Theory of machine

If you have a smart project, you can say "I'm an engineer"

Lecture 6

Instructor

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Theory of machine
MDP 234

- **Lecture aims:**

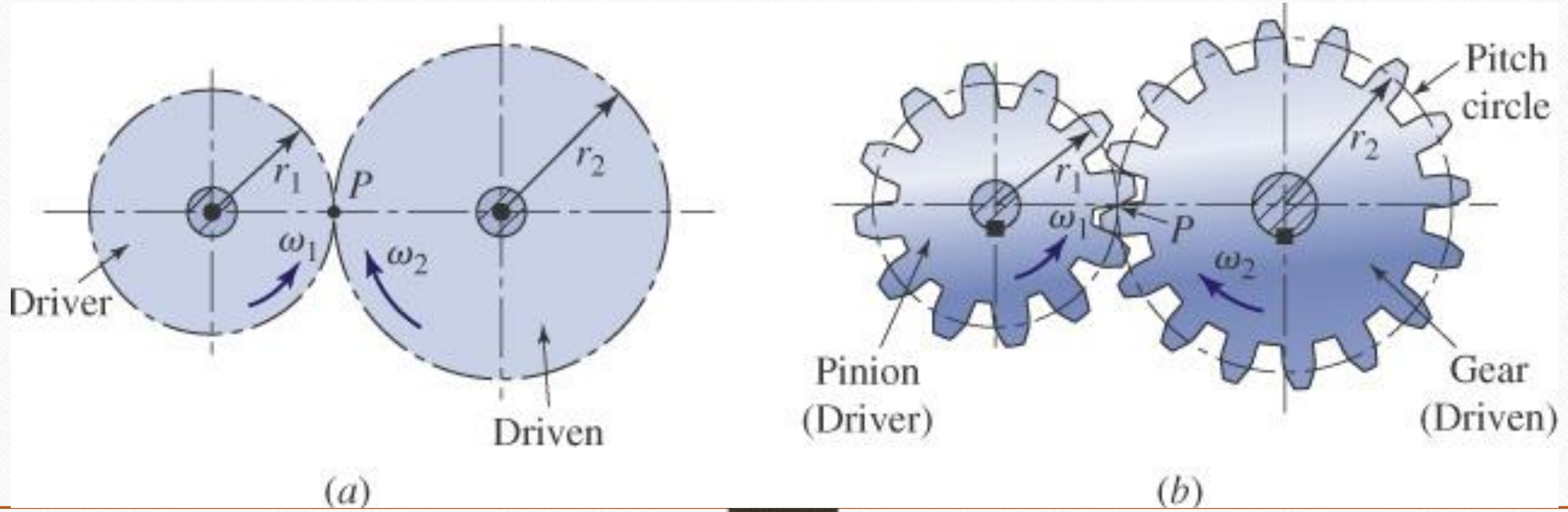
1. Learn fundamental concepts and terminology.
2. Learn how to design of a gear box and Forces transmitted.

Introduction

Nomenclature

Smaller Gear is Pinion and Larger one is the gear

In most application the pinion is the driver, This reduces speed but it increases torque.



Introduction

Nomenclature

pitch circle, theoretical circle upon which all calculation is based

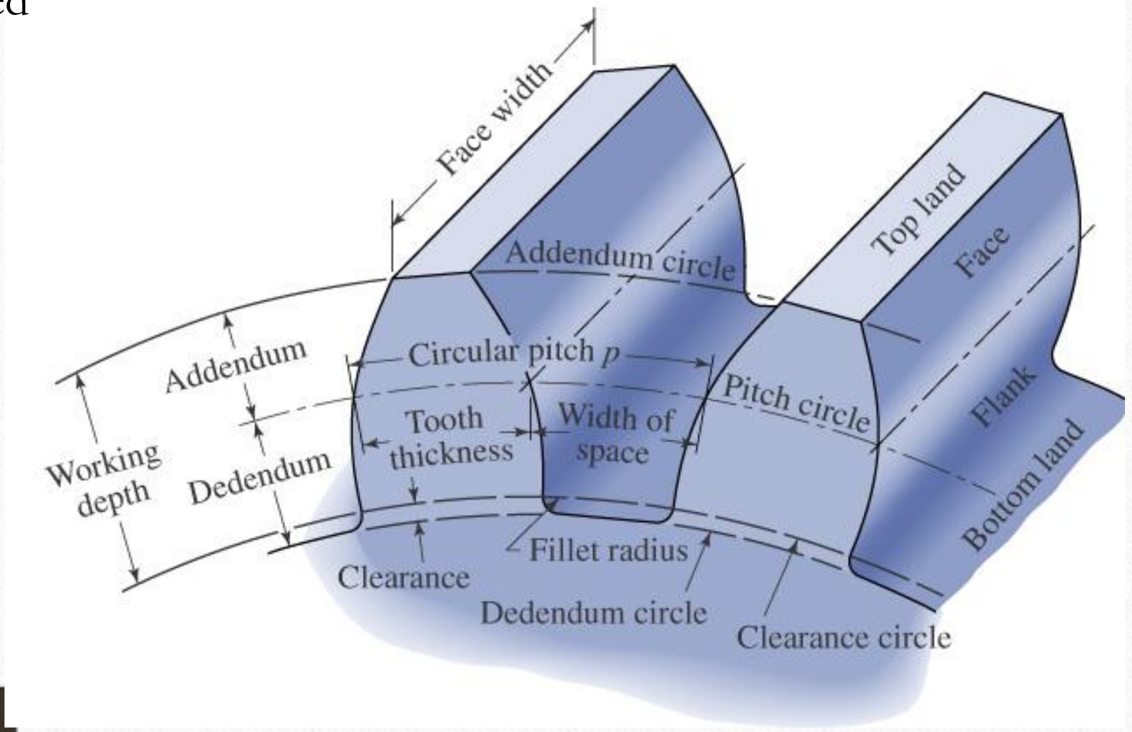
p, Circular pitch, p the distance from one teeth to the next, along the pitch circle. p=πd/N

m, module=d/N pitch circle/number of teeth

p= πm

P, Diametral Pitch P=N/d

pP= π

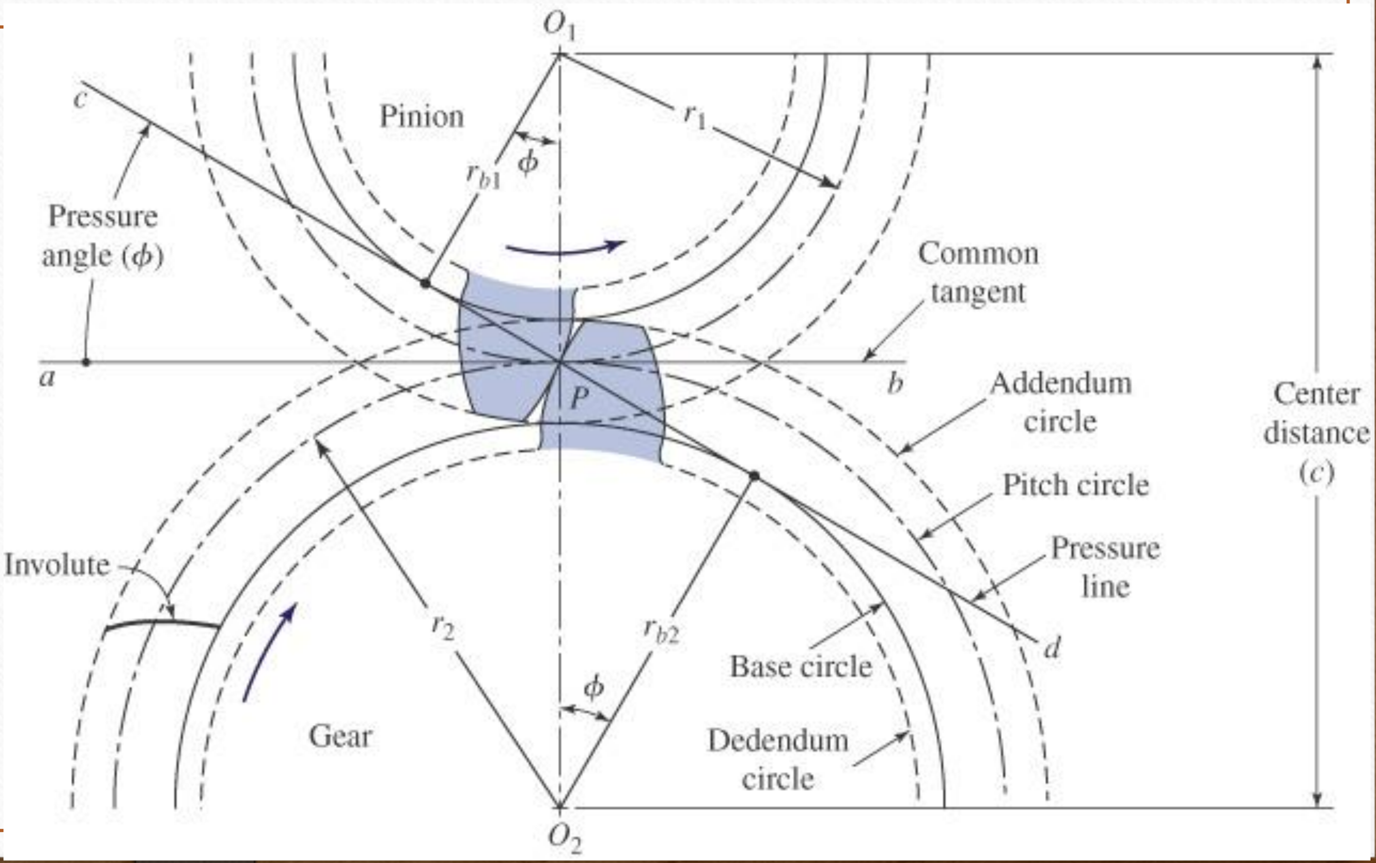


Introduction

Nomenclature

Angle Φ has the values of 20 or 25 degrees.
Angle 14.5 have been also used.

Gear profile is constructed from the base circle.
Then additional clearance are given.



Standard Gear Teeth

Item	20^o full depth	20^o Stub	25^o full depth
Addendum a	1/P	0.8/P	1/P
Dedendum	1.25/P	1/P	1.25/P
Clearance f	0.25/P	0.2/P	0.25/P
Working depth	2/P	1.6/P	2/P
Whole depth	2.25/P	1.8/P	2.25/P
Tooth thickness	1.571/P	1.571/P	1.571/P
Face width	$9/P<b<13/P$	$9/P<b<13/P$	$9/P<b<13/P$

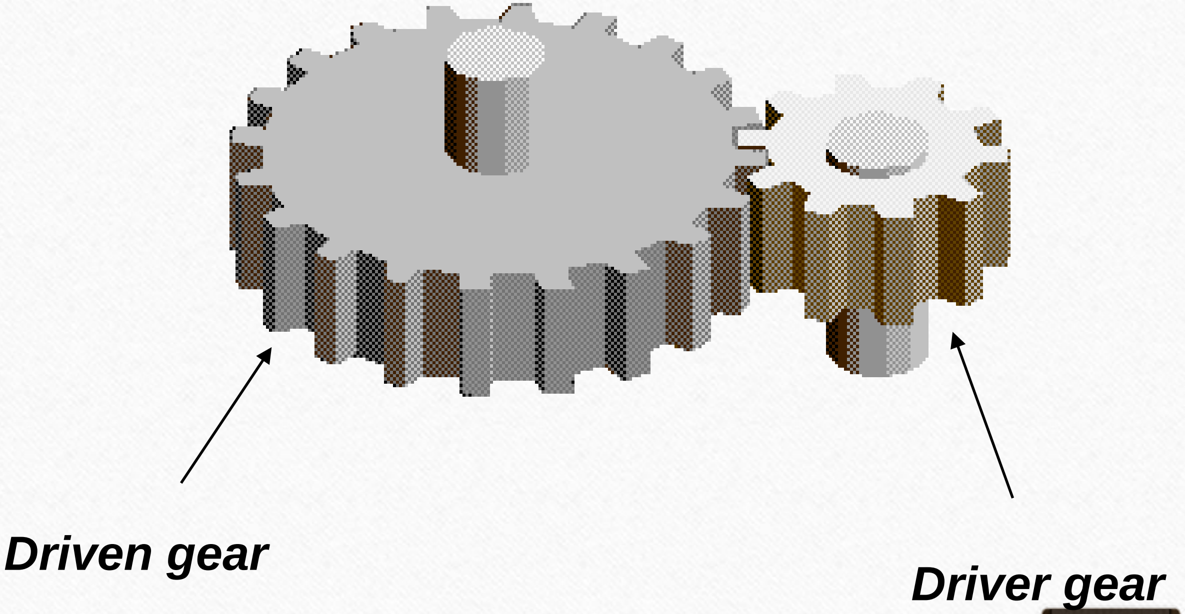
Type of Gears

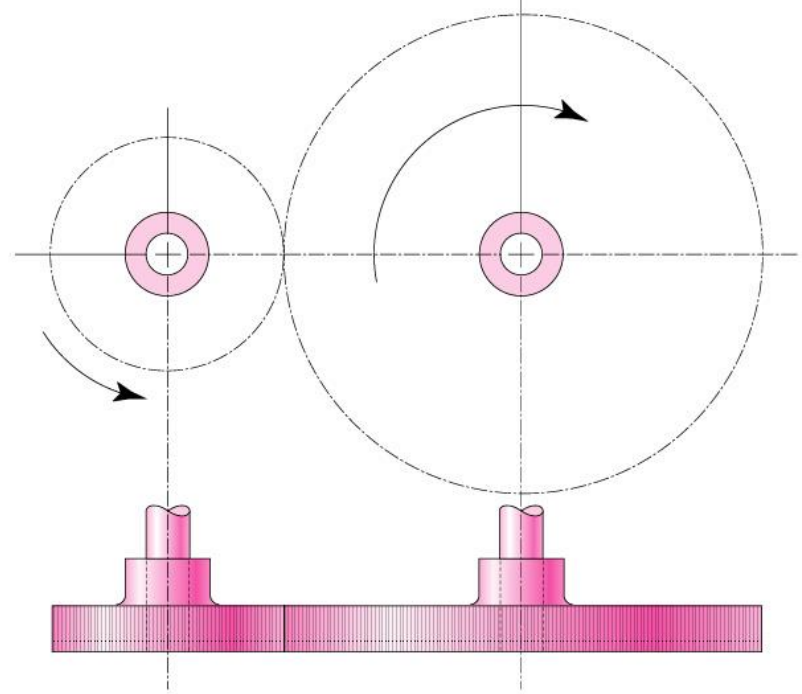
- Spurs
- Helical
- Bevel
- And Worm Gears

Type of Gears

Spur Gears

Are used in transmitting torque between parallel shafts

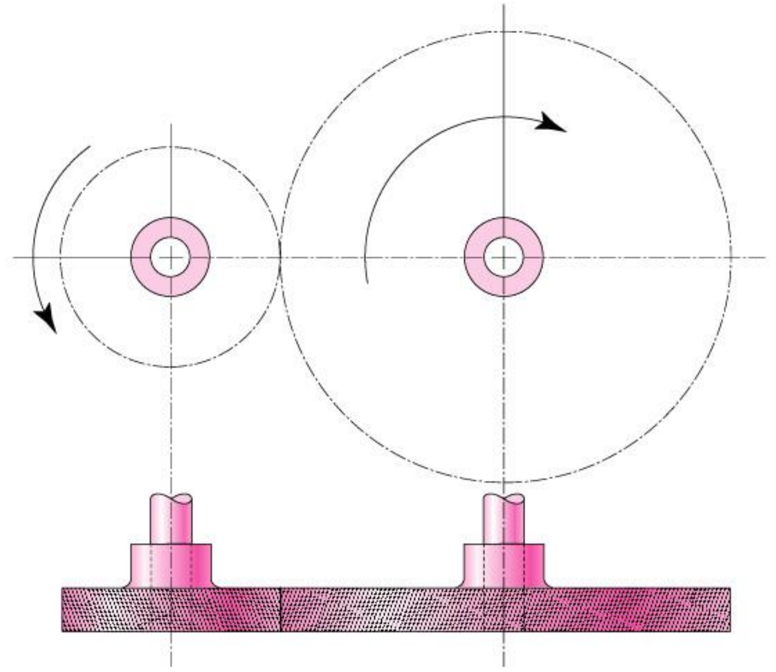




Type of Gears

Helical Gears

Are used in transmitting torques between parallel or non parallel shafts, they are not as noisy as spur gears

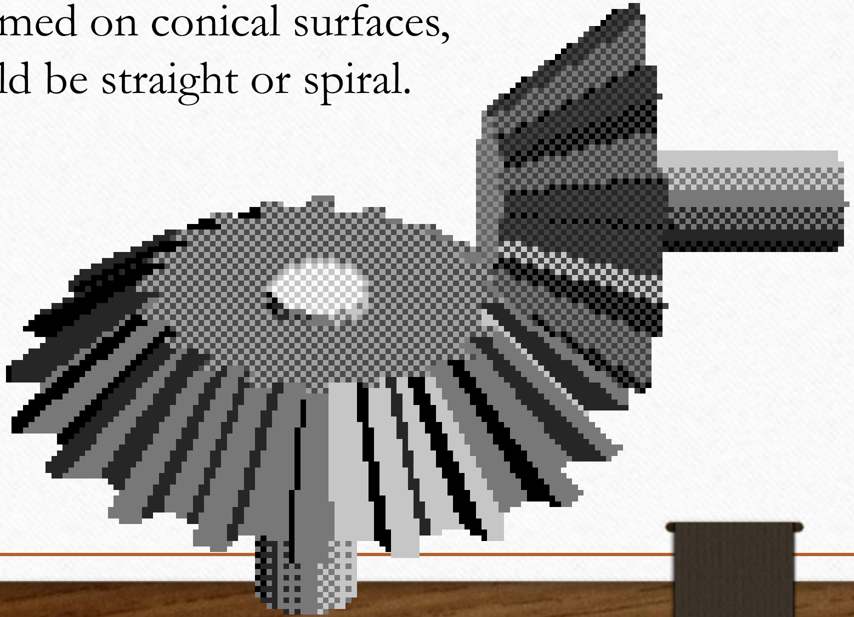


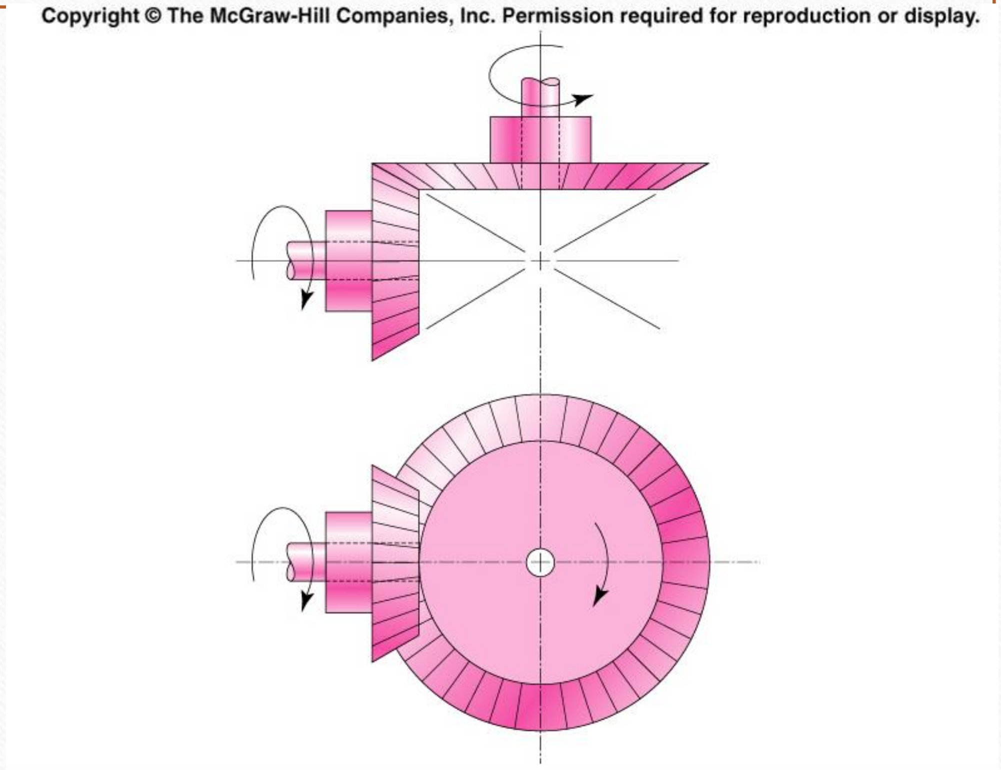
Type of Gears

Bevel Gears

- Are used to transmit rotary motion between intersecting shafts

Teeth are formed on conical surfaces, the teeth could be straight or spiral.



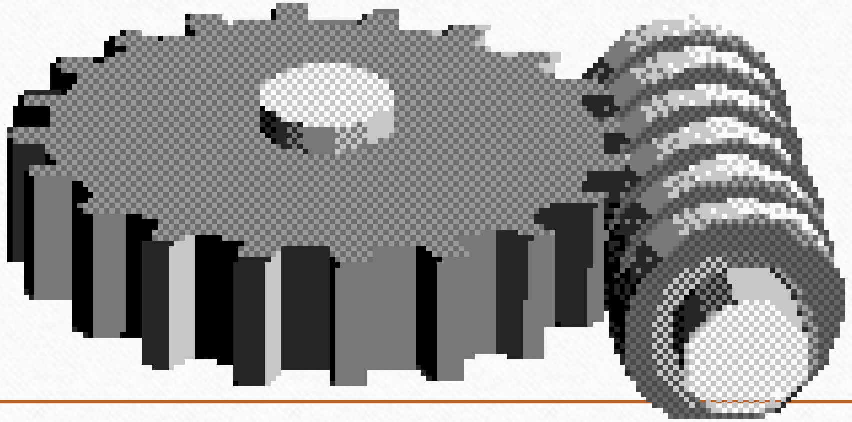


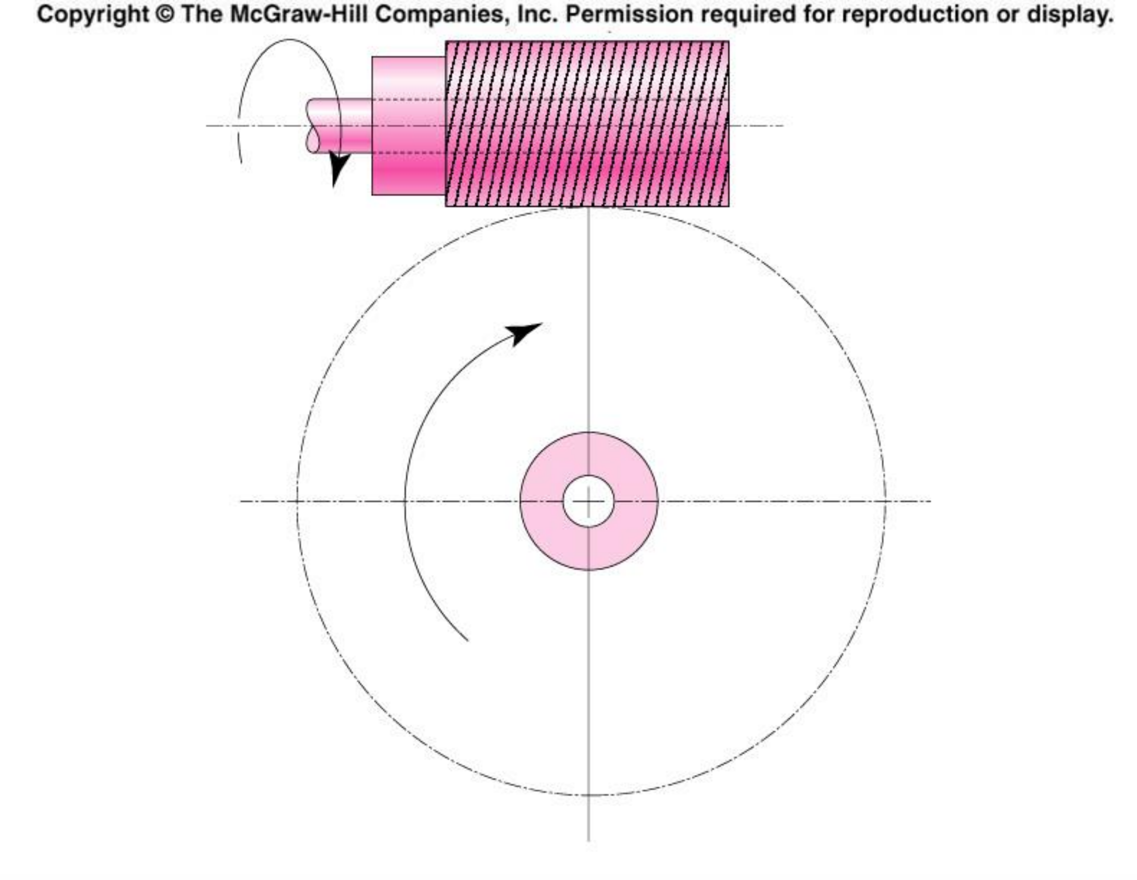
Type of Gears

Worm Gears

Are used for transmitting motion between non parallel and non transmitting shafts, Depending on the number of teeth engaged called single or double.

- ***The worm gear is always the drive gear***

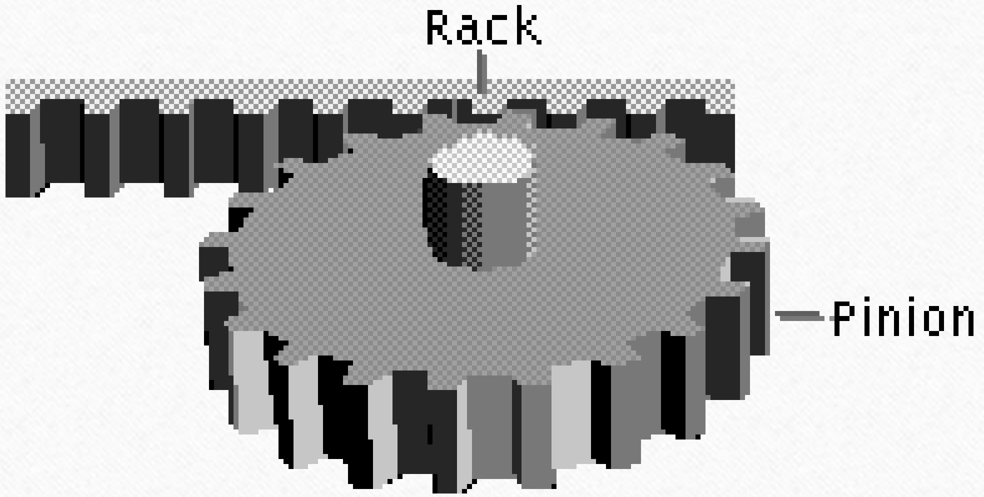


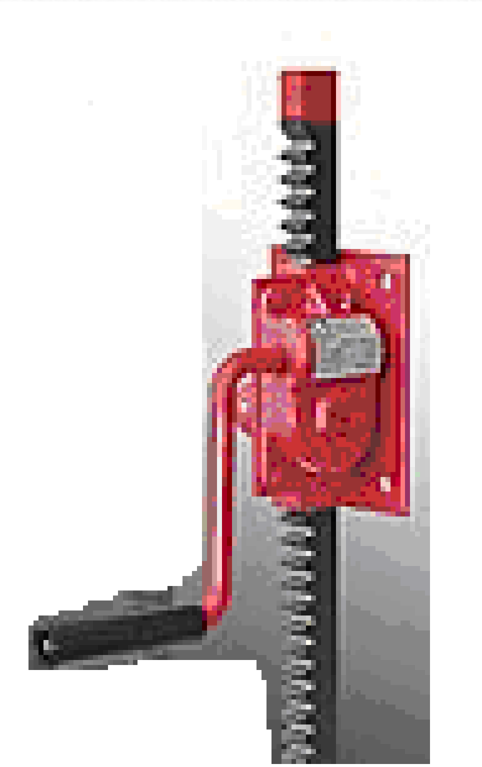


Type of Gears

Rack and Pinion

- The rack and pinion gear is used to convert between rotary and linear motion.





Heavy Duty

Car Jack

Introduction to the gear train

❑Definition:
When two or more gears are made to mesh with each other to transmit power from one shaft to another, such a combination is called '*gear train or train of toothed wheels*'.

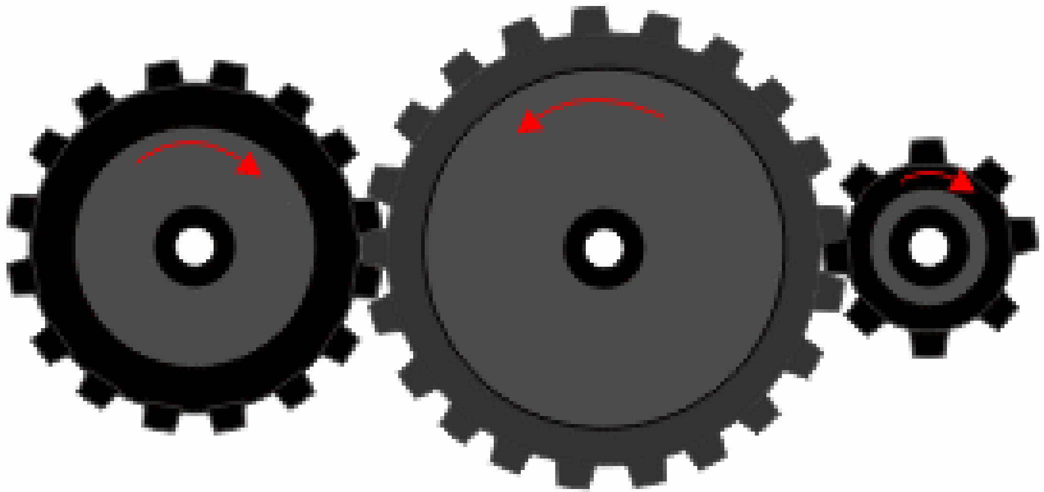
❑Types of gear trains:
1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

Gear Trains

SIMPLE GEAR TRAIN

- Multiple gears can be connected together to form a gear train.

Intermediate gears are known as Idler Gears.



Each shaft carries only one gear wheel.

Gear Trains

SIMPLE GEAR TRAIN

- It may be noted that when the number of intermediate gears odd, the motion of both gears is like but if the number of intermediate gears even, the motion of both gears is unlike.

- These intermediate gears are called *idle gears*, as they do not effect the speed ratio or train value of the system.

- The idle gears are used for the following two purposes :

1. To connect gears where a **large distance** is required, and
2. To obtain the **desired direction** of motion of the driven gear(i.e. clockwise or anticlockwise).

Gear Trains

SIMPLE GEAR TRAIN

- Since the speed ratio of gear train is the ratio of the speed of the driver to the speed of the driven or follower and the ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{\text{speed of driver}}{\text{speed of driven}} = \frac{\text{No of teeth on driven}}{\text{No of teeth on driver}}$$

- It may be noted that ratio of the speed of the driven to the speed of the driver is known as train value of the gear train. Mathematically,

$$\text{Train value} = \frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{No of teeth on driver}}{\text{No of teeth on driven}}$$

Gear Trains

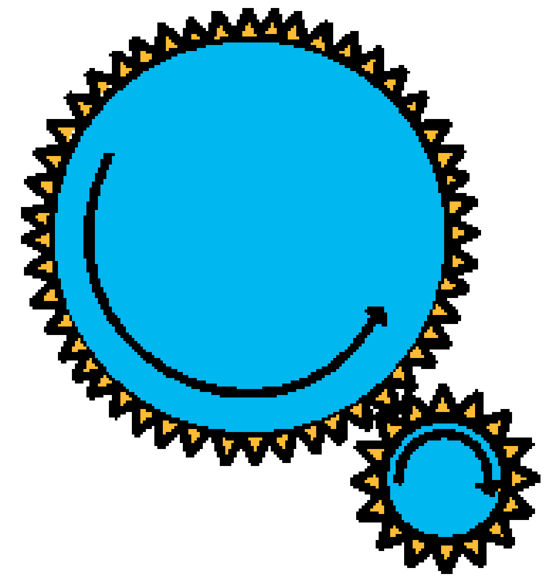
ADVANTAGES OF SIMPLE GEAR TRAIN

- to connect gears where a large center distance is required
- to obtain desired direction of motion of the driven gear (CW or CCW)
- to obtain high speed ratio

Gear Trains

GEAR RATIO - CALCULATION

A 100 tooth gear drives a 25 tooth gear.
Calculate the gear ratio for the meshing teeth.



Gear ratio = $\dfrac{\textbf{Number of teeth on driven gear}}{\textbf{Number of teeth on driver gear}}$

Gear ratio = $\dfrac{\text{driven}}{\text{driver}}\ \dfrac{25}{100} = \dfrac{1}{4}$

This is written as 1:4

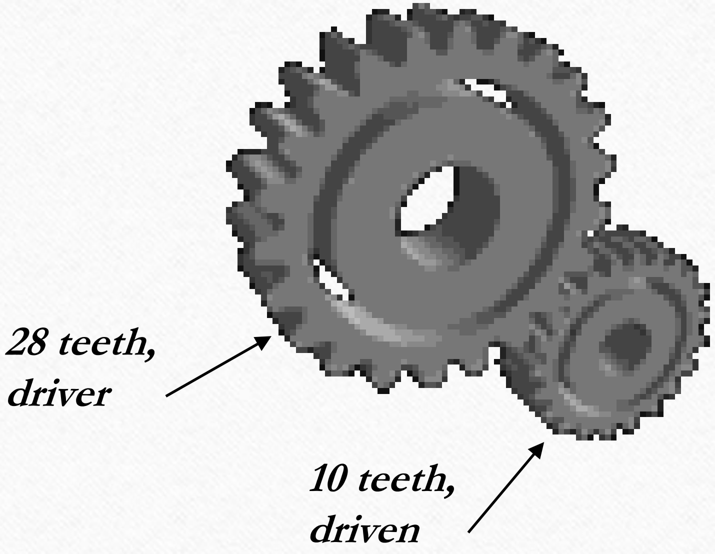
Gear Trains

GEAR SPEED :- CALCULATION

A motor gear has 28 teeth and revolves at 100 rev/min.
The driven gear has 10 teeth. What is its rotational speed?

Speed of driven gear = $\frac{\text{Number of teeth on driver gear}}{\text{Number of teeth on driven gear}}$ x 100

Speed of driven gear = $\frac{\text{driver}}{\text{driven}} = \frac{28}{10} \times 100 = 280$ rev/min



Gear Trains

IMPORTANT CALCULATIONS

Work Done = ***Force x Distance moved in the direction of the force***

$$\textbf{Power} = \frac{\text{Total Work Done}}{\text{Total Time Taken}}$$

$$\textbf{Efficiency \%} = \frac{\text{Power Output}}{\text{Power Input}} \text{ X } 100$$

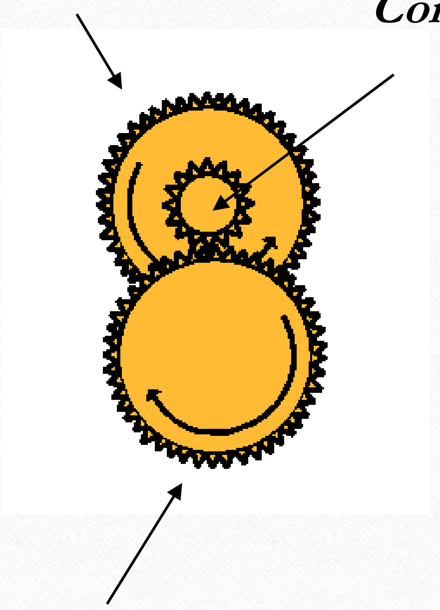
$$\textbf{\textit{Efficiency}} = \frac{\textbf{\textit{Mechanical advantage}}}{\textbf{\textit{Velocity ratio}}}$$

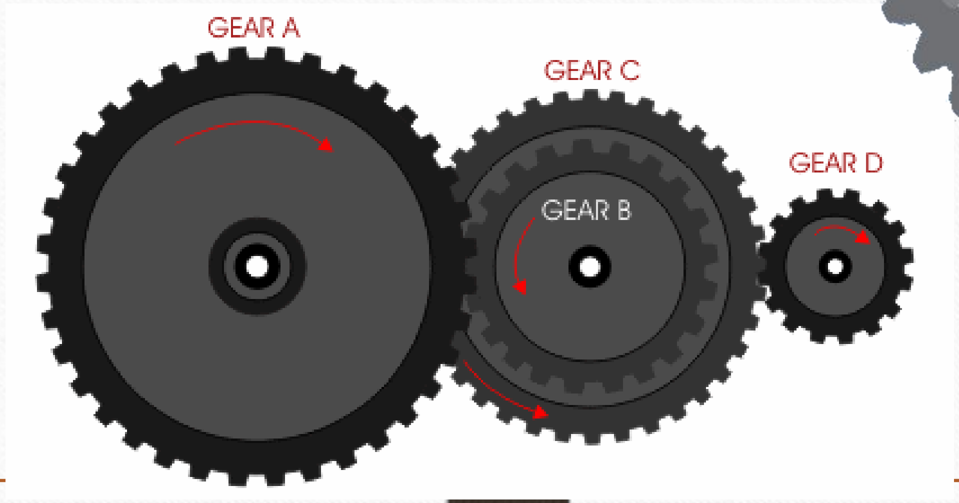
Friction: - *Resists the movement of one surface over another*

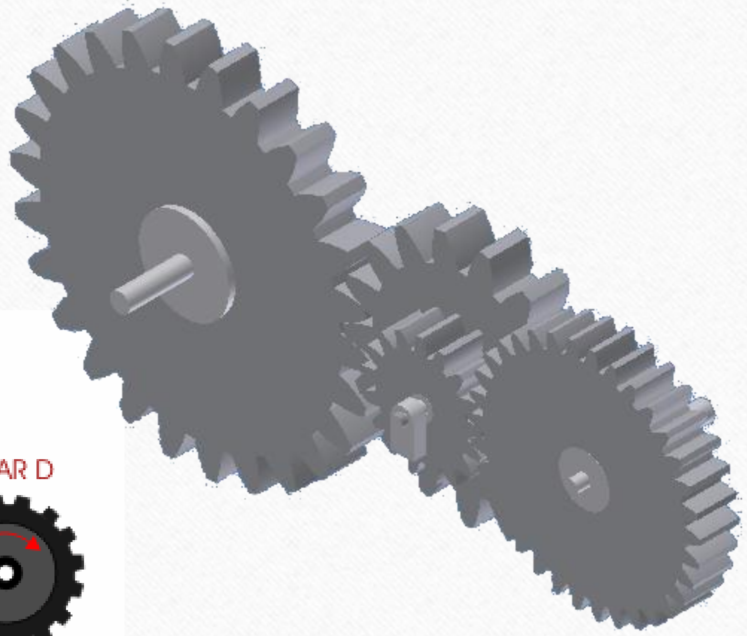
Gear Trains

COMPOUND GEAR TRAIN

If two gear wheels are mounted on a common shaft then it's a Compound Gear train.







Gear Trains

COMPOUND GEAR TRAIN

Speed ratio of compound gear train is given by,

$$\text{Speed ratio} = \frac{\textit{speed of the first driver}}{\textit{speed of the last driven}}$$

$$= \frac{\textit{product of the number of teeth on the drivens}}{\textit{product of the number of teeth on the drivers}}$$

Train value of compound gear train is given by,

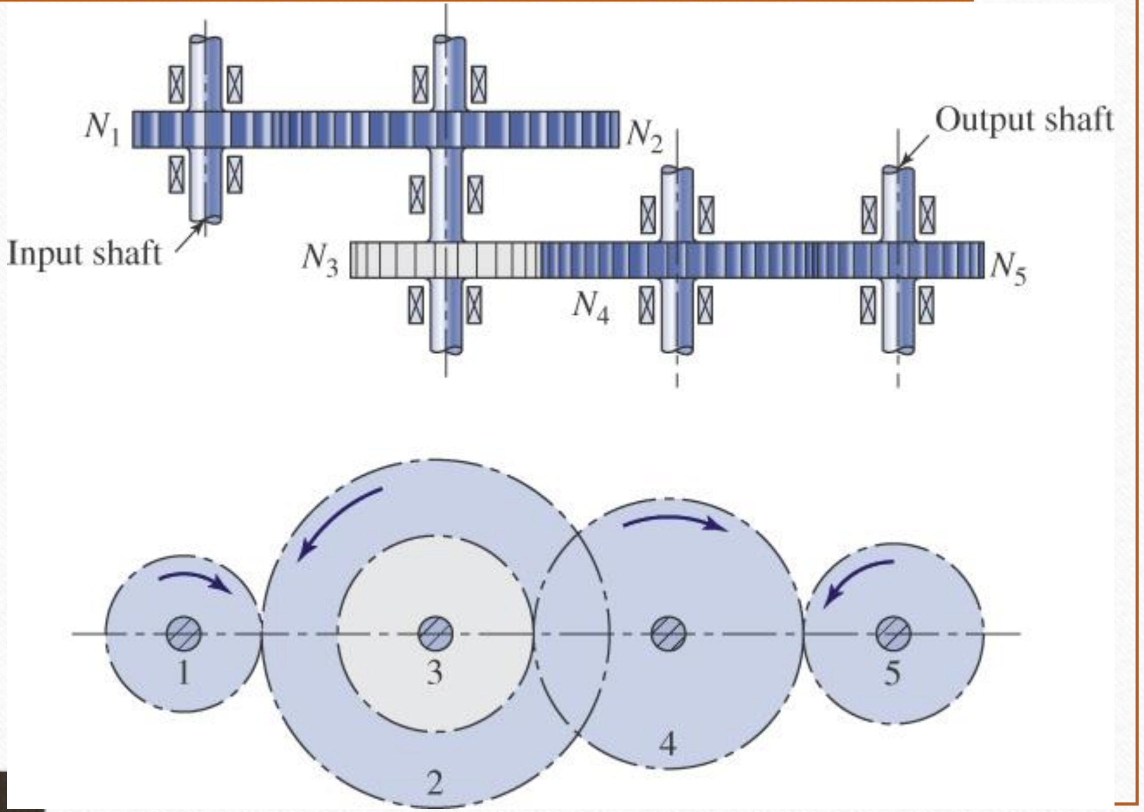
$$\text{Train value} = \frac{\textit{speed of the last driven}}{\textit{speed of the first driver}}$$

$$= \frac{\textit{product of the number of teeth on the drivers}}{\textit{product of the number of teeth on the drivens}}$$

- The advantage of compound train over a simple train is that a much longer speed reduction from one shaft to the last shaft can be obtained with small gears.

Gear Trains

$$\frac{n_5}{n_1} = \left(-\frac{N_1}{N_2}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_4}{N_5}\right)$$



Gear Trains

ADVANTAGES OF COMPOUND GEAR TRAIN

- A much larger speed reduction from the first shaft to the last shaft can be obtained with small gear.
- If a simple gear trains used to give a large speed reduction, the last gear has to be very large.

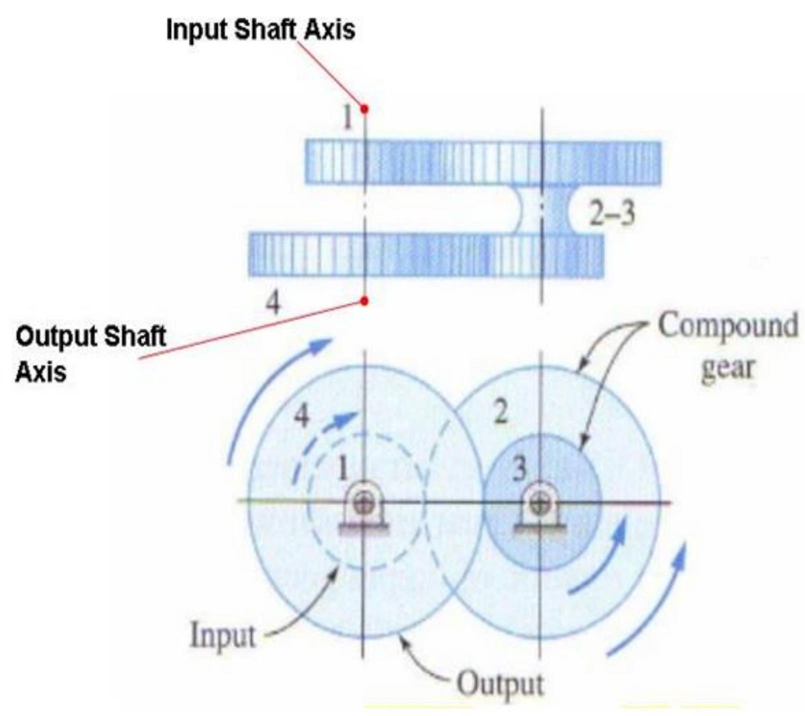
Gear Trains

REVERTED GEAR TRAIN

- When the axes of the first driver and the last driven are co-axial, then the gear train is known as

  reverted gear **train.**

- In a reverted gear train, the motion of the first gear and the last gear is ***same.***



Gear Trains

ADVANTAGES OF RIVERTED GEAR TRAIN

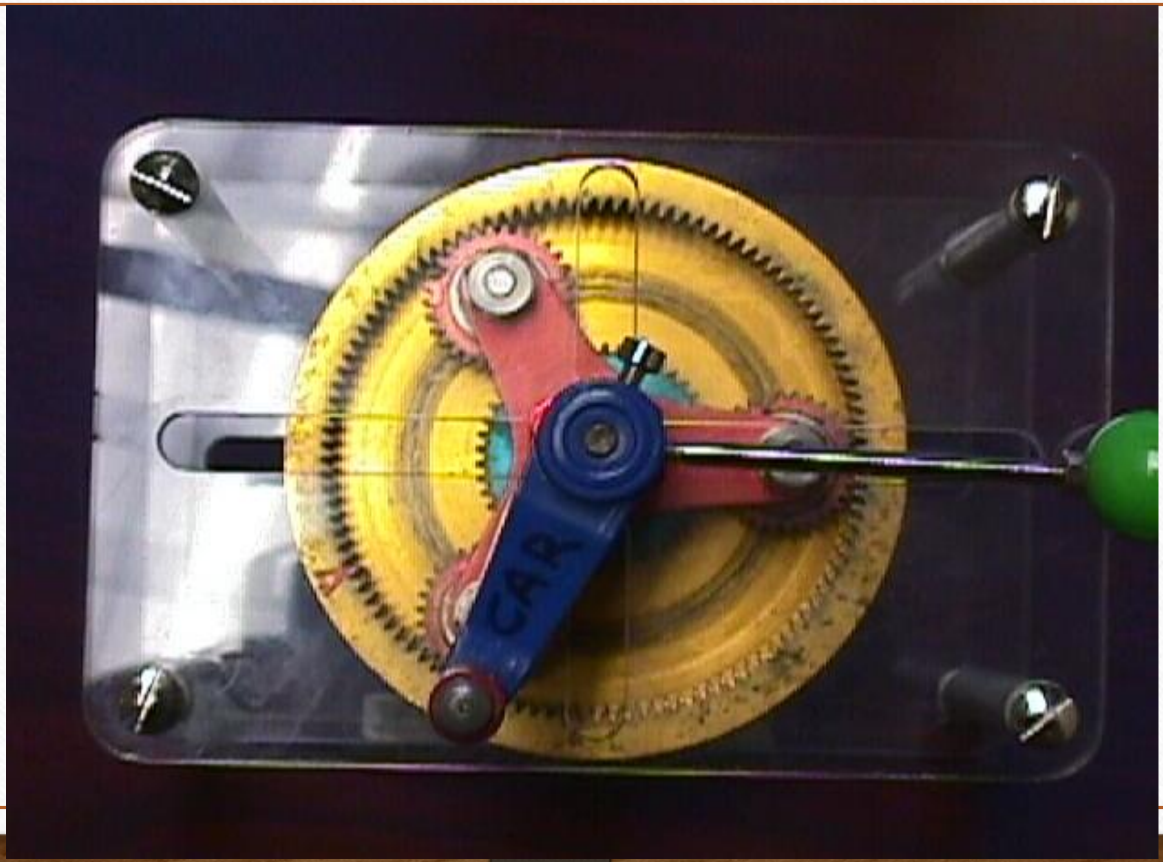
- The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Gear Trains

PLANETARY (OR EPICYCLIC) GEAR TRAIN

- Gears whose centers can move
- Used to achieve large speed reductions in compact space
- Can achieve different reduction ratios by holding different combinations of gears fixed
- Used in automatic transmissions of cars

Planetary gear

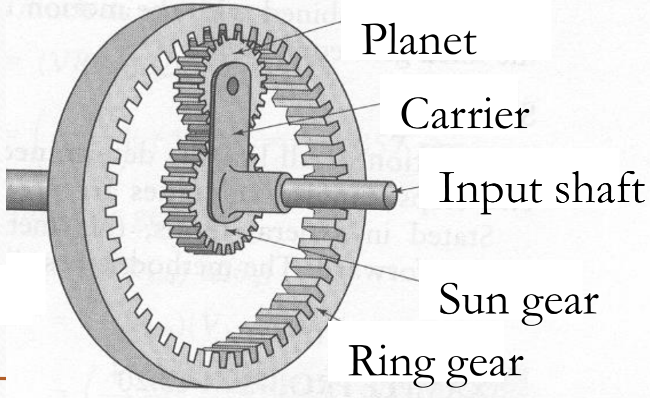


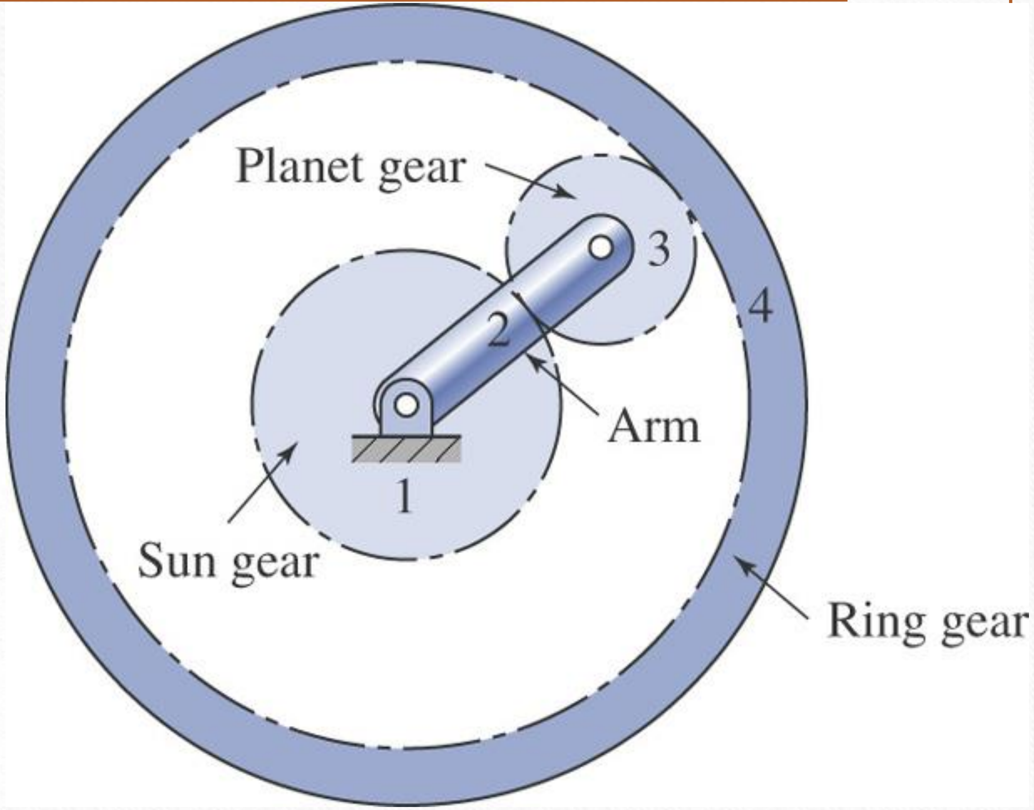
Gear Trains

Planetary Gear train

You can get high torque ratio in a smaller space

There are two inputs to the planetary gears, RPM of sun and Ring, The out put is the speed of the arm.





Gear Trains

ADVANTAGES OF EPICYCLIC GEAR TRAIN

- They have higher gear ratios.
- They are popular for automatic transmissions in automobiles.
- They are also used in bicycles for controlling power of pedaling automatically or manually.
- They are also used for power transmission between internal combustion engine and an electric motor.

Gear Trains

VELOCITY RATIO OF EPICYCLIC GEAR TRAIN

- **velocity ratio**

velocity ratio of epicyclic gear train is the ratio of the speed of the driver to the speed of the driven or follower.

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

- **Tabular method**
- **Algebraic method**

Gear Trains

1. Tabular Method

T_A = Number of teeth on gear *A*

T_B = Number of teeth on gear B.

Suppose that the arm is fixed.

Therefore, the axes of both the gears are also fixed relative to each other.

When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B

$$N_B / N_A = T_A / T_B$$

Since N_A = 1 revolution, therefore

$$N_B = T_A / T_B$$

Assuming the **anticlockwise** rotation as **positive** and **clockwise** as **negative**.

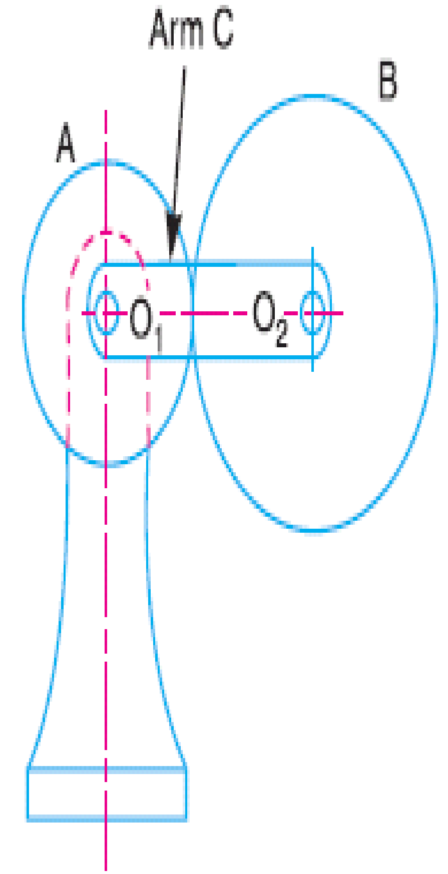


Fig. 13.6. Epicyclic gear train.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Gear Trains

2. Algebraic Method

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations.

The number of equations depends upon the number of elements in the gear train. Let the arm C be fixed in an epicyclic gear train as shown in Fig

Therefore,

Speed of the gear A relative to the arm C= N(A)-N(C)

Speed of the gear B relative to the arm C= N(B) –N(C)

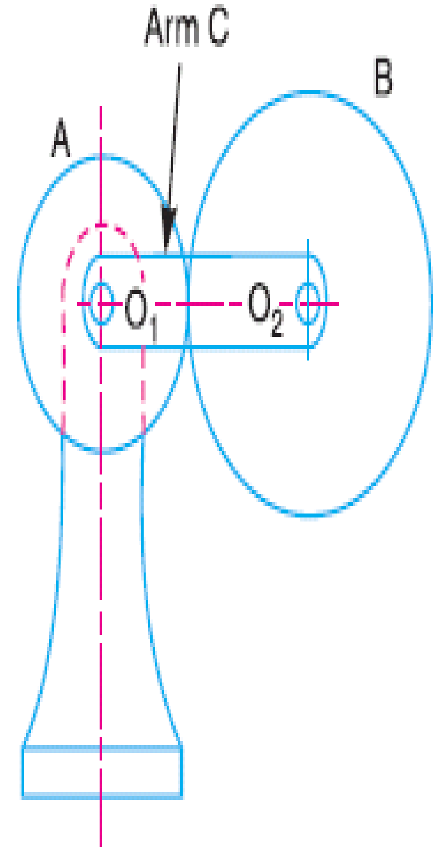


Fig. 13.6. Epicyclic gear train.

Since the gears A and B are meshing directly, therefore they will revolve in *opposite* directions.

$$\therefore \qquad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

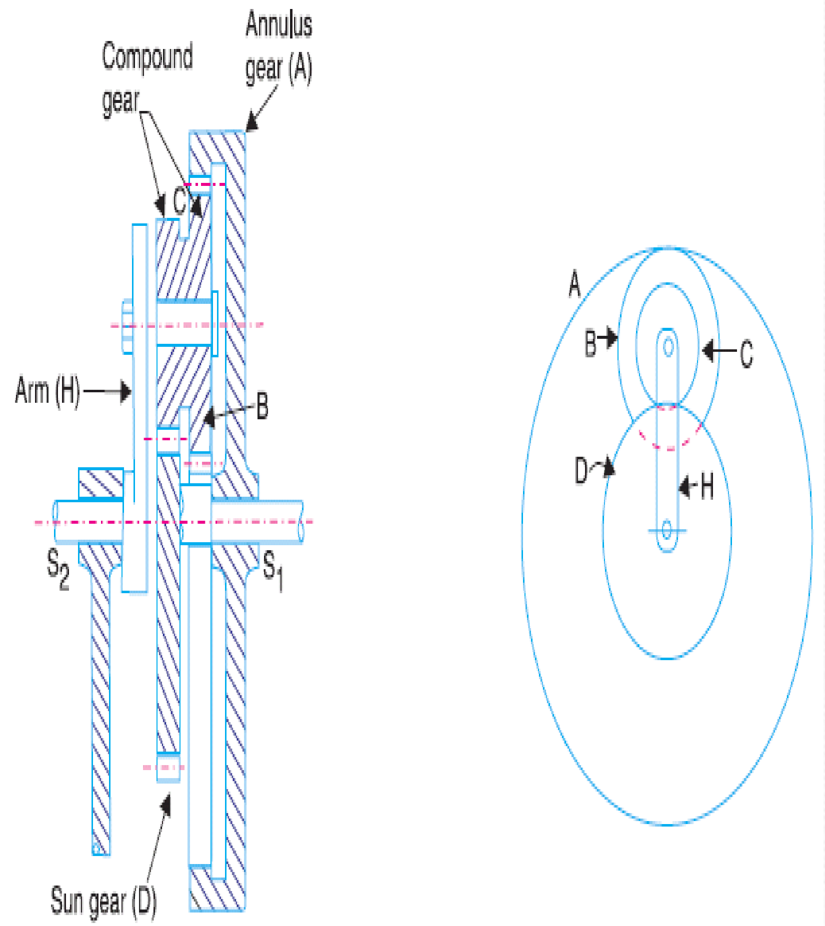
$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

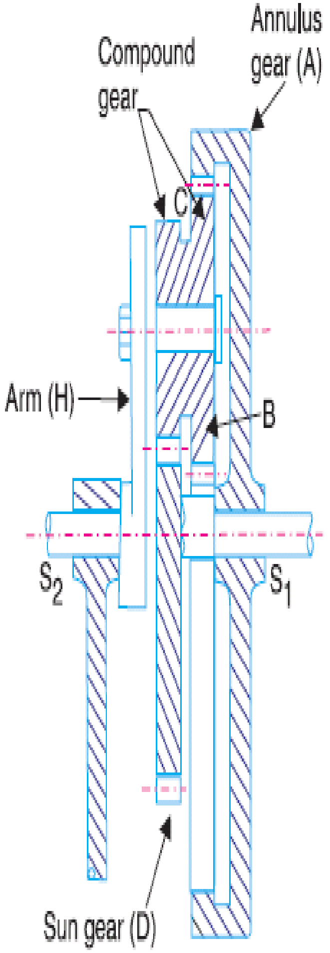
Gear Trains

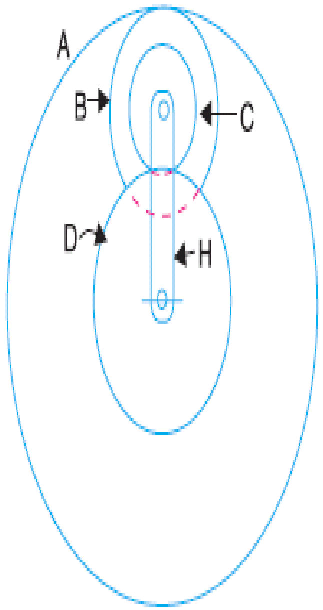
COMPOUND EPICYCLIC GEAR TRAIN (SUN AND PLANET GEAR)

It consists of two co-axial shafts S_1 and S_2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them.



The annulus gear *A* meshes with the gear *B* and the sun gear *D* meshes with the gear *C*. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases,





Note : The gear at the centre is called the *sun gear* and the gears whose axes move are called *planet gears*.

Let T_A, T_B, T_C, and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A*, *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwise direction.

The motion of rotations of the various elements are shown in the table below.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear *D* rotates through + 1 revolution	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear *D* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

Gear Trains

Example of planetary Gear train

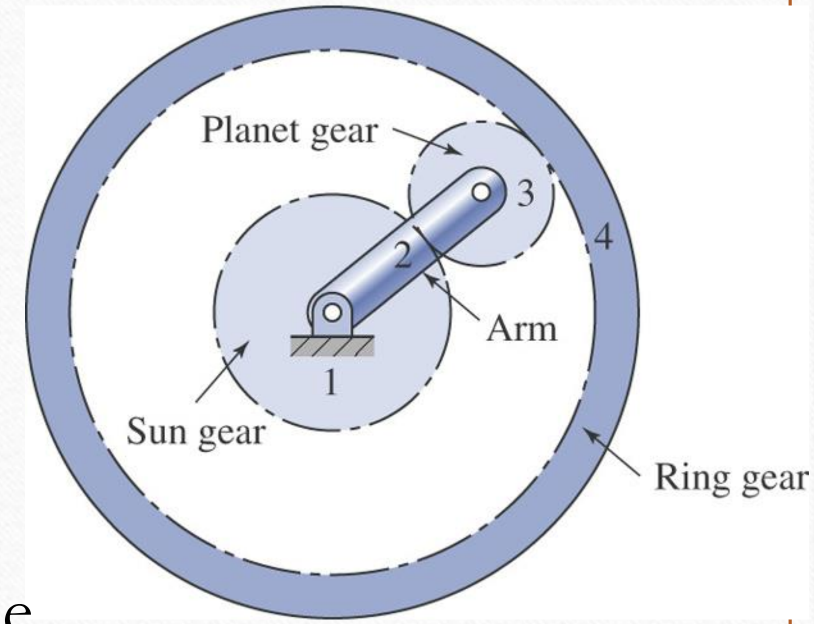
Gear 1, sun , RPM 1200, Number of teeth 20,

Planet Gear , Number of teeth 30

Ring Gear, Rotates RPM 120, and teeth of 80,

¼ horse power, find the speed of the arm and torque on the ring.

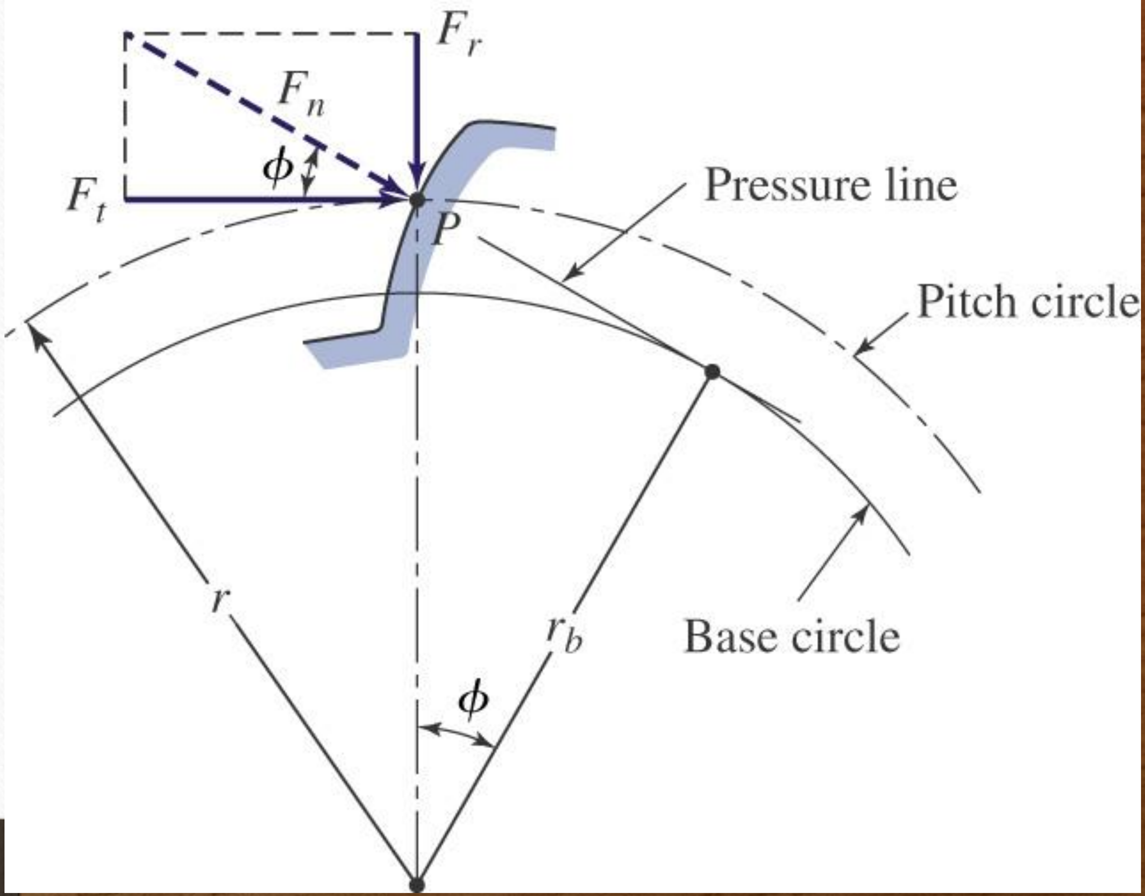
Alternatively you may have Certain Output Torque requirements



Some Useful Relations

Transmitted Load

- With a pair of gears or gear sets, Power is transmitted by the force developed between contacting Teeth



Some Useful Relations

$$F_t = F_n \cos\phi$$

$$F_r = F_n \sin\phi$$

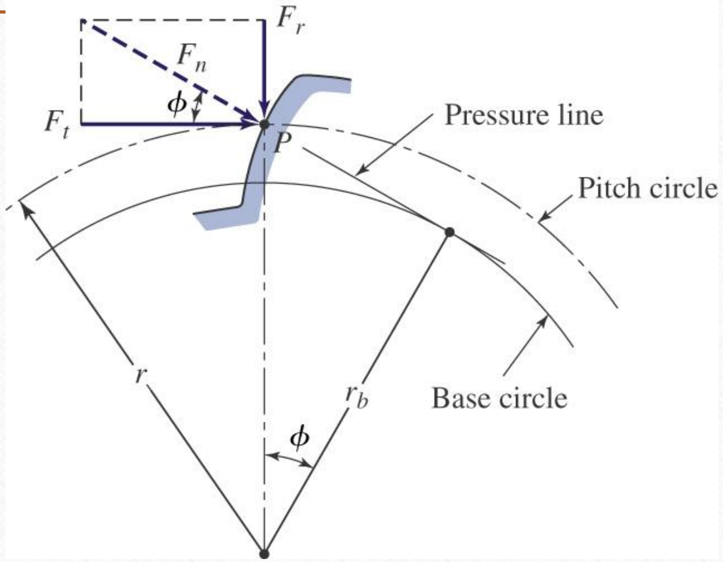
$$V = d/2\omega = d * \frac{2\pi RPM}{60}$$ d in, RPM rev./min, V in/sec

$$V = \frac{\pi d n}{12}$$ d in, n rpm, V fpm

$$hp = \frac{Tn}{63000}$$ Toque lb-in

$$F_t = \frac{33000 hp}{V}$$ V fpm

$$KW = \frac{F_t V}{1000} = \frac{Tn}{9549}$$ T= N.m, V m/s, F Newton



These forces have to be corrected for dynamic effects , we discuss later, considering AGMA factors

Some Useful Relations

- F=33000hp/V   V fpm   English system

- Metric System
- KW=(FV)/1000=Tn/9549
- F newton, V m/s, n rpm, T, N.m
- hp= FV/745.7=Tn/7121

Gear Box Design