon the transfer function of "the DC Motor" and "the Pulley, Belt & Cart".

PULLEY, BELT & CART

Load-Inertia to the motor consists of pulley (of radius *r*) and masses of cart and pendulum. The load-torque, to be delivered by the motor is given as:

$$T_L = (M+m).r^2.D\boldsymbol{w}$$

Note: $\boldsymbol{T_L \mu r^2}$ *and* $\boldsymbol{F \mu r}$

MOTOR

The dynamics of motor will also affect the transfer function of the actuation mechanism. Experimental transfer function for the armature-controlled servo dc-motor is given as:

$$\boldsymbol{w} = K_m \frac{E}{\boldsymbol{t}D+1}$$

Where **t** is the time constant, and it depends upon the load drive. That is, heavier the load higher will be the value of **t**. K_M (radians/second/volts) is steady-state gain.

So the overall transfer function of the actuation mechanism is:

$$\frac{U(s)}{E(s)} = K_m \frac{(M+m)rs}{(\boldsymbol{t}_m s+1)}$$

Transfer Function of the Whole System

Open loop and linearized transfer function for the whole (uncontrolled) system can be given as:

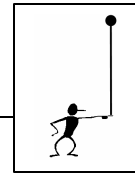$$\frac{\Phi(s)}{E(s)} = K \frac{s}{(\boldsymbol{t}_m s+1)(\frac{s^2}{A_p^2}-1)}$$

where $K = K_F K_P K_M r (M+m),$
E (s) = Error Voltage, and
Φ (s) = Angular Position of the Pendulum.

Most of the modelling and system analysis has been taken from [3],[4] and [13].

SYSTEM PARAMETERS

The physical parameters of the system prototype are tabulated as follows:

M	Mass of the Cart	*900 gm*
m	Mass of the Pendulum	*100 gm*
b	Friction of the Cart	*0.000 N/m/sec*
L	Length of pendulum to Center of Gravity	*23.5 cm*
I	Moment of Inertia (Pendulum)	*5.3 gm-m²*
R	Radius of Pulley,	*2.3 cm*
$\boldsymbol{t}_M$	Time Constant of motor	*0.5 second*
$\mathbf{K}_M$	Gain of Motor	*17 rad/sec/V*
$\mathbf{K}_F$	Gain of Feedback	*9/$\boldsymbol{p}$ V/rad/sec*
F	Force applied to the cart	
x	Cart Position Coordinate	
$\boldsymbol{q}$	Pendulum Angle with the vertical	

SECTION 3

# ANALYSIS OF UNCOMPENSATED SYSTEM
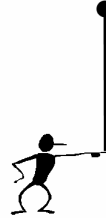
ANALYSIS OF UNCOMPENSATED SYSTEM
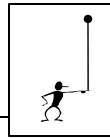
POLE ZERO MAP OF UNCOMPENSATED OPEN LOOP SYSTEM

The poles position of the linearized model of Inverted Pendulum (in open loop configuration) shows that system is unstable, as one of the poles of the transfer function lies on the Right Half Side of the *s*-plane. Thus the system is absolutely unstable.

Pole-Zero Map of Open-Loop Uncompensated Inverted Pendulum System
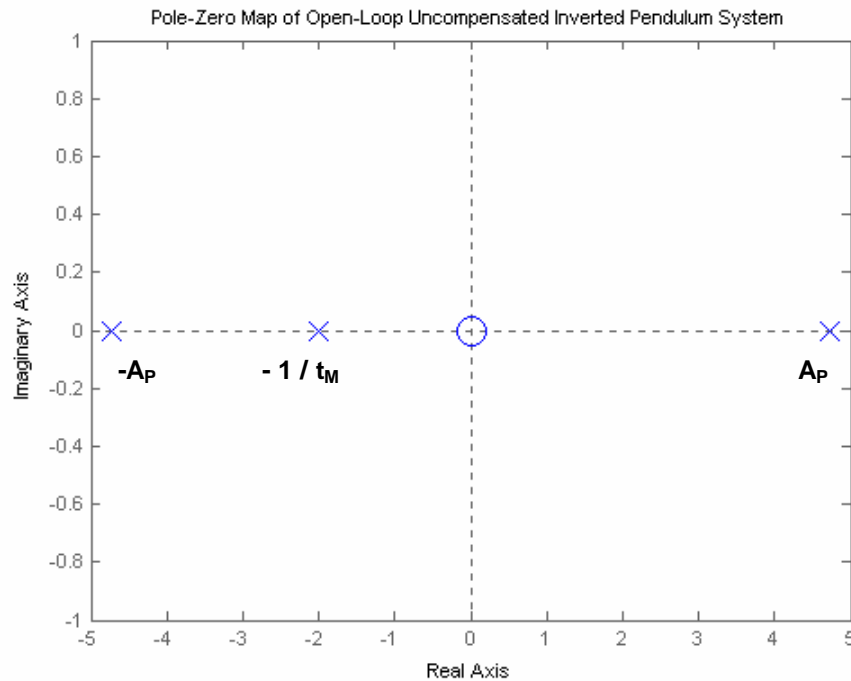
IMPULSE RESPONSE OF UNCOMPENSATED OPEN LOOP SYSTEM

An impulse response of the system is shown in following figure. The system is highly unstable as theta diverges very rapidly. The runaway nature of the response indicates instability.

Impulse Response of Open Loop Uncompensated Inverted Pendulum System
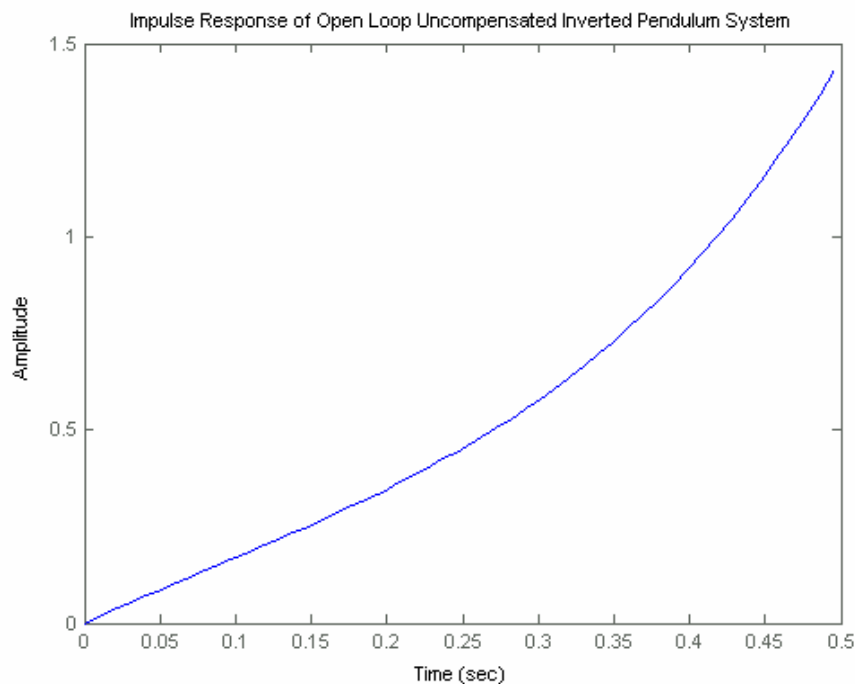
STEP RESPONSE OF UNCOMPENSATED OPEN LOOP SYSTEM

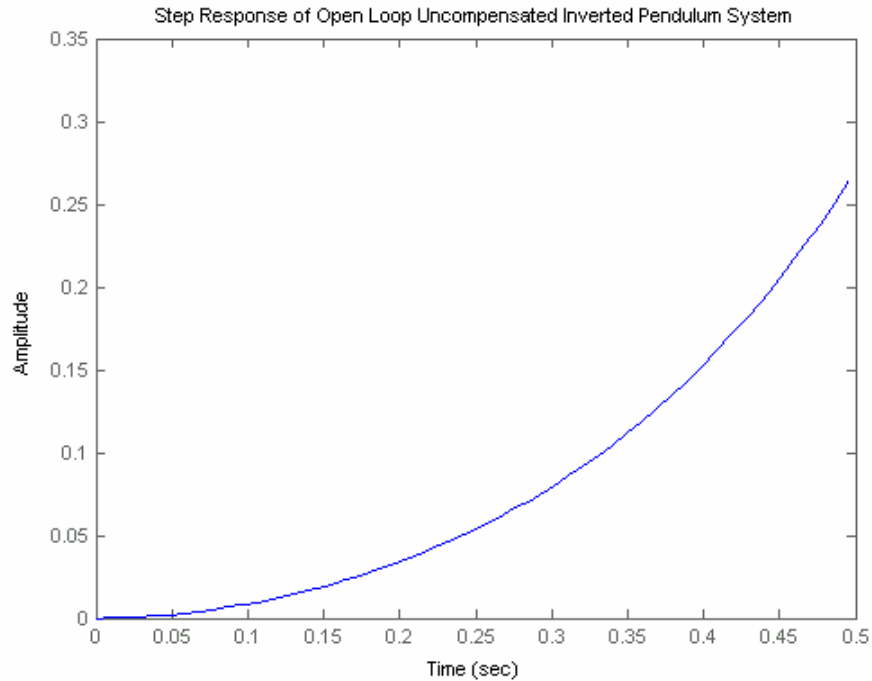A step response of the system is shown in following figure. Here also, theta diverges very rapidly as the system is highly unstable. The runaway nature of the response indicates instability.

Step Response of Open Loop Uncompensated Inverted Pendulum System

ROOT LOCUS OF THE UNCOMPENSATED SYSTEM

The first step in designing compensation for any plant is to observe the closed loop unity feedback response to check for stability. Many systems are unstable in open loop but stable in closed loop configuration. The other way round is also possible that the system is stable in the open loop but unstable in closed loop, although this case is rare.

The closed loop uncompensated system can be studied by viewing the root locus plot of the system. Following figure shows the root locus plot of the system.

Root Locus of Uncompensated Inverted Pendulum System
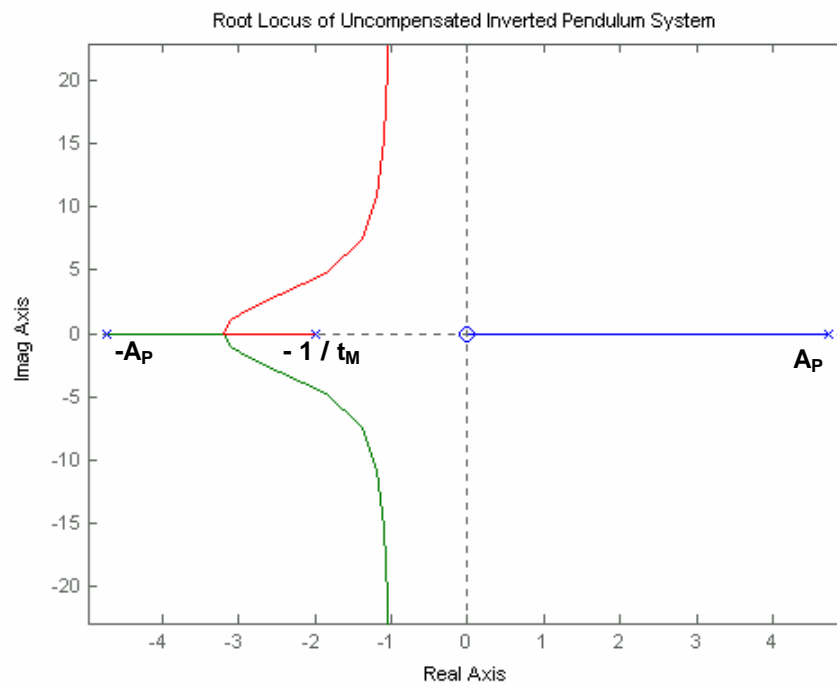
The plot reveals that the system cannot be controlled by a simple unity feedback loop. Whatever be the value of loop gain, ($\forall$K), one branch of the locus remains on RHS (in unstable region) of s-plane. This makes control impossible by unity feedback.

The root locus has a branch on the right hand side of the imaginary axis, which indicates that the system is unstable in closed loop for all values of K.

From the above analysis, it is concluded that using only the gain compensation in closed loop cannot control the IP. RESHAPING OF THE SYSTEM ROOT LOCUS is necessary so that for certain range of gains, the system has all its roots in the left half plane (stable region) of the s-plane.

(1) SIMULINK MODEL FOR THE OPEN LOOP IMPULSE RESPONSE OF THE INVERTED PENDULUM SYSTEM
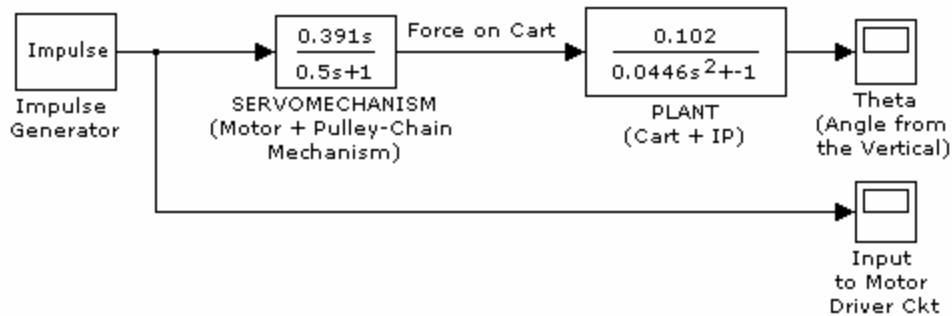
FILE: `ipolimpulse.mdl`

Here the transfer function of Servomechanism is evaluated from the `U_E` variable and that of the Plant from the `Th_U` variable.

These variables are evaluated in `trans_func_ip.m` file.

The impulse is applied at 0.5 s.

SIMULATION PARAMETERS:	
Start Time:	0
Stop Time:	1.5
Solver Algorithm:	Variable-step
	ODE45 (Dormand-Prince)
Maximum Step Size:	0.05

Open Loop Impulse Response of Uncompensated Inverted Pendulum System

Impulse — Impulse Generator

$\dfrac{0.391s}{0.5s+1}$ — SERVOMECHANISM (Motor + Pulley-Chain Mechanism) — Force on Cart

$\dfrac{0.102}{0.0446s^2+-1}$ — PLANT (Cart + IP)

Theta (Angle from the Vertical)

Input to Motor Driver Ckt

Click Here for more Information on IP

This model shows the impulse response of Inverted Pendulum System. The system is highly unstable as Theta diverges very rapidly.

The impulse input to the Inverted Pendulum (Motor Driver Card) is:

Impulse Input to the Inverted Pendulum System

The impulse response of open-loop uncompensated system is shown below:
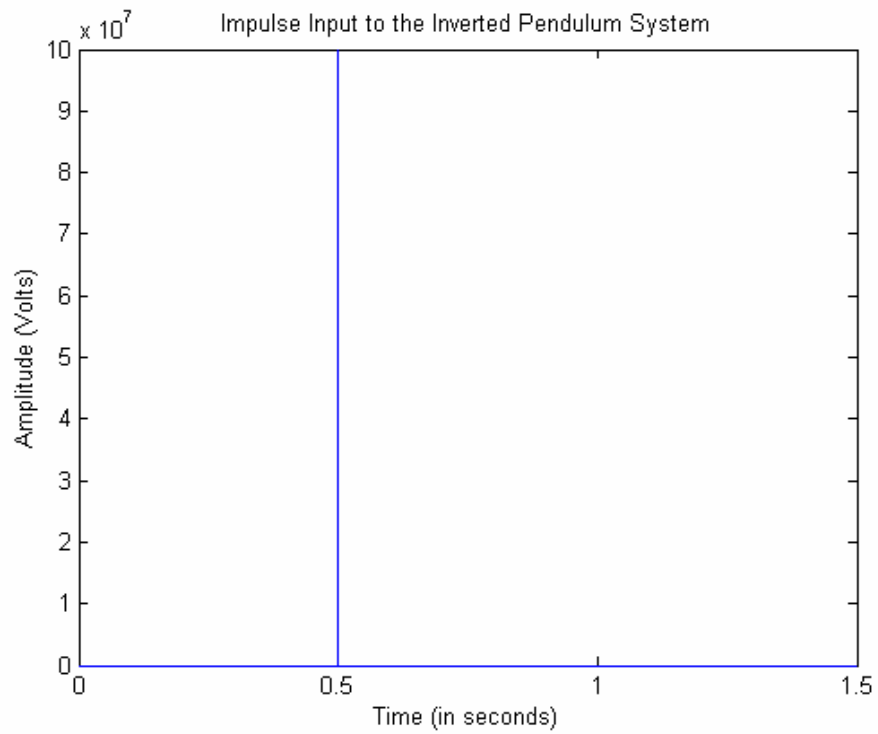
Impulse Response of Open Loop Uncompensated System

(2) SIMULINK MODEL FOR THE OPEN LOOP STEP RESPONSE OF THE INVERTED PENDULUM SYSTEM

FILE: ipolstep.mdl

Here the transfer function of Servomechanism is evaluated from the U_E variable and that of the Plant from the Th_U variable.

These variables are evaluated in trans_func_ip.m file.

The step is applied at 1 s.

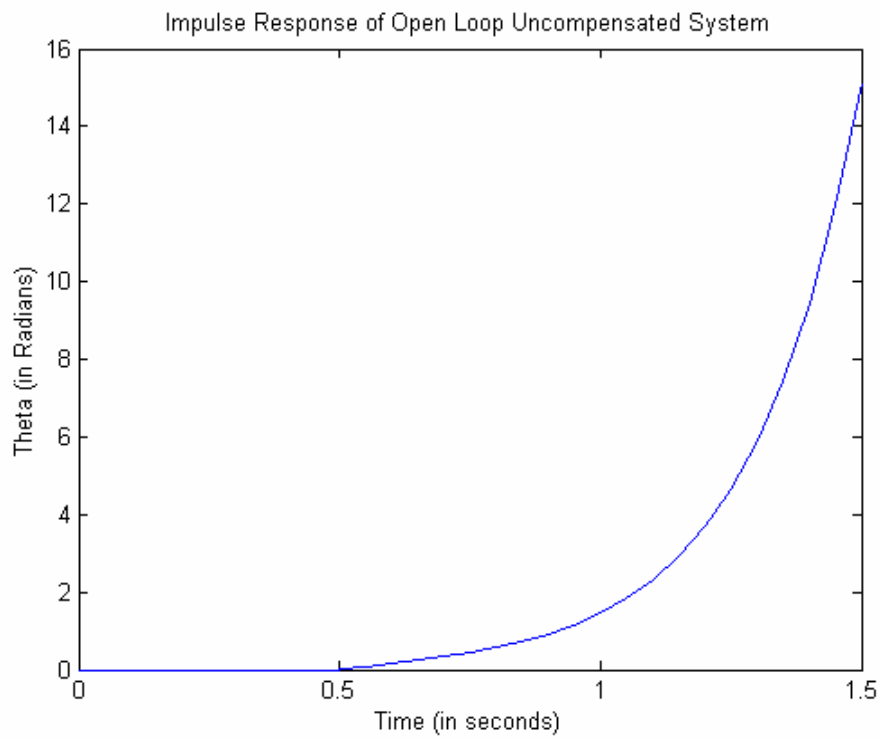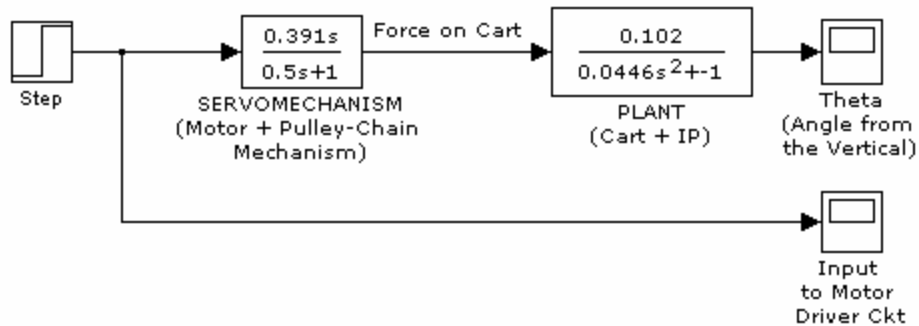| SIMULATION PARAMETERS:	
Start Time:	0
Stop Time:	1.5
Solver Algorithm:	Variable-step ODE45 (Dormand-Prince)
Maximum Step Size:	0.05

Open Loop Step Response of Uncompensated Inverted Pendulum System

Step

$$\frac{0.391s}{0.5s+1}$$

SERVOMECHANISM
(Motor + Pulley-Chain
Mechanism)

Force on Cart

$$\frac{0.102}{0.0446s^2+-1}$$

PLANT
(Cart + IP)

Theta
(Angle from
the Vertical)

Input
to Motor
Driver Ckt

Click Here for more
Information on IP

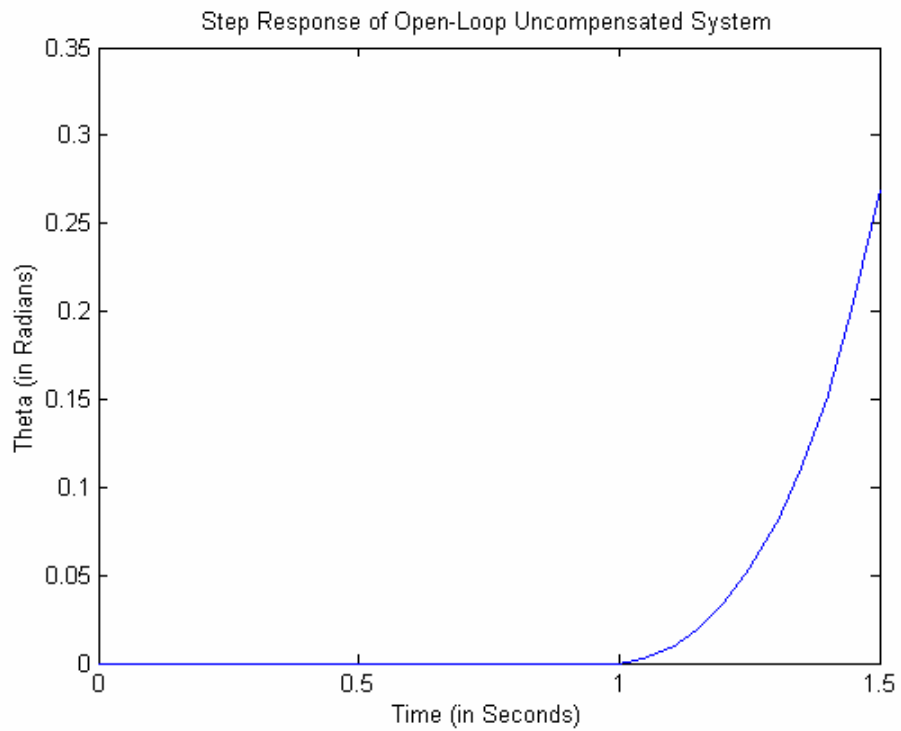This model shows the step response of Inverted Pendulum System. Here initially the Inverted Pendulum is at vertical position. It also shows that the system is highly unstable as Theta diverges very rapidly, when step input comes.

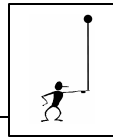The step input to the Inverted Pendulum (Motor Driver Card) is:
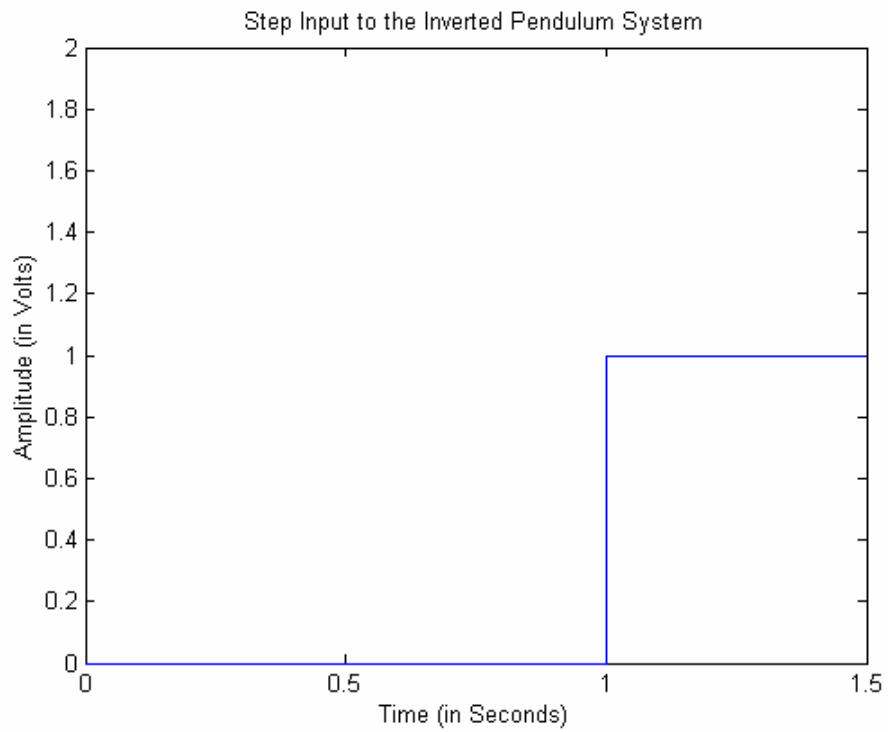
The step response of open-loop uncompensated system is shown below:

SECTION 4

COMPENSATION DESIGN

HOW CAN THE COMPENSATION BE DESIGNED?

The compensation for the Inverted Pendulum System can be designed using any of the following control analysis and design techniques. These are:

- ✓ ROOT-LOCUS Method
- ✓ BODE-PLOTS
- ✓ NYQUIST Diagrams
- ✓ NICHOLS Charts

Out of these techniques, the Root-Locus technique is time domain technique, whereas the later three are frequency domain techniques.
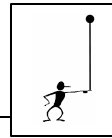
We have used the root-locus techniques because they permit accurate computation of the time-domain response in addition to yielding readily available frequency response information.

SECTION 5

ROOT LOCUS COMPENSATION DESIGN
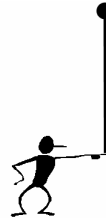
ROOT LOCUS COMPENSATION DESIGN

WHY COMPENSATION IS REQUIRED?

The system analysis indicates that using only the gain compensation in closed loop cannot control the IP. **RESHAPING OF THE SYSTEM ROOT LOCUS** is necessary so that for certain range of gains, the system has all its roots in the left half plane (stable region) of the s-plane.

That is, as the given system is unstable for all values of gain, so the root locus must be reshaped so that the part of each branch falls in the left half s-plane, thereby making the system stable.

Also the desired performance specifications established for the system must be achieved.

COMPENSATION GOALS

The desired <u>TRANSIENT</u> response for the system has following characteristics:
- ➤ **Transient (settling) time** of **½ second**
- ➤ **Overshoot** should be **< 20%**, it implies that
- ➤ The **damping ratio z > 0.5**

The desired <u>STEADY-STATE</u> response for the system has following characteristics:
- ➤ **Steady-state error** must be **zero**.

COMPENSATION DESIGN

The compensation of the system by the introduction of poles and zeroes is used to improve the operating performance. However, each additional compensator pole increases the number of roots of the closed-loop characteristics equation.

The compensator designing has been done with **MATLAB SISO DESIGN TOOL**.

THE SISO DESIGN TOOL

WHAT IS THE SISO DESIGN TOOL

The SISO Design Tool is a graphical user interface (GUI) that facilitates the design of compensators for single-input, single-output feedback loops. The SISO Design Tool allows you to iterate rapidly on your designs and perform the following tasks:

• Manipulate closed-loop dynamics using root locus techniques.
• Shape open-loop Bode responses.
• Add compensator poles and zeros.
• Add and tune lead/lag networks and notch filters.
• Inspect closed-loop responses (using the LTI Viewer).
• Adjust phase and gain margins.
• Convert models between discrete and continuous time.

So this Graphical User Interface allows you to design single-input/single-output (SISO) compensators by interacting with the root locus, Bode, and Nichols plots of the open-loop system.

OPENING THE SISO DESIGN TOOL

To open the SISO Design Tool, type

```
>> sisotool
```

at the MATLAB prompt. SISOTOOL opens the SISO Design Tool.

IMPORTING MODELS INTO THE SISO DESIGN TOOL
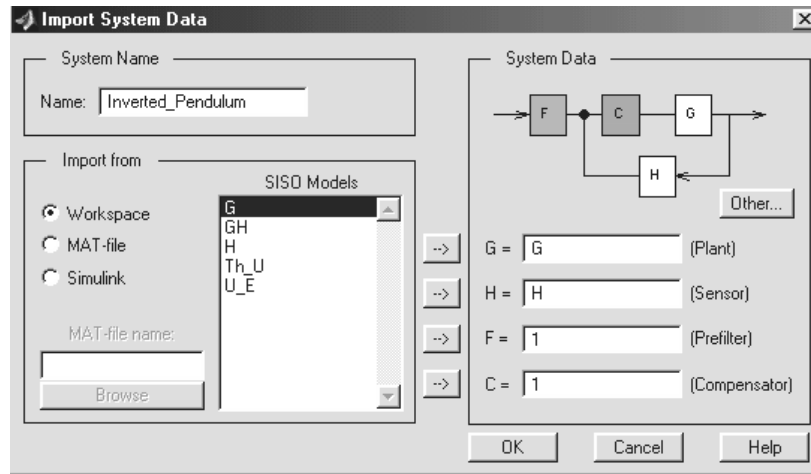
METHOD 1
If you type

```
>> sisotool
```

at the MATLAB prompt, an empty SISO Design Tool opens.

To import the plant data into the SISO Tool, select "**File >> Import**". That is, you can import the IP model by selecting **Import Model** under the **File** menu. This opens the **Import System Data** dialog box, which is shown below. (See next page)

Follow these steps to import the IP model:

> Select **G** under **SISO Models**.
> Place it into the **G** field under **Design Model** by clicking the right arrow button to the left of **G**.
> Select **H** under **SISO Models**.
> Place it into the **H** field under **Design Model** by clicking the right arrow button to the left of **H**.
> Click **OK**.

METHOD 2

SISOTOOL(G,C,H,F) specifies the models of plant G, compensator C (more precisely, initial value for C), sensor H and the prefilter F to be used in the SISO Tool. Here G, C, H, and F are any linear models created with TF, ZPK, or SS.

This command opens the SISO Design Tool with the root locus and open-loop Bode diagrams for the IP model plotted by default.

If you type

```
>> sisotool (G,1,H,1)
```

at the MATLAB prompt, this opens the SISO Design Tool with the IP plant imported.
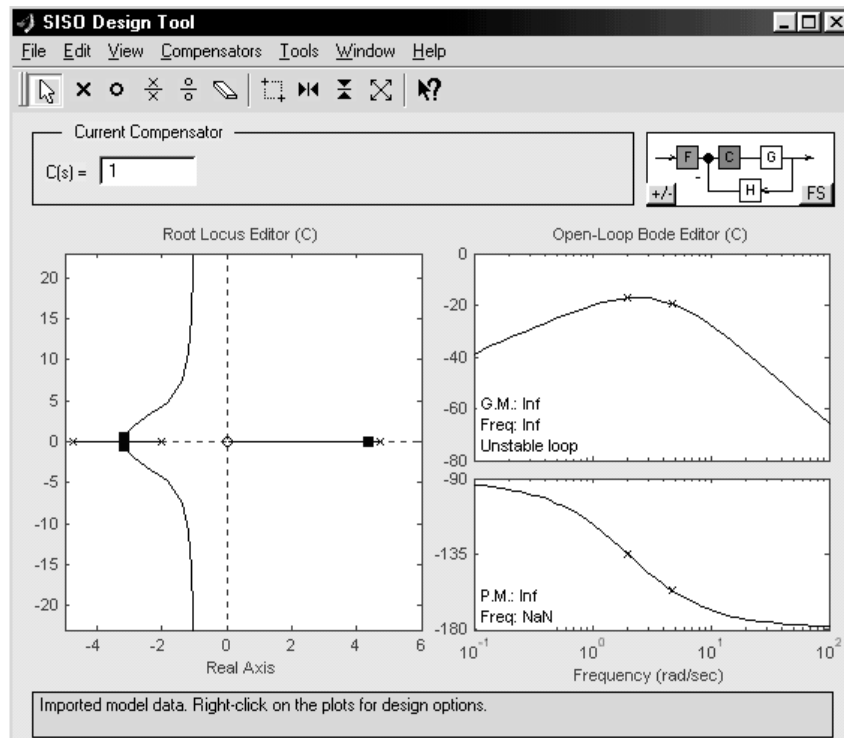
DESIGN SPECIFICATIONS

We have to design a controller so that the step response of the closed-loop system meets the following specifications:

- The 5% settling time is less than 0.5 second.
- The maximum overshoot is less than 20%.

ROOT LOCUS DESIGN WITH SISO DESIGN TOOL

A common technique for meeting design criteria is root locus design. This approach involves iterating on a design by manipulating the compensator gain, poles, and zeros in the root locus diagram.

The root locus diagram shows the trajectories of the closed-loop poles of a feedback system as a single system parameter varies over a continuous range of values. Typically, the root locus method is used to tune the loop gain of a SISO control system by specifying a feedback gain the closed-loop pole locations.

The root locus technique consists of plotting the closed-loop pole trajectories in the complex plane as *k* varies. You can use this plot to identify the gain value associated with a desired set of closed-loop poles.

ADDING POLES AND ZEROS TO THE COMPENSATOR

You may have noticed that whatever be the value of loop gain, ($\forall$K), one branch of the locus remains on RHS (in unstable region) of s-plane. This makes meeting the design requirements with only a gain in the compensator is not possible. The root locus has a branch on the right hand side of the imaginary axis, which indicates that the system is unstable in closed loop for all values of K.

So poles and zeroes will be added in the compensator so as to reshape the root locus to meet our design requirements.

> **Transient (settling) time** of **½ second**
> For this, the horizontal component (real) of roots in negative real axis must be
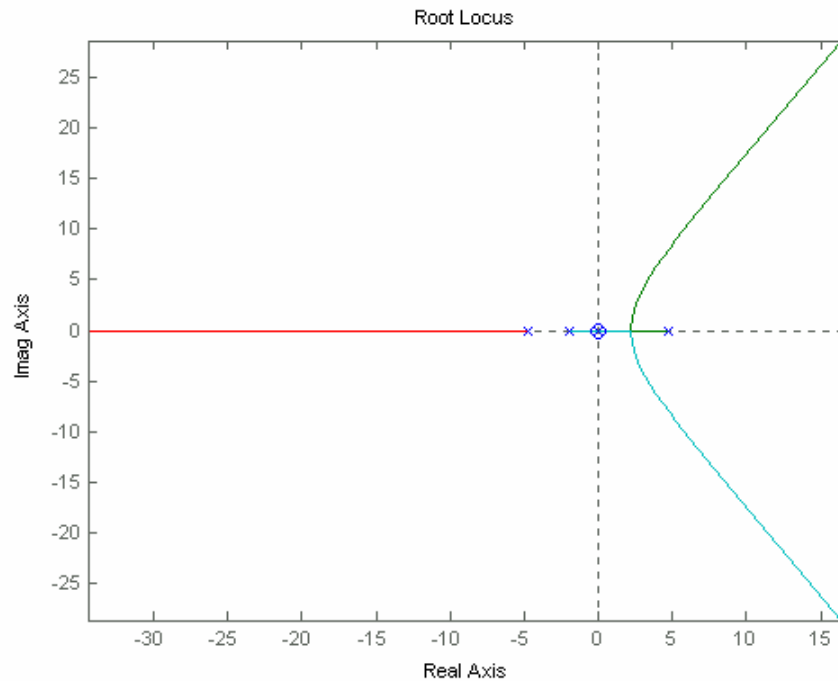>
> $T_S = 4 / \zeta \, w_N$
> or $(\zeta \, w_N)_{MIN} = 4 / T_{S \, MAX} = 4 / 0.5 = 8$

> **Overshoot** should be **< 20%**.

> The **damping ratio z > 0.5**
> The β angle should be less than 60 degrees.

> **Steady-state error** must be **zero**.
> The compensator must have integral control.

PROCEDURE

1. We introduce a pole at origin to cancel the effect of zero of the plant at the origin. The integral has cancelled the zero at origin. The root locus now becomes as

2. Now add two zeros of the controller. These zeroes will provide finite terminus points to the two branches of locus on the LHS of the s-plane which were, otherwise, approaching to infinity. The zeroes have to be added such that we can achieve the design requirements of settling time and the damping ratio.

3. We add two real poles at -10. The resulting root locus has the following form. $K_{MIN} = 0.8$. Below K_{MIN}, roots shift in right-half and the system becomes unstable.
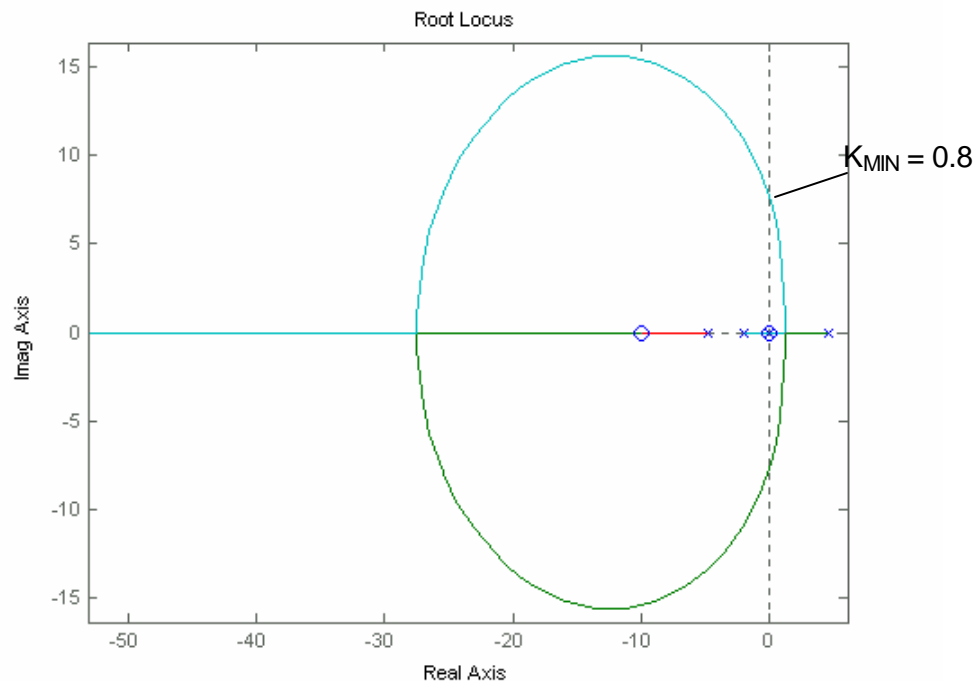
For the loop stability it is required that the loop gain should be greater than the given value (i.e., $K > K_{MIN} = 0.8$).

4. Now we set the gain such that roots of the characteristics equation correspond to $\zeta = 0.75$ and $T_{S\ MAX} = 0.5$

So we select roots (selected by cross) as shown in figure.
These roots correspond to value of k = 8.
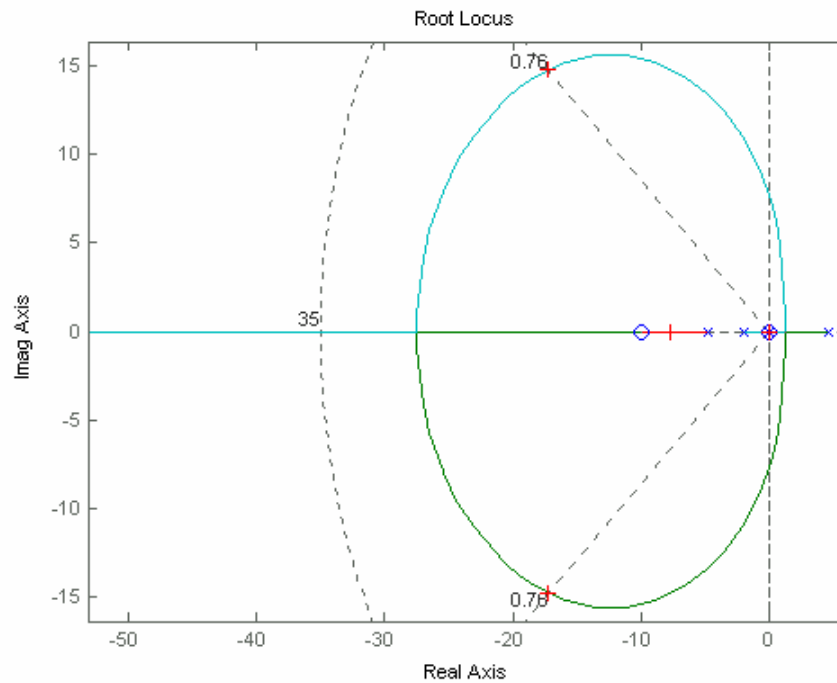
The compensator equation is found to be:

$$G_C = 8 * (\frac{s^2 + 20\ s + 100}{s})$$

Comparing it with following equation,

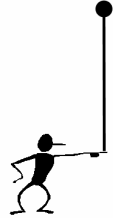$$G_C = K_C * (\frac{K_D\ s^2 + K_P\ s + K_I}{s})$$

yields
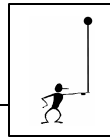
$$K_C = 8$$
$$K_P = 20$$
$$K_I = 100$$
$$K_D = 1$$

However, after simulating this PID compensated system in SIMULINK, the gain K_C was found to be best when it is 30 (minimum overshoot). So the final values of the gains are

$$K_C = 30 \qquad K_P = 20 \qquad K_I = 100 \qquad K_D = 1$$

SECTION 6

ANALYSIS OF COMPENSATED SYSTEM

ANALYSIS OF COMPENSATED SYSTEM

POLE-ZERO MAP OF COMPENSATED OPEN LOOP SYSTEM

The poles position of the compensated model of Inverted Pendulum (in open loop configuration) is shown in the following figure.

In addition to the plant's poles and zeroes, we have PID Controller's pole at origin to cancel out the plant's zero at the origin.

We have introduced two zeroes of the compensator (at   10) that pull the root locus towards them (left) so we obtain a part of root locus, that has all its roots in the stable region of the s-plane.

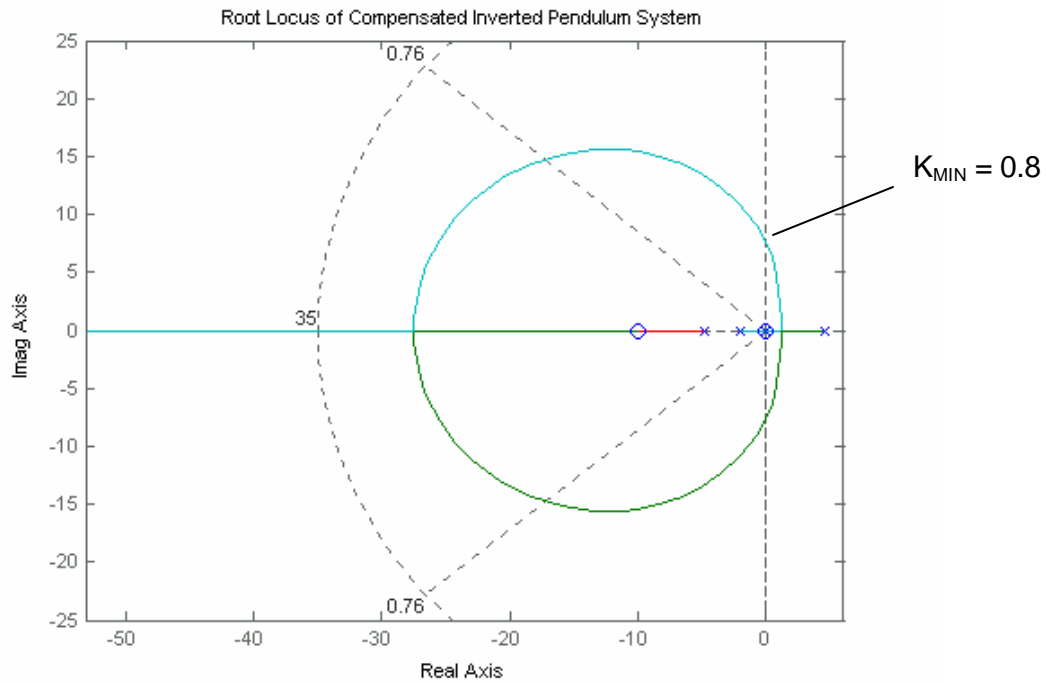Pole-Zero Map of Open-Loop Compensated Inverted Pendulum System
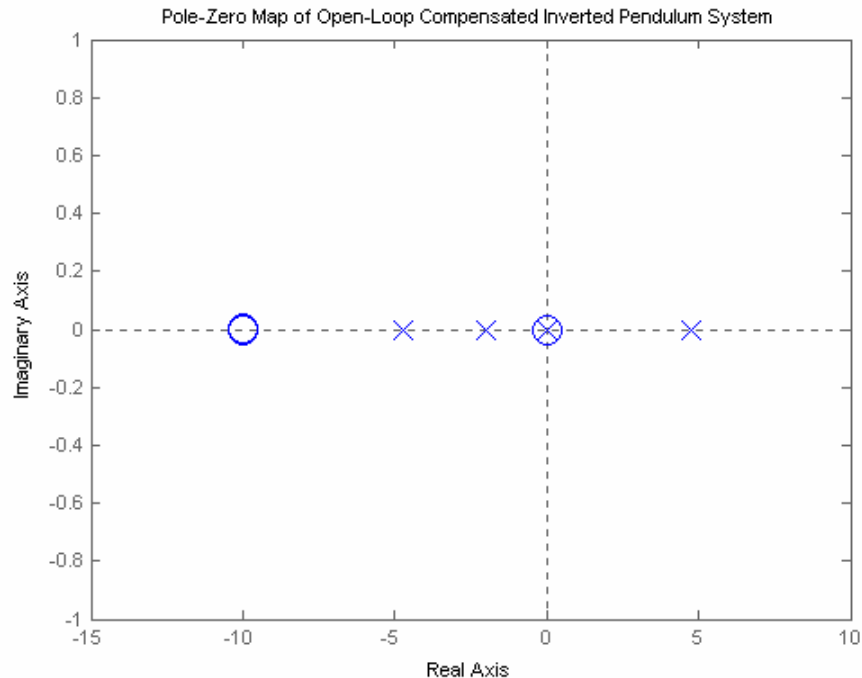
ROOT LOCUS OF THE COMPENSATED SYSTEM

As can be seen in the root locus of the compensated system shown below, a PID controller can stabilize the loop. The integral has cancelled the zero at origin. The two zeros of the controller have provide finite terminus points to the two branches of the locus on LHS of the s-plane which were, otherwise, approaching to infinity. However for the loop stability it is required that the loop gain should be greater than the given value (i.e., $K > K_{MIN}$).

For our given data, $K_{MIN} = 0.8$

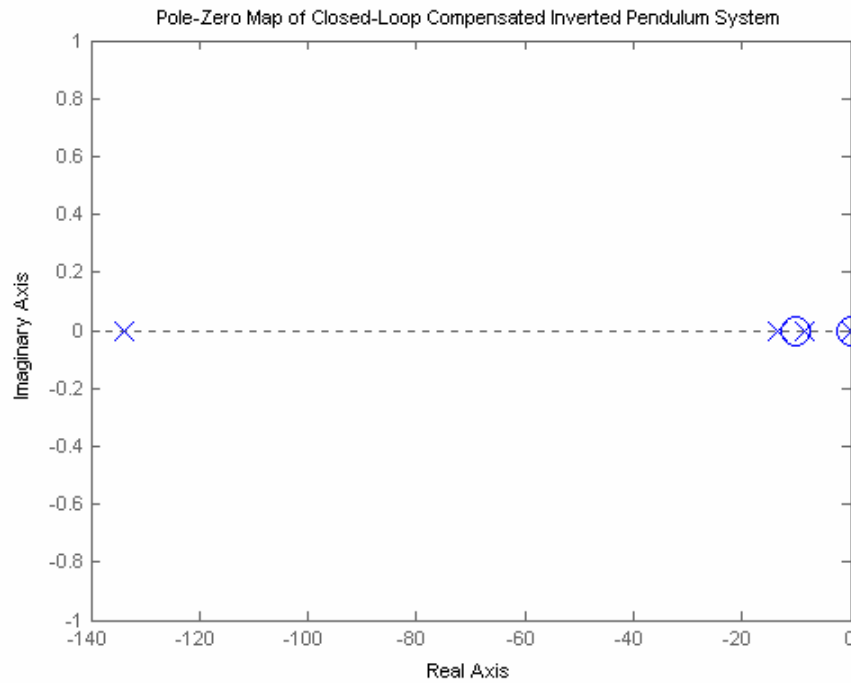So for **K > 0.8**; the system is stable since all the roots are on the left-hand-side of the imaginary axis.

ROOT LOCUS PLOT OF THE PID COMPENSATED LOOP

## POLE-ZERO MAP OF COMPENSATED CLOSED-LOOP SYSTEM

The poles-zeroes position of the compensated model of Inverted Pendulum (in closed-loop configuration) is shown in the following figure.

POLES: 0, -8.5359, -13.4232, -133.8242
ZEROES: 0, -10, -10

IMPULSE RESPONSE OF PID COMPENSATED SYSTEM

An impulse response of the PID compensated system is shown in following figure. The impulse response of the compensated system is shown in the figure. The response of the system is very fast. The settling time is very small i.e. **0.028 s.**
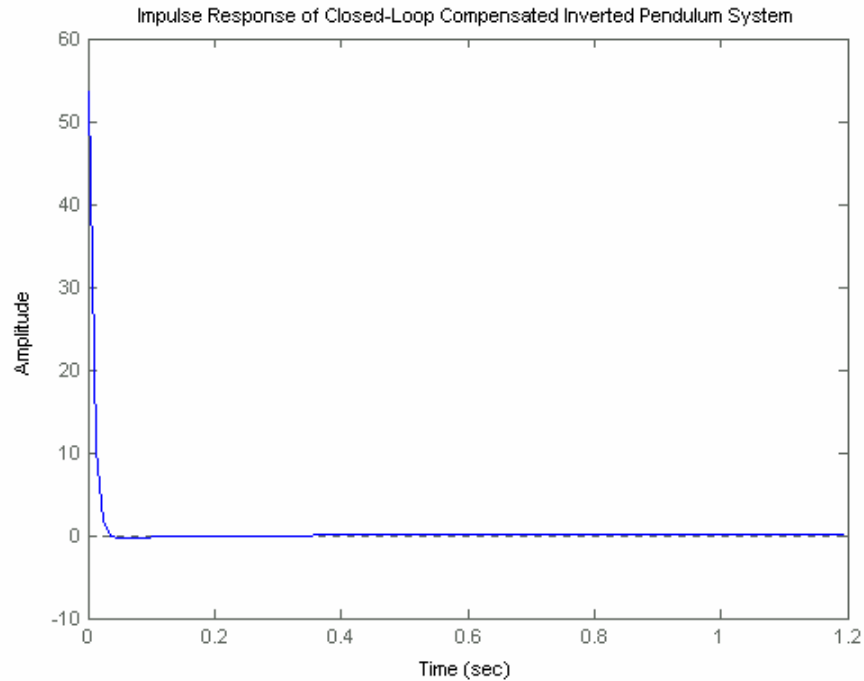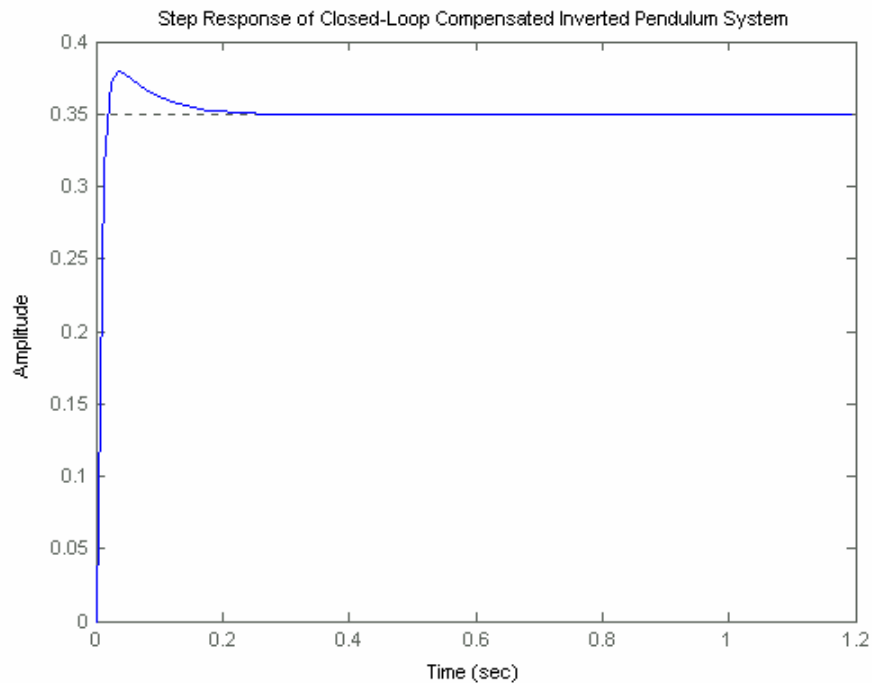

STEP RESPONSE OF PID COMPENSATED SYSTEM

The step response of the PID compensated system is shown in following figure. The DC Gain of the Closed-Loop Compensated Inverted Pendulum System is 0.3501. The percent overshoot is 8.48% (less than 20%). The steady-state error is zero.

CONCLUSION OF COMPENSATION ANALYSIS

Following compensation goals have been achieved.

The desired TRANSIENT response for the system has following characteristics:

- ➢ **Transient (settling) time < ½ second**
  0.028 SEC FOR IMPULSE RESPONSE
  0.127 SEC FOR STEP RESPONSE
- ➢ **Overshoot < 20%**
  8.48% FOR STEP RESPONSE
- ➢ The **damping ratio z > 0.5**

The desired STEADY-STATE response for the system has following characteristics:

- ➢ **Steady-state error = zero**.

(3) SIMULINK MODEL FOR THE CLOSED-LOOP STEP RESPONSE OF THE COMPENSATED INVERTED PENDULUM SYSTEM

FILE: ipclstep.mdl

The step is applied at 1 s.

```
SIMULATION PARAMETERS:

Start Time:             0
Stop Time:              10
Solver Algorithm:       Variable-step
                        ODE45 (Dormand-Prince)
Maximum Step Size:      auto
```

Closed Loop Step Response of Compensated Inverted Pendulum System

This model shows the step response of the Compensated Inverted Pendulum
System. It shows that the system has become stable as the output stabilizes for certain value.
The steady state error is zero and % Overshoot is 8.48%.

The step input to the Inverted Pendulum is:

Step Input to the Closed-Loop Compensated System

The step response of closed-loop compensated system is shown below:

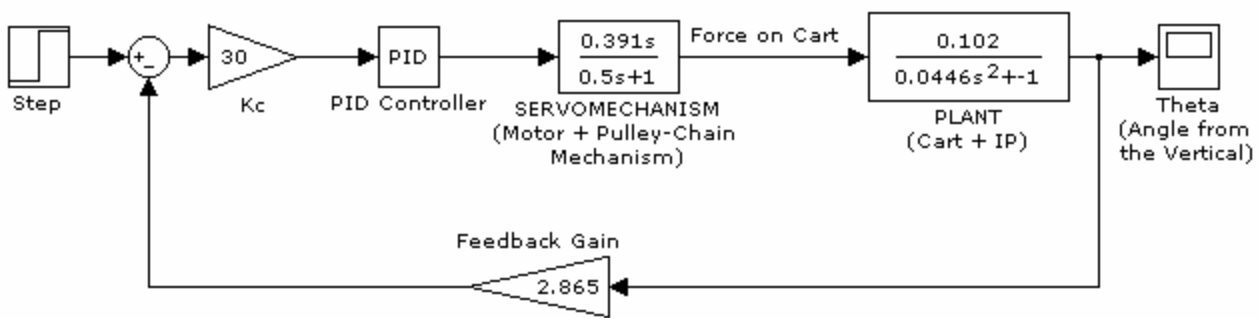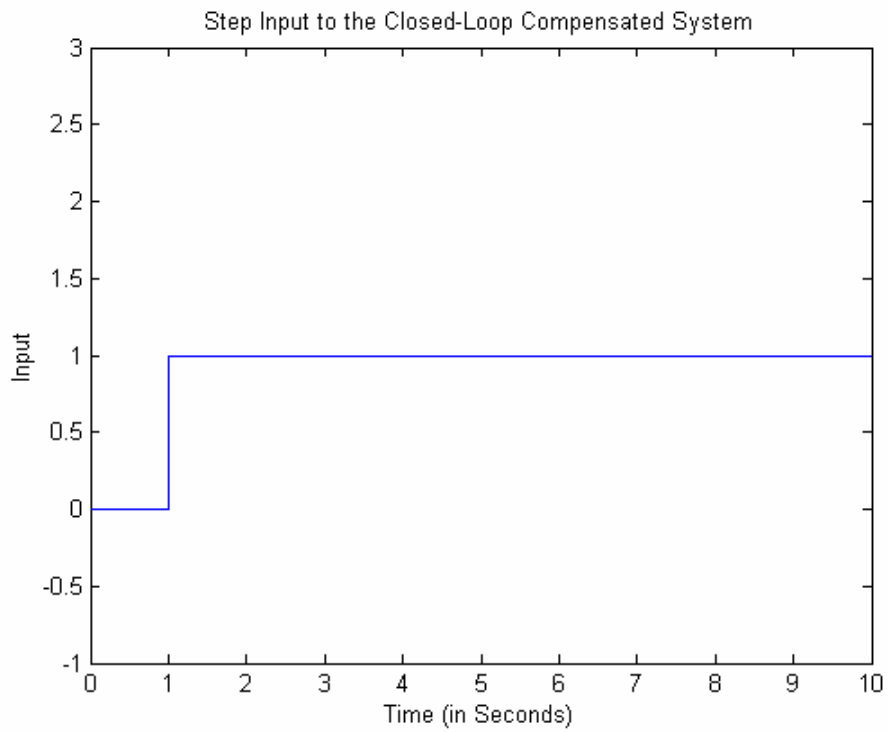Step Response of Closed-Loop Compensated System

(4) SIMULINK MODEL FOR THE CLOSED-LOOP IMPULSE RESPONSE OF THE COMPENSATED INVERTED PENDULUM SYSTEM

FILE: `ipclimpulse1.mdl`

The impulse is applied at 0.5 s.

SIMULATION PARAMETERS:	
Start Time:	0
Stop Time:	5
Solver Algorithm:	Variable-step ODE45 (Dormand-Prince)
Maximum Step Size:	0.05

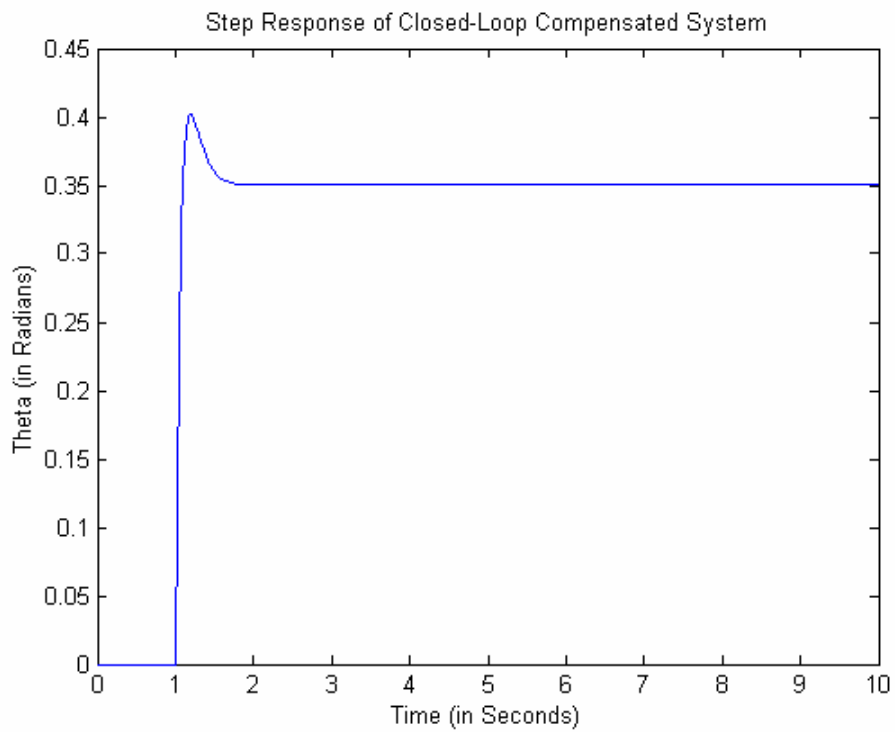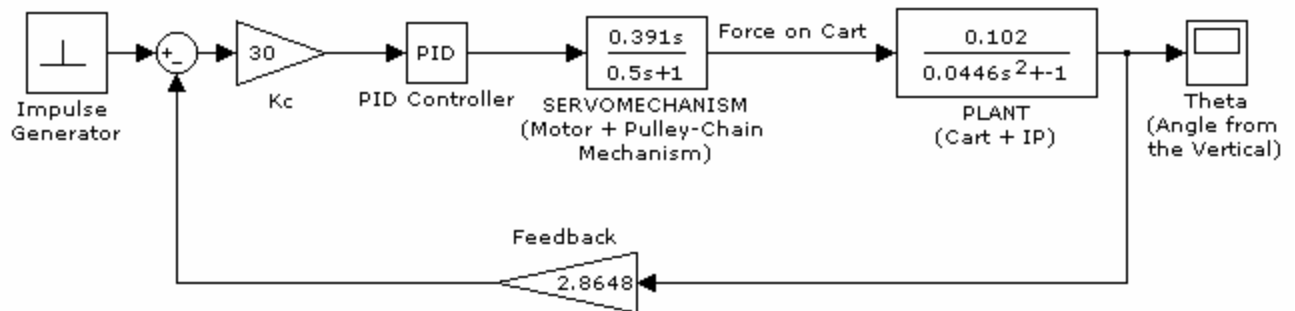**Closed Loop Impulse Response of Compensated Inverted Pendulum System**

Impulse Generator → (+ -) → 30 / Kc → PID / PID Controller → $\dfrac{0.391s}{0.5s+1}$ / SERVOMECHANISM (Motor + Pulley-Chain Mechanism) → Force on Cart → $\dfrac{0.102}{0.0446s^2+-1}$ / PLANT (Cart + IP) → Theta (Angle from the Vertical)

Feedback: 2.8648

This model shows the impulse response of the Compensated Inverted Pendulum System. Here initially the Inverted Pendulum is at vertical position. It also shows that the system has become stable as Theta returns back to zero, when impulse input comes.

The impulse input to the Inverted Pendulum is:
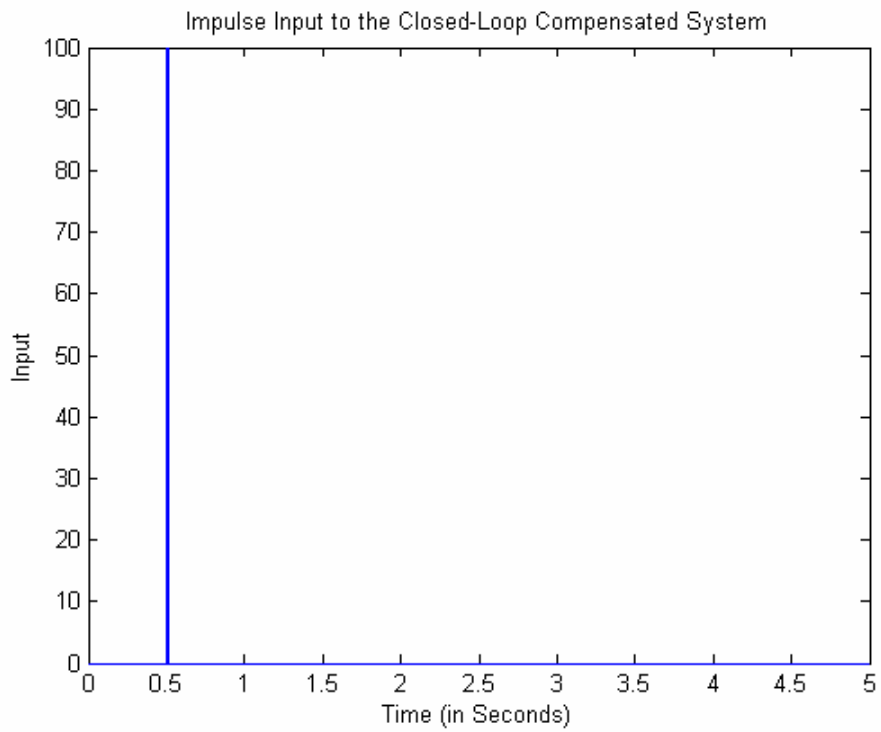
Impulse Input to the Closed-Loop Compensated System

The impulse response of the closed-loop compensated system is shown below:

Impulse Response of Closed-Loop Compensated System
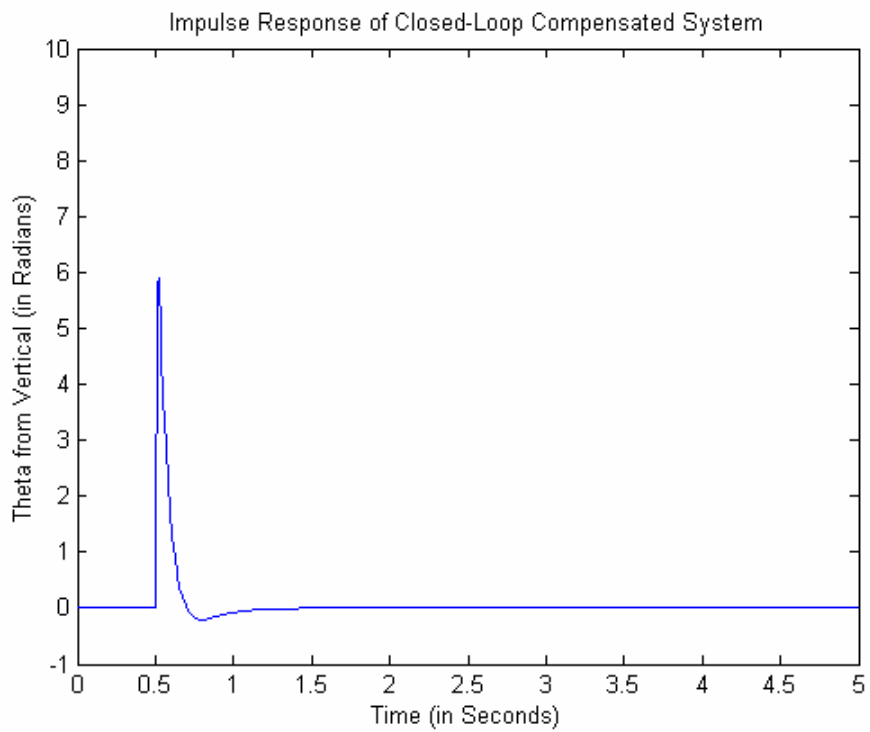
(5) SIMULINK MODEL FOR THE RESPONSE TO DISTURBANCE IN THE FORCE ON THE CART OF THE COMPENSATED INVERTED PENDULUM SYSTEM

FILE: `ipclimpulse2.mdl`

The impulse is applied at 0.5 s.

SIMULATION PARAMETERS:	
Start Time:	0
Stop Time:	5
Solver Algorithm:	Variable-step
	ODE45 (Dormand-Prince)
Maximum Step Size:	0.05

Response to Disturbance in Force on the Cart of Compensated Inverted Pendulum System
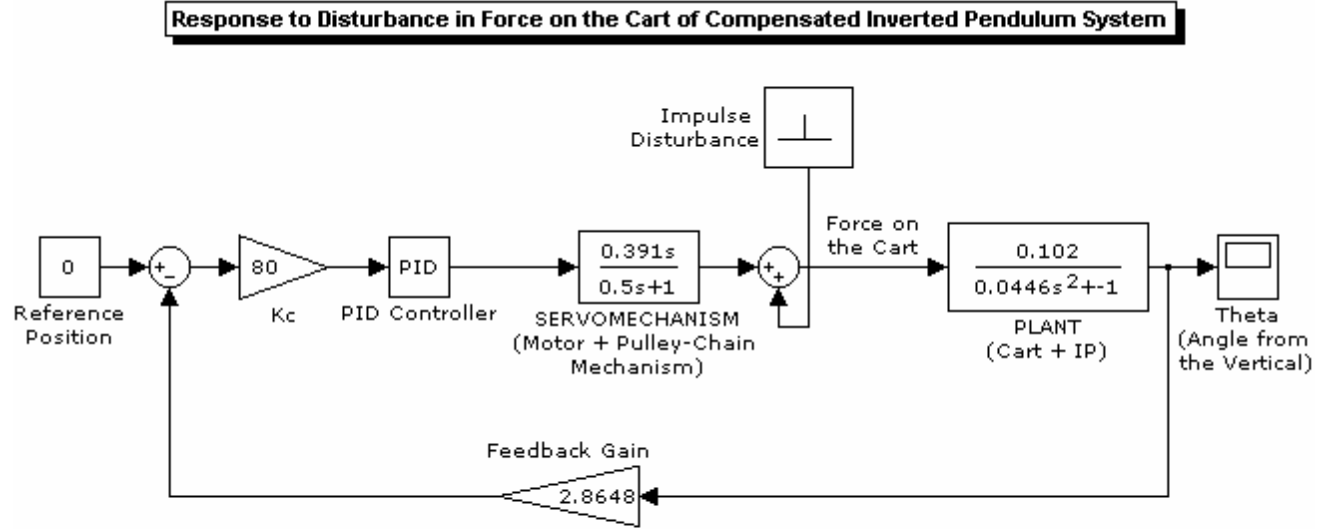
This model shows the response to disturbance in force on the cart of compensated Inverted Pendulum System. It shows that how the system rejects the disturbance in the force in the cart.

The impulse disturbance in the force of the cart of the Inverted Pendulum is:
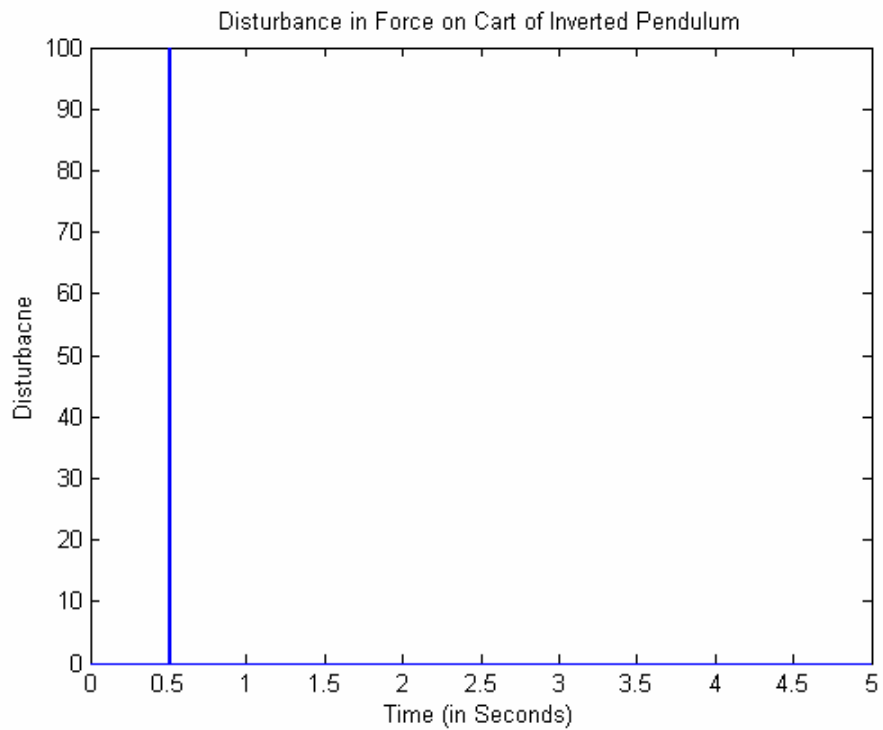
Disturbance in Force on Cart of Inverted Pendulum

The response of the closed-loop compensated system to disturbance in the force on the cart is shown below:

Response to Disturbance in Force on Cart

(6) SIMULINK MODEL FOR THE RESPONSE TO THE DISTURBANCE IN THE IN POSITION OF INVERTED BROOM OF THE COMPENSATED INVERTED PENDULUM SYSTEM

FILE: ipclimpulse3.mdl

The impulse is applied at 0.5 s.

SIMULATION PARAMETERS:	
Start Time:	0
Stop Time:	5
Solver Algorithm:	Variable-step
	ODE45 (Dormand-Prince)
Maximum Step Size:	0.05

Response to Disturbance in Position of Inverted Broom of Compensated Inverted Pendulum System
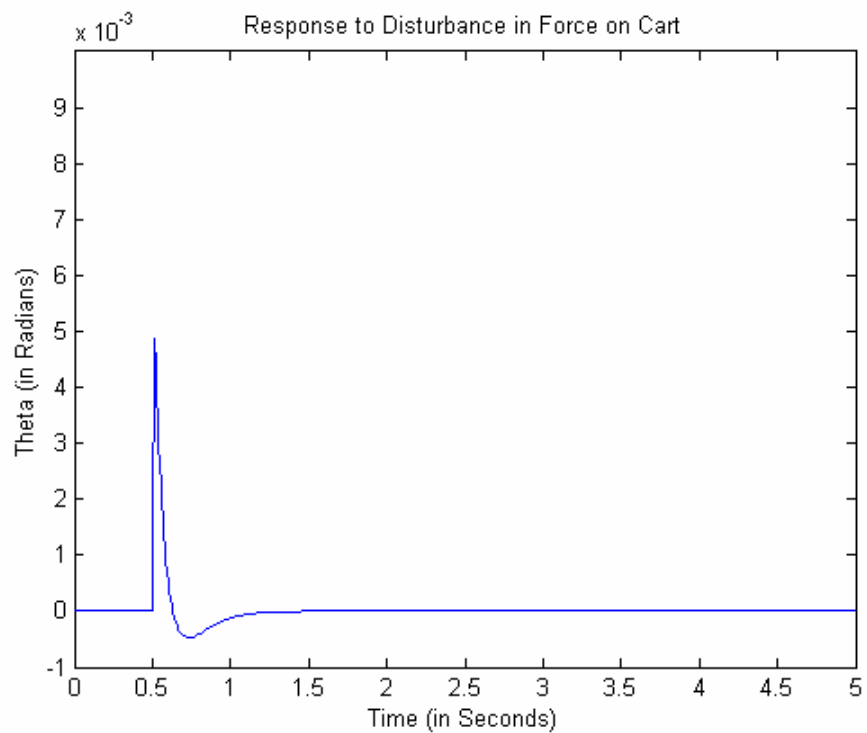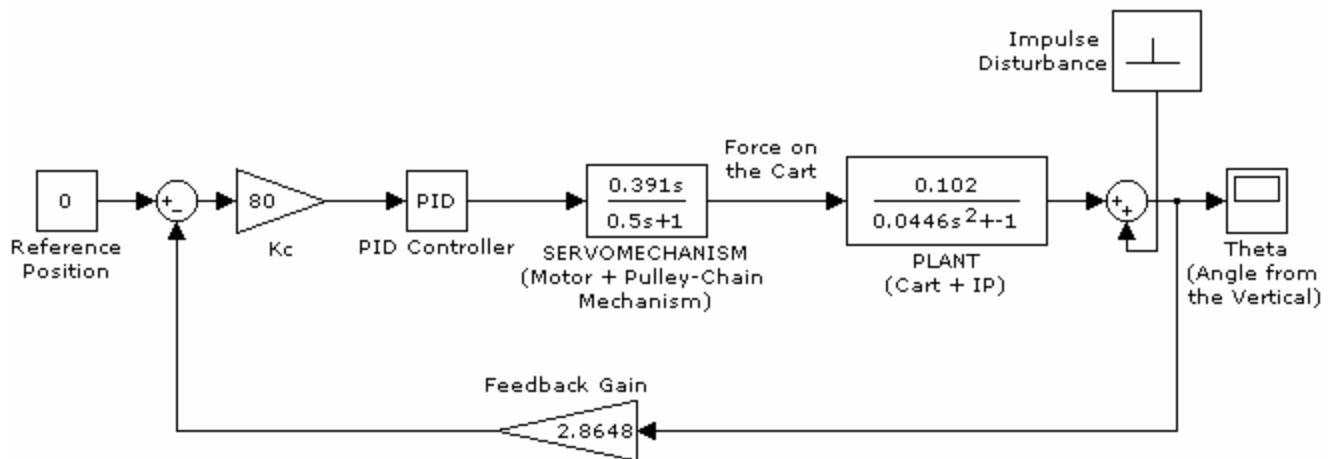

This model shows the response to disturbance in Position of Inverted Broom of compensated Inverted Pendulum System. It shows that how the system rejects the disturbance in the position of the inverted broom.

The impulse disturbance in the position of the Inverted Broom of the Inverted Pendulum is:
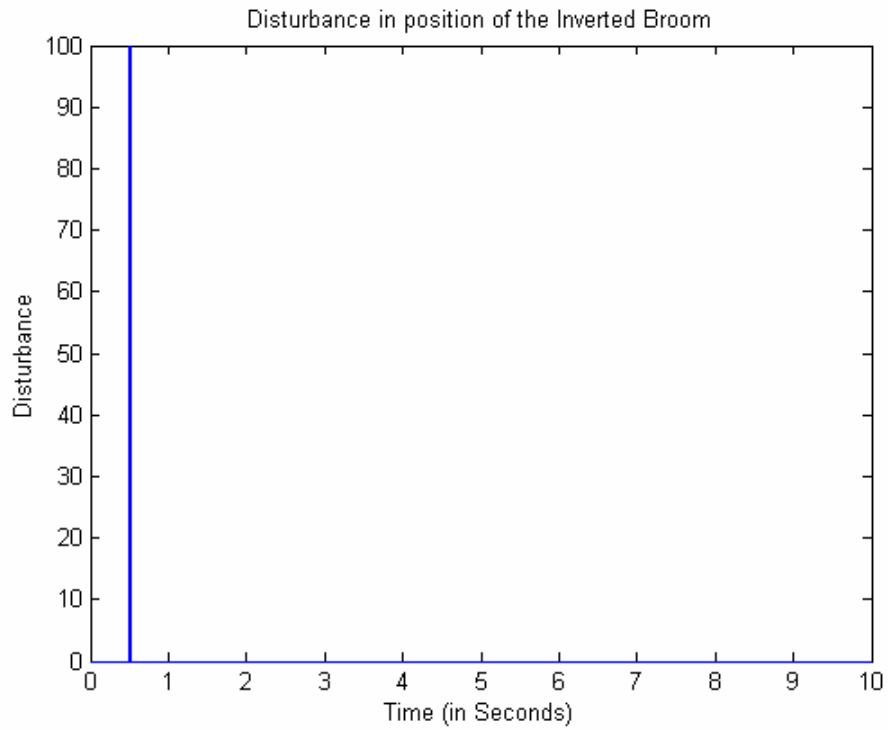
Disturbance in position of the Inverted Broom

The response of the closed-loop compensated system to disturbance in the position of the Inverted Broom is shown below:

Response to disturbance in position of the Inverted Broom
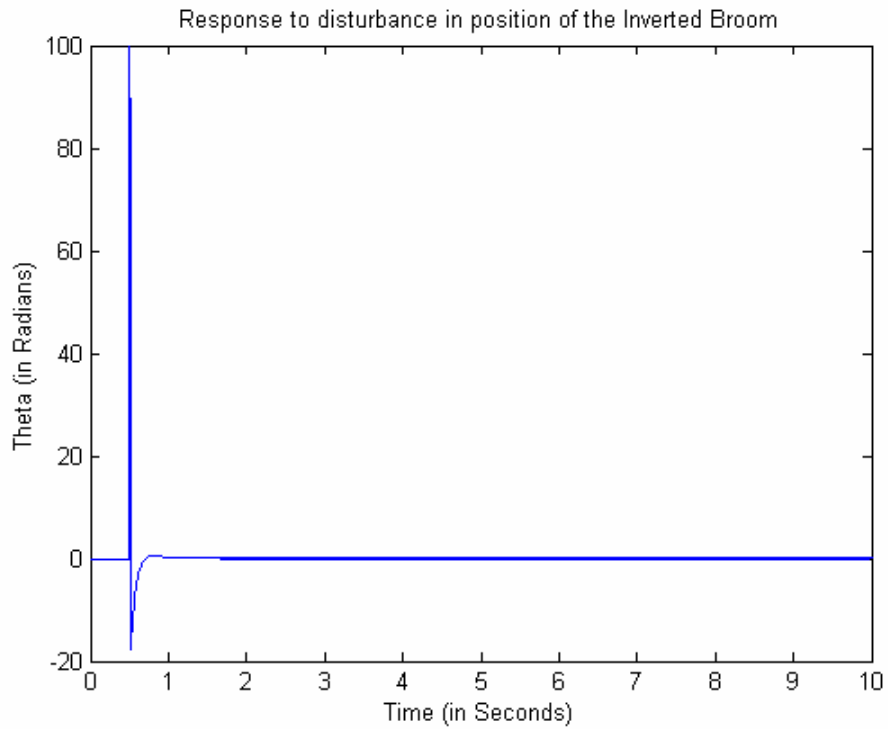
SECTION 7
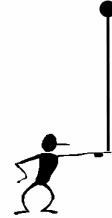
PRACTICAL IMPLEMENTATION
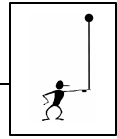
The PID controller, that was implemented, was Analog in nature and was implemented by Analog Circuitry.

ANALOG CONTROLLER can be realized by a number of different Op-Amp based circuits. These networks are discussed in the following section.

Besides Control-Electronics, there are Error-Detector, Servo-Amplifier, and a Feedback-Sensor (Linear Rotary Potentiometer) in the Control Loop.

ANALOG PID CONTROLLER DESIGNS

DESIGN 1: IDEAL PID ALGORITHM

We will start describing this design from the general operational amplifier based computer circuit and derive its equation of operation. Then we will extend this general derivation for treatment of PID Controller circuit.

GENERAL OPERATIONAL AMPLIFIER CIRCUIT:
Following is a schematic representation of an operational amplifier in which input impedance is Z_1 and the feedback or parallel impedance is Z_2.
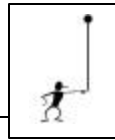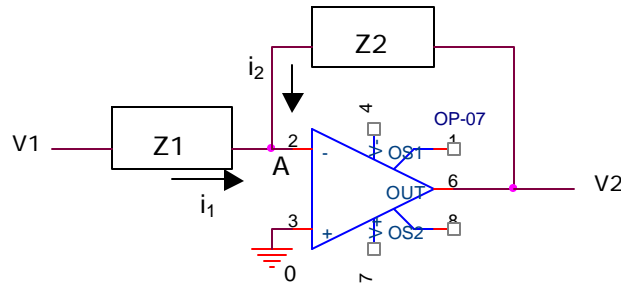

At point A (the virtual ground), by Kirchoff's current law,

$$i_1 + i_2 = 0$$
or $\quad i_1 = -i_2$
or $\quad V_1 / Z_1 = -V_2 / Z_2$

So the equation (transfer function) of this amplifier is

$$V_2 / V_1 = -Z_2 / Z_1$$
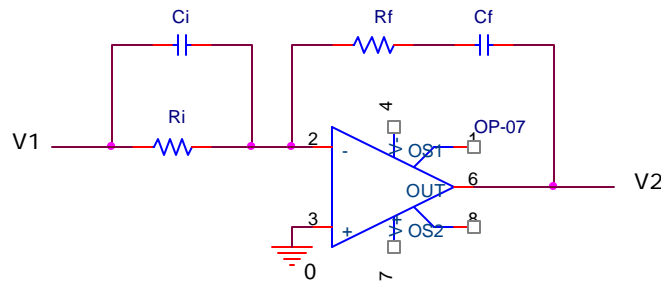
Here $\qquad Z_1 = R_i \parallel C_i = \{R_i * (1 / C_i D)\} / \{Ri + (1 / C_i D)\} = R_i / (R_i C_i D + 1)$

$\qquad Z_2 = (R_f + 1 / C_f D) = (R_f C_f D + 1) / C_f D$

So the transfer function of this proportional plus derivative controller is

$$V_2 / V_1 = -\{(R_f C_f D + 1) / C_f D\} / \{R_i / (R_i C_i D + 1)\}$$

$$V_2 / V_1 = -(1 / R_i C_f D) * \{(R_f C_f D + 1) * (R_i C_i D + 1)\}$$

$$V_2 / V_1 = -(1 / R_i C_f D) * \{(R_i R_f C_i C_f) D^2 + (R_i C_i + R_f C_f) D + 1\}$$

$$\mathbf{V_2 / V_1 = -\{(R_f / R_i + C_i / C_f) + (1 / R_i C_f D) + (R_f C_i) * D\} = -\{K_p + K_i / D + K_d D\}}$$

So here

PROPORTIONAL GAIN is	$K_p = (R_f / R_i + C_i / C_f)$,
INTEGRAL GAIN is	$K_i = (1 / R_i C_f)$, and
DERIVATIVE GAIN is	$K_d = R_f C_i$.

DESIGN 2: PARALLEL PID ALGORITHM

This design may be regarded as the simplest approach to design an Analogue PID Circuit. This design is based on the following configuration to achieve PID action.
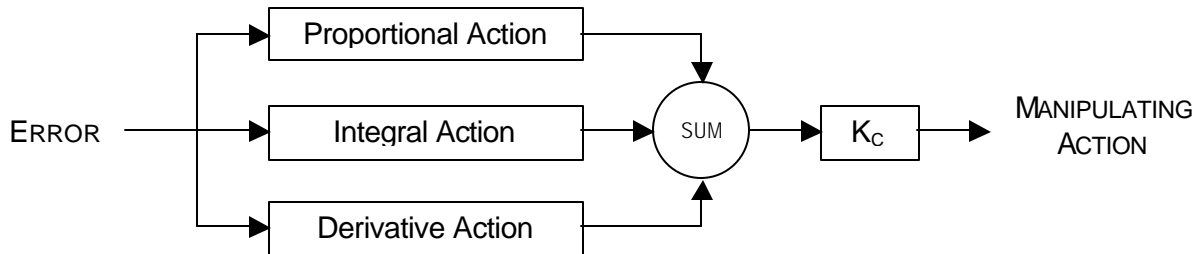
The advantage of using this configuration is that we can have separate controls to vary K_P, K_I, K_D and K_C (not possible in any of the other design). So a Tunable PID controller can be made using this design, by using potentiometers (preferably) & (or) variable capacitors. We have also adopted this design to make a TUNABLE ANALOGUE PID CONTROLLER, with different gain ranges as follows:

$$K_P = 1 \text{ TO } 1001; K_I = 1 \text{ TO } ; K_D = 0 \text{ TO } 10; K_C = 0 \text{ TO } 100$$

The schematic representation of this design is as follows:
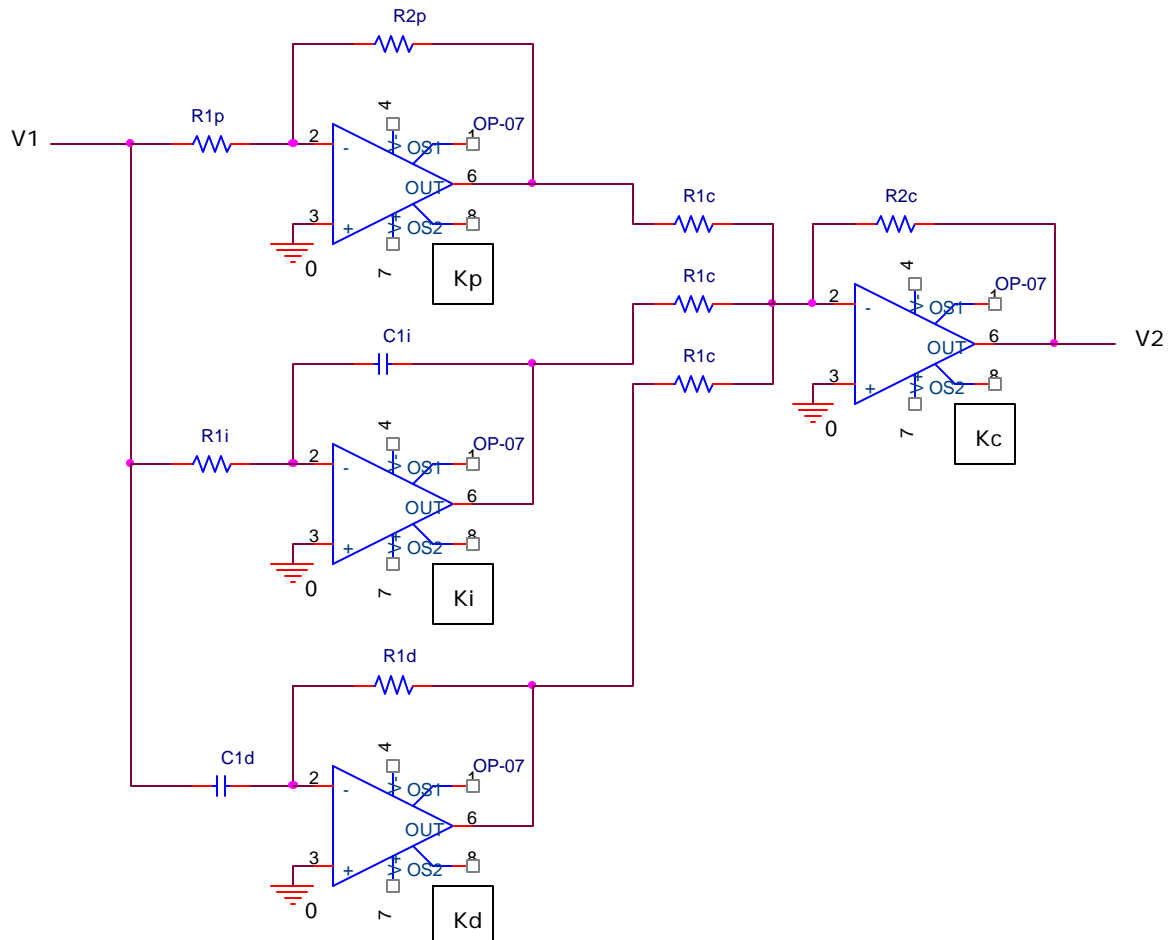
So here

PROPORTIONAL GAIN is	$K_P = (R_{2p} / R_{1p})$,
INTEGRAL GAIN is	$K_I = 1 / (R_{1i} C_{1i})$, and
DERIVATIVE GAIN is	$K_D = (R_{1d} C_{1d})$, and
CASCADED GAIN is	$K_C = (R_{2C} / R_{1C})$.

DESIGN 3: SERIES PID ALGORITHM

Here the transfer function of PID Controller is achieved as product of PI controller and PD controller. So following is this configuration of PI and PD in cascade to achieve PID action.
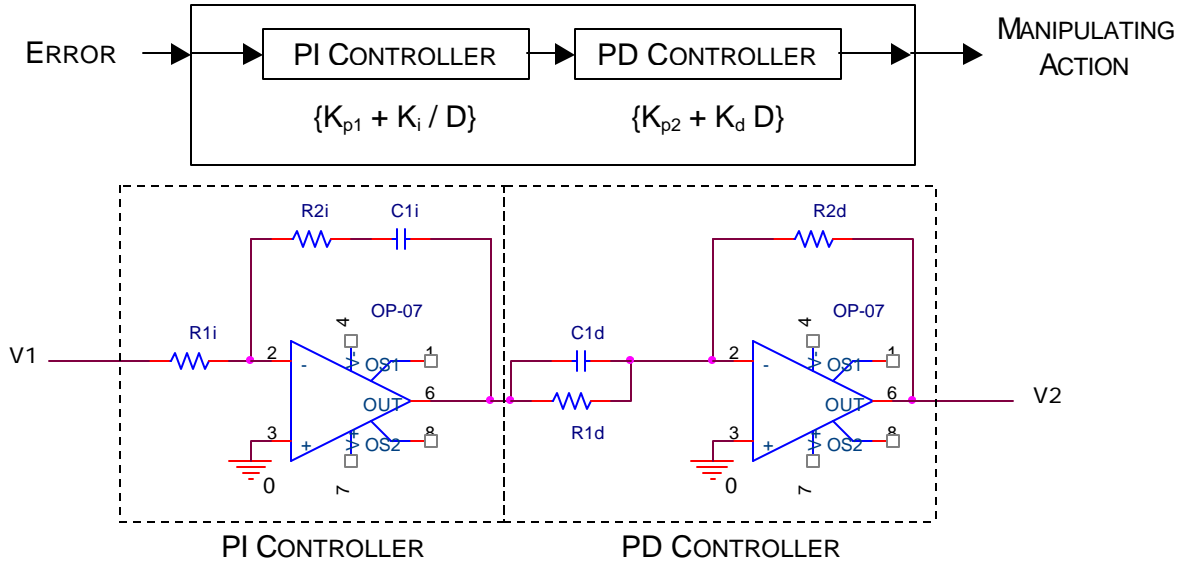
The PID Controller is defined by the equation:

$$G_C(s) = G_{PI}(s) * G_{PD}(s)$$

$$= \{K_{p1} + K_i / D\} * \{K_{p2} + K_d D\}$$

where
$$K_{p1} = R_{2i} / R_{1i} \ ,$$
$$K_i = 1 / R_{1i} C_{1i} \ ,$$
$$K_{p2} = R_{2d} / R_{1d} \ , \text{and}$$
$$K_d = R_{2d} C_{1d} \ .$$

So

$$G_C(s) = K_{p1} K_{p2} + K_{p1} K_d D + K_{p2} K_i / D + K_i K_d$$

$$= (K_{p1} K_{p2} + K_i K_d) + (K_{p2} K_i) / D + (K_{p1} K_d) D$$

$$= \{K_P + K_I / D + K_D D\}$$

So here
$$K_P = (K_{p1} K_{p2} + K_i K_d) = (R_{2i} R_{2d} / R_{1i} R_{1d} + R_{2d} C_{1d} / R_{1i} C_{1i}),$$
$$K_I = (K_{p2} K_i) = R_{2d} / (R_{1d} R_{1i} C_{1i}) \text{ and}$$
$$K_D = (K_{p1} K_d) = (R_{2i} R_{2d} C_{1d}) / R_{1i}$$

So here

PROPORTIONAL GAIN is	$K_P = (R_{2i} R_{2d} / R_{1i} R_{1d} + R_{2d} C_{1d} / R_{1i} C_{1i})$,
INTEGRAL GAIN is	$K_I = R_{2d} / (R_{1d} R_{1i} C_{1i})$, and
DERIVATIVE GAIN is	$K_D = (R_{2i} R_{2d} C_{1d}) / R_{1i}$.

SECTION 8
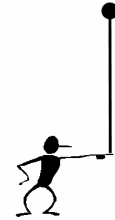
PRACTICAL RESULTS
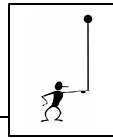
EXPERIMENTAL DATA

Below is the plot of the smoothened (filtered) experimental data recorded using a 8051-based Data Acquisition Card.

The Inverted Pendulum was given an initial condition THETA_{IC}, as indicated by an initial 20 magnitude of pendulum's angular displacement.

As is shown in the plot, the **settling time** of the system is **0.06 seconds**.
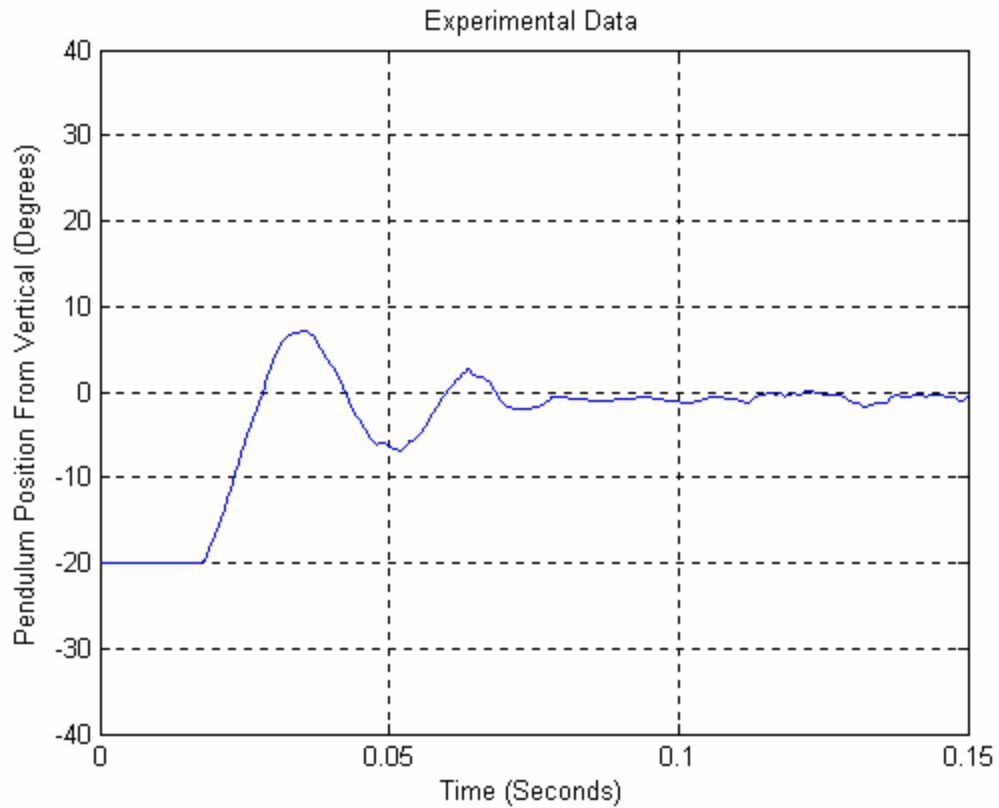
SECTION 9

CONCLUSION

YES!
I DID IT!
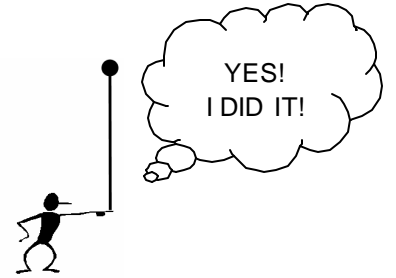
CONCLUSION

The experience of working on the classical problem of Inverted Pendulum is great. It is an ideal exercise to show one's talent as Control Engineer. The practical work has gone a long way in helping us understand and develop an insight into the designing of control systems for SISO (Single Input Single Output) systems.

This exercise provides a chance of designing a controller for a system that has a good dynamic behavior and hence the consideration for the transient response is accentuated.

The power of MATLAB and Simulink becomes more evident to one as all these designing would not have been possible without these tools.

The practical implementation of the controller familiarizes one with the use of analog computers and their importance in the field of Control Engineering.

(1) M-FILE FOR DATA OF THE INVENTED PENDULUM SYSTEM

```
%-------------------------------------------------------------------------
% data_ip.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Data of the Inverted Pendulum System
%-------------------------------------------------------------------------

% Mass of the Cart = 900 gm
M = 0.9;        % in Kg
% Mass of the Pendulum = 100 gm
m = 0.1;        % in Kg
% Length of Pendulum = 47 cm
Lp = 0.47;      % in m
% Length of pendulum to Center of Gravity = 23.5 cm
l = 0.235;      % in m
% Moment of Inertia  of Pendulum = 5.3 gm-m^2
I = 0.0053;     % in Kg.m^2
% Radius of Pulley = 2.3 cm
r = 0.023;      % in m
% Time Constant of Motor = 0.5 second
tau = 0.5;      % in seconds
% Gain of Feedback = 9/pi V/rad/sec
Kf = 2.8648;    % in V/rad/sec
% Gain of Motor = 17 rad/sec/V
Km =17;         % in rad/sec/V
% Friction of the Cart = 0.000 N/m/sec
b = 0;          % in N/m/sec
% Acceleration due to Gravity = 9.8 m/sec^2
g = 9.8;        % in m/sec^2

% Force applied to the cart by the pulley chain mechanism = u
% Cart Position Coordinate = x
                  % Pendulum Angle with the vertical = theta
```

(2) M-FILE FOR OPEN LOOP & CLOSED LOOP (UNCOMPENSATED) TRANSFER FUNCTION OF IP SYSTEM

```
%-----------------------------------------------------------------------------
% trans_func_ip_uc.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Open Loop & Closed Loop (Uncompensated) Transfer Function of IP System
%-----------------------------------------------------------------------------
%
%                     E(s)           U(s)
%         Vpot --->O--->[ G2(s) ]--->[ G1(s) ]---+---> Theta (s)
%               -  ^                              |
%                  |                              |            Theta = SYS * E
%                  +----------[ H(s) ]<----------+
%                    V viper

% ip_data is a MAT File (MATLAB specific binary file),
% with variables I, Kf, Km, Lp, M, b, g, l, m, r, tau
load ip_data

Kp = 1 / ((M + m) * g);
K  = Kf * Kp * Km * r * (M + m);
Ap = sqrt ((M + m) * m * g * l / ((M + m)*(I + (m * (l ^ 2)))- ((m * l)^2)));

% G1(s) = Theta(s) / U(s)
% ø represents a small angle from the vertical upward direction,
% u represents the input (impulse) force on the cart by pulley chain mechanism.
num_Th_U = [0 0 Kp];
den_Th_U = [Ap^(-2) 0 -1];
Th_U = tf (num_Th_U, den_Th_U);

% G2(s) = U(s) / E(s)
% u represents the input force on the cart by the pulley chain mechanism,
% e represents the input to the motor driving pulley-chain mechanism.
num_U_E = [((Km * (M + m))*r) 0];
den_U_E = [tau 1];
U_E = tf (num_U_E, den_U_E);

disp ' '

% G(s) = Theta(s) / E(s)
% Open Loop Transfer Function (Without Feedback)
disp 'Forward Path Transfer Function of Inverted Pendulum System is:'
G = series (U_E, Th_U)

% H(s) (Feedback)
num_H = Kf;
den_H = 1;
H = tf (num_H, den_H);

% Closed Loop (Unity Feedback) Transfer Function
% Gc(s) = G(s) / (1 + G(s) * H(s))
disp 'Closed-Loop Transfer Function of Inverted Pendulum System is:'
Gc = feedback (G, H)

% GH(s)
% Open Loop Transfer Function
GH = series (G, H);
```

(3) M-FILE FOR ANALYSIS OF THE UNCOMPENSATED INVERTED PENDULUM SYSTEM

```
%--------------------------------------------------------------------------
% analysis_uc_ip.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Analysis of the Uncompensated Inverted Pendulum System
%--------------------------------------------------------------------------
%

% func_ip is a MAT File (MATLAB specific binary file),
% with variables G, GH, H, Th_U, U_E
load func_ip

% Locations of Poles and Zeroes of Open-Loop Transfer Function in Complex Plane
figure
pzmap (G)
title ('Pole-Zero Map of Open-Loop Uncompensated Inverted Pendulum System')

% Impulse Response
figure
impulse (G)
title ('Impulse Response of Open Loop Uncompensated Inverted Pendulum System')

% Step Response
figure
step (G)
title ('Step Response of Open Loop Uncompensated Inverted Pendulum System')

% Locations of Poles and Zeroes of Closed-Loop Transfer Function in Complex
Plane
figure
pzmap (feedback (G, H))
title ('Unity Feedback Closed-Loop Uncompensated Inverted Pendulum System')

% Root Locus Plot of Uncompensated System
figure
rlocus (GH)
title ('Root Locus of Uncompensated Inverted Pendulum System')
```

(4) M-FILE FOR CLOSED LOOP COMPENSATED TRANSFER FUNCTION OF IP SYSTEM

```
%-------------------------------------------------------------------------------
% trans_func_ip_comp.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Closed Loop Compensated Transfer Function of the Inverted Pendulum System
%-------------------------------------------------------------------------------
%
%                    E(s)         U(s)
%        Vpot --->O---[ C(s) ]--->[ G2(s) ]--->[ G1(s) ]---+---> Theta (s)
%               - ^                                         |
%                 |                                         |  Theta = SYS * E
%                 +----------------[ H(s) ]<--------------+
%                         V viper

% func_ip_uc is a MAT File (MATLAB specific binary file),
% with variables G, GH, Gc, H, Th_U, U_E, num_G, den_G
load func_ip_uc

% C(s) = ( Kd * s^2 + Kp * s + Ki ) / s
% PID Controller to reshape the root locus
Kp = 20;
Ki = 100;
Kd = 1;
Kc = 30;
num_PID = Kc * [Kd Kp Ki];
den_PID = [1 0];
disp ('The transfer function of the PID Controller is:')
PID = tf (num_PID, den_PID)

% G_comp(s) = PID(s) * G(s)
% Overall Forward Transfer Function.
num_Gcomp = conv (num_PID, num_G);
den_Gcomp = conv (den_PID, den_G);
G_comp = series (PID, G);

% Open Loop Transfer Function of the Compensated System
% G_comp_H(s) = G_comp(s) * H(s)
G_comp_H = series (G_comp, H);

% Closed-Loop Transfer Function of the Compensated System
% Gc_comp
disp 'Closed Loop Transfer Function of the Compensated IP System is:'
Gc_comp = feedback (G_comp, H)

% DC Gain
disp ('The DC Gain of the Closed Loop Compensated IP System is:')
disp (dcgain (Gc_comp))
```

(5) M-FILE FOR ANALYSIS OF THE COMPENSATED INVERTED PENDULUM SYSTEM

```
%-------------------------------------------------------------------------------
% analysis_comp_ip.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Analysis of the Compensated Inverted Pendulum System
%-------------------------------------------------------------------------------
%

% func_ip_comp is a MAT File (MATLAB specific binary file),
% with variables G_comp, G_comp_H, Gc_comp, PID
load func_ip_comp

% Locations of Poles and Zeroes of Open-Loop Compensated Transfer Function in
Complex Plane
figure
pzmap (G_comp_H)
axis ([-15 10 -1 1])
title ('Pole-Zero Map of Open-Loop Compensated Inverted Pendulum System')

% Root-Locus Plot of Compensated Inverted Pendulum System
figure
rlocus (G_comp_H)
sgrid (0.76,35)
title ('Root Locus of Compensated Inverted Pendulum System')

% Locations of Poles and Zeroes of Closed-Loop Transfer Function in Complex
Plane
figure
pzmap (Gc_comp)
title ('Pole-Zero Map of Closed-Loop Compensated Inverted Pendulum System')

% Impulse Response of Compensated Inverted Pendulum System
figure
impulse (Gc_comp)
title ('Impulse Response of Closed-Loop Compensated Inverted Pendulum System')

% Step Response of Compensated Inverted Pendulum System
figure
step (Gc_comp)
title ('Step Response of Closed-Loop Compensated Inverted Pendulum System')
```

(6) M-FILE FOR PLOT OF EXPERIMENTAL DATA OBTAINED THRU 8051-BASED DAQ CARD

```
%-------------------------------------------------------------------------
% data_exp_ip.m
% Design and Development of Closed Loop Control for INVERTED PENDULUM
% By IIEE Visionaries
% Copyright 2003
% Experimental Data of the Inverted Pendulum System
%-------------------------------------------------------------------------

% expdata is a MAT File (MATLAB specific binary file),
% with variables exp_data & tout
load expdata

plot (tout,exp_data);
xLabel ('Time (Seconds)');
yLabel('Pendulum Position From Vertical (Degrees)');
Title('Experimental Data');
axis ([0 0.15 -40 40]);
Grid;
```

BIBLIOGRAPHY

BIBLIOGRAPHY
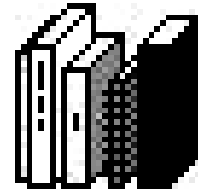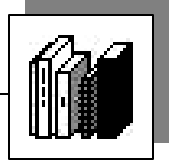
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