INFLUENCE OF FUEL DISTRIBUTION AND HEAT TRANSFER ON ENERGY CONSUMPTION IN TUNNEL KILNS

Hassanein A. Refaey1*, E. Specht2, M.R.Salem3
1,3Mechanical Power Engineering Department, Shoubra Faculty of Engineering, Benha University, 108 Shoubra street, Cairo, Egypt.
2Institute of Fluid Dynamics and Thermodynamics, Otto von Guericke University, Universitätsplatz 2 D-39106 Magdeburg, Germany.

ABSTRACT
Reducing energy consumption is one of the most important priorities of nowadays life. Therefore, it is of interest to reduce energy consumption in industrial processes, especially of ceramic products. In this paper, a 1D simple mathematical model to predict gas and solid temperature profiles in tunnel kilns has been developed. For this purpose, three different cases were studied to simulate the process in preheating and burning zones. In each case, the ordinary differential equations are obtained to demonstrate the heat transfer and energy consumption along the kiln. In addition, the influence of fuel distribution along the firing zone on the temperature profiles was studied. The results showed that the energy consumption is always increases progressively with the firing zone length. Therefore, up to approximately 30% of the firing zone, the increase in energy is relatively low and as a consequence, the firing zone length should be as short as possible for low energy consumption. The MATLAB program was used especially the bvp4c code to solve all the ordinary differential equations (ODEs) systems to obtain the temperature profiles for all cases.

KEYWORDS: Tunnel kilns; Ceramics products; Mathematical model; Energy consumption; Fuel distribution; Preheating zone; Firing zone.

I. INTRODUCTION
Nowadays the whole world concerns on energy saving due to energy crisis. Energy consumption during the production process of any products’ effects on the price of the final products. So that, there is a great concern on kilns specially tunnel kiln design. Obtaining accurate numerical information about tunnel kiln allows the development of the kiln. Tunnel kiln is one of the basic continuously operated kilns in the firing process. It is a long structure in which only the central portion is directly heated. The ware enters the kiln and is slowly transported through it, and then the temperature of ware is increased steadily as it approaches the central of the kiln which considered the highest temperature part of the kiln. The temperature is reduced until the products exit from the kiln near room temperature. Therefore, developing of specific codes and models to simulate the heat transfer and fuel distribution involved in the tunnel kiln is important
A laboratory model of a section of a tunnel kiln has been built to investigate a correlation to calculate convective heat transfer rates in the firing of refractory by Dugwell and Oakley [1]. A 1/10th scale model in which chrome-magnetite blocks have been used to represent the blades of ware set on cars in the kiln. Karaush et al. [2] established an experimental study which involves a model unit conducted on heat absorption from the radiating walls of a kiln by the ceramic ware set up on a kiln car. Mathematical and numerical models have been done by former researchers [3-10]. Regarding the mathematical models, Dugwell and Oakley [3] made a mathematical model for a firing refractory ware in gas-fired tunnel kilns which can be used to predict gas temperature and composition and ware temperature profiles during firing.
A one dimensional model to simulate the drying bricks in the preheating zone of a tunnel kiln was developed by Mancuhan et al. [4]. Furthermore, two different profiles and vent locations for the ambient air into the preheating zone were used to achieve the desired gas temperature for high quality bricks. A mathematical model that could be used to maintain the process parameters in real-time as a function of the amount of brick manufactured was examined by Gol'tsova et al. [5]. In the model, they divided the entire kiln into 48 zones equal to process zones. In addition, they assumed that the temperature in each zone was constant, i.e., was not a function of time. The results showed a great degree excessive power consumption. An approach to determine the temperature fields in a tunnel kiln for brick production together with different possibilities of decreasing specific fuel consumption were investigated by Durakovic and Delalic [6]. They developed software to execute a simulation of temperature distribution in the furnace during brick production process in real conditions.

Regarding the numerical simulations, the convective heat transfer under superimposed longitudinal and cross flow conditions in a fast firing tunnel kiln for glost firing of porcelain was investigated by Becker et al. [7]. They used a commercial CFD-code for the simulation. In addition, a 1:5 cold model of a real existing tunnel kiln for firing flat tableware was used in order to determine the convective heat transfer coefficient by similarity examinations Numerical simulation of the thermal process inside a ceramic Frits melting kiln with CFD and a global FORTRAN code were developed by Possamai et al. [8]. The used domain in the analysis was consisted of a rectangular refractory kiln. The comparison of the CFD results with experimental data proved that the mathematical models used in the numerical simulation were consistent with the real phenomena.

An optimization of the fuel and air used around a tunnel kiln producing bricks with a low calorific value (LCV) coal as an additive was presented by Mancuhan and Kucukada [9]. They concluded that the use of admixed coal for both cases which using pulverized coal or natural gas in firing zone is considered an advantage. They recommended the high calorific value (HCV) of admixed coal when the plant uses natural gas as the fuel supplied to the firing zone. An optimization of the firing zone to minimize the fuel cost as being the objective function was made by Kaya et al. [10]. Furthermore, a mathematical model representing in the simplest form the phenomena of heat transfer, combustion of admixed coal and pulverized coal, together with gas flow was used. The optimization was realized by considering the tunnel kiln to be composed of a number of increments. The optimized value of the PC was found to be lower than the existing tunnel kiln.

A mathematical model to analyze the heat transfer in tunnel kilns for burning of ceramics was done by Hassanein A. Refaey [11]. Recently, flue field visualization in the Sanitaryware kiln has been done by Hassanein Refaey and Specht [12]. A calculation methodology, based on certain kiln operating parameters, for quantifying the energy saving obtained in the tile kiln when part of the cooling gases were recovered in the firing chamber and do not exhausted into the atmosphere have been studied by Ana Mezquita et al. [13]. An energy savings up to 17% have been reported in the studied case. A comparison between the theoretical results and the experimental data have been done to validity the proposed methodology. From the study there is a need to improve combustion process control. A thermal analysis with three dimensional numerical model for a roof tiles kiln, fueled by firewood and shale oil has been studied by R. Oba et al. [14]. A comparison between an experimental data and numerical results have been done and a good agreement was observed.

The models achieved by former searches are only marginal and not clear enough. Therefore, this paper describes the simulation of thermal process in tunnel kilns and the effect of fuel distribution along the firing zone. Furthermore, from the previous studies, there is less research on general simplified model for all tunnel kilns. Therefore, in the present study a simple 1D mathematical model is representative to understand the main process in these kilns regardless type and shape of products. The following sections describe the model, energy balance, obtaining the ordinary differential equations for each case, results, discussion, conclusions and finally outlook for future work. The schematic description of the tunnel kiln and its mathematical model are now briefly described in the following sections.

### II. Model Description

The heat transfer mechanism is different from one kiln to another. The complexity of the mechanism depends on the type of products and its interaction with kiln car and kiln furniture. Therefore, it is important to establish a simplified model to understand the principal behavior of the kiln process and
to analyze the heat transfer. The tunnel kiln basically consists of three zones; preheating, firing and cooling. In the present paper the mathematical model was done for the preheating and firing zone. Therefore, the model describes three basic cases; case (A) with zero firing length, case (B) with zero preheating length and case (C) with preheating and firing zones with equal length. In addition, the effect of firing length on the temperature profiles was being studied. Moreover, the effect of different fuel additions on the temperature profiles was also studied.

The schematic diagram of the tunnel kiln with gas distribution along the firing zone length is shown in figure 1. From the shown figure, the combustion of fuel with air has occurred in an adiabatic combustion chamber. Therefore, the outlet gas temperature of this combustion process is the adiabatic flame temperature. As illustrated in the figure, the kiln car carries the product through the kiln in counter direction to gas flow. Therewith, the kiln process is reduced to a simple counter current heat exchanger.

2.1 Energy balance

The tunnel kiln has a complex heat transfer mechanism because of the interaction of kiln car, kiln furniture, product types and product arrangements in the kiln. The present model represents only two zones; preheating and burning zone. In the model the solid materials (product, furniture and car) are heated up to the required sintering temperature. Actually, in the majority of tunnel kilns, the cooling air is sucked away from the kiln and then used in the dryer. It is assumed that the enthalpy of the cooling can covers the energy for the drying process. As a consequence, the energy of the fossil fuel of the kiln is the required energy for the total process. This is the case for the most processes of ceramics burning. Therefore, the following assumptions have to be made to the present model.

Assumptions:
- The process is assumed to be a steady state process.
- The temperature of the product and of gas is assumed to be constant at any cross-section. As a consequence, the temperature depends only on the length.
- The temperature of products and transportation materials are the same.
- The heat transfer coefficient is constant.
- The heat losses through the walls are neglected.
- The material properties (specific heat capacity, density) are assumed as constant.

![Fig.1: Schematic description of tunnel kiln with heating and firing zones.](image)

Basic of all kiln processes is an energy balance. The enthalpies in heat balances are always referred to the reference temperature (0°C). In energy balance analysis, the energy inserted by fuel is equal to the
heat gain by solid, heat removed from gas and the heat losses through walls. The following equation is obtained with heat losses \( Q_w = 0 \) by applying the energy balance [11].

\[
M_s c_s (T_{s,f} - T_{s,in}) + M_F c_{pg} (T_{g,out} - T_a) + Q_w = M_F h_u
\]

Herein, \( M_s \) is the mass flow rate of solid (product \( M_{pro} \)) and transportation (\( M_T \)). And it could be expressed as in the following equation:

\[
M_s = M_{pro} \left( 1 + X_T \frac{c_{pro}}{c_s} \right)
\]

Where, the ratio between the product materials to the transportation material is defined as

\[
X_T = \frac{M_T c_T}{M_{pro} c_{pro}}
\]

Energy consumption is an important part in the production process because ceramic production is an intense energy, so that it is necessary to know the amount of energy consumption due to emissions in the flue gas and then for the whole production process. The energy consumption controls the final price of the product. All sectors which used to produce ceramic in the industry are energies intensive, as a key part of the process involves drying followed by firing to temperatures of between 800 and 2000°C. For example, the manufacture of porcelain has \( CO_2 \) emissions consumes energy between less than 10 to 18% of the total cost. Another example, the manufacture of bricks, the share of the energy costs varies between 17% and 25% with maximum levels of up to 30%. Table 1 shows the specific energy consumption in the ceramics industry in Europe [15].

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>Brick and roof tiles</td>
<td>GJ/t</td>
<td>2.65</td>
<td>2.45</td>
<td>2.19</td>
<td>2.06</td>
<td>2.38</td>
<td>2.31</td>
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<td>Wall and roof tiles</td>
<td>GJ/t</td>
<td>11.78</td>
<td>9.16</td>
<td>6.76</td>
<td>5.45</td>
<td>5.74</td>
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<td>Refractory products</td>
<td>GJ/t</td>
<td>4.88</td>
<td>4.96</td>
<td>6.51</td>
<td>4.91</td>
<td>5.41</td>
<td>5.57</td>
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<tr>
<td>Sanitaryware</td>
<td>GJ/t</td>
<td>26.56</td>
<td>24.21</td>
<td>22.27</td>
<td>22.76</td>
<td>20.88</td>
<td>21.87</td>
</tr>
<tr>
<td>Vitrified clay pipes</td>
<td>GJ/t</td>
<td>5.75</td>
<td>5.77</td>
<td>6.10</td>
<td>5.23</td>
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<tr>
<td>Table- and ornamental</td>
<td>GJ/t</td>
<td>47.56</td>
<td>38.91</td>
<td>43.46</td>
<td>45.18</td>
<td></td>
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<td>Technical ceramics</td>
<td>GJ/t</td>
<td>34.72</td>
<td>50.39</td>
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</table>

The specific energy consumption for the tunnel kiln is calculated from the following equation with neglecting the heat losses.

\[
E_s = \frac{M_F h_u}{M_s} = \frac{c_s (T_{s,f} - T_{s,in})}{1 - \frac{c_{pg}(1-\lambda L)(T_{g,out} - T_a)}{h_u}}
\]

This specific energy consumption, \( E_s \), cannot be calculated, because \( T_{g,out} \) is not known. The outlet temperature depends on area and heat transfer coefficient. But, in industry the specific energy consumption is referred to the product flow as in the following equation:

\[
E_{pro} = \frac{M_F h_u}{M_{pro}}
\]

By joining Eq. 2, Eq. 4 together with Eq. 5 the specific energy consumption per kg of products can be presented as

\[
E_{pro} = E_s \left( 1 + X_T \frac{c_{pro}}{c_s} \right)
\]

From the previous equation it is clearly obvious that, the energy consumption related to the product is therewith always higher than the energy consumption related to the solid.

### III. TEMPERATURE PROFILES

In the present study, the temperature profile of the solid and the gas is calculated by using an energy balance for a differential kiln length \( dz \). The basic analysis has been done for simple case A. The general enthalpy change for both gas and solid and the convective heat transfer between the gas and solid are represents as follow:

\[
d\dot{H} = M c_p dT = dQ_{conv}
\]

\[
dQ_{conv} = \alpha (T_g - T_s) dA
\]
Where,
\[ dA = \frac{A_t}{L_k} \, dz \]  
(9)

The analysis has been done for the kiln to obtain the equations. The following two first order ordinary differential equations (ODEs) for both gas and solid are obtained with these two boundary conditions \( T_{g(z=0)} = T_{s,in} \) and \( T_{s(z=L_k)} = T_{s,f} \).

\[ \dot{M}_g \, c_{pg} \, dT_g = \frac{\alpha A_t}{L_k} \left( T_g - T_s \right) dz \]  
(10)

\[ \dot{M}_s \, c_s \, dT_s = \frac{\alpha A_t}{L_k} \left( T_g - T_s \right) dz \]  
(11)

Where, from the basic knowledge of combustion the gas mass flow rate is specified as a function of fuel mass flow rate, air demand (L) and excess air number (\( \lambda \)); \( \dot{M}_g = \dot{M}_f \left( 1 + \lambda L \right) \).

Furthermore, the analytical solution is obtained for the foregoing (ODEs) for different values of the specific heat capacity ratio (i.e. \( \Omega \neq 1 \)) and the two equations are:

\[ T_g(Z) = \frac{\Omega' \, T_{s,in}(1 - e^{St_s(\Omega' - 1)Z}) + T_{g,out}(\Omega' e^{St_s(\Omega' - 1)Z - 1})}{\Omega' - 1} \]  
(12)

\[ T_s(Z) = \frac{T_{s,in}(\Omega' - e^{St_s(\Omega' - 1)Z}) + T_{g,out}(e^{St_s(\Omega' - 1)Z - 1})}{\Omega' - 1} \]  
(13)

Wherein, \( Z = \frac{z}{L_k} \), \( \Omega' = \frac{1}{\Omega} \),

\[ T_{g,out} = \frac{T_{s,f}(\Omega' - 1) - T_{s,in}(\Omega' - e^{St_s(\Omega' - 1)})}{(e^{St_s(\Omega' - 1)} - 1)} \]  
(14)

And the Stanton number for solid (\( St_s \)) and the specific heat capacity ratio (\( \Omega \)) are defined as:

\[ St_s = \frac{\alpha A_t}{\dot{M}_s \, c_s} \quad \text{and} \quad \Omega = \frac{\dot{M}_g \, c_{pg}}{\dot{M}_s \, c_s} \]  
(15)

For a special case in which the heat capacity is equal to unity (\( \Omega = 1 \), \( \dot{M}_s \, c_s = \dot{M}_g \, c_{pg} \)) the temperature profiles for solid and gas are linear and parallel to each other. Regarding the other cases (B and C), the difference between them and simple case A, is the gas ODE. Therefore, to obtain this equation from the basic analysis the following assumption is added.

- The fuel is distributed uniformly along the firing zone of length \( L_f \).

From figure 1 it is seen that there is a homogeneous inlet of combustion gas along the kiln and the axial distribution of gas flow is given as:

\[ \dot{M}_g(z) = \frac{\dot{M}_{g,max}}{L_f} (L_k - z) \]  
(16)

Furthermore, the gas enthalpy equation will be different rather than before and it is expressed as follow:

\[ \dot{H}_g = \dot{M}_g(z) c_{pg} \, T_g(z) \]  
(17)

After the analysis the ODE for gas is represented as:

\[ \dot{M}_g(z) \frac{dT_g}{dz} = \frac{\alpha A_t}{L_k c_{pg}} \left( T_g - T_s \right) - \frac{\dot{M}_{g,max}}{L_f} \left( T_{ad} - T_g \right) \]  
(18)

Where, \( T_{g,in} = T_{ad} \), and \( T_{ad} \) is the adiabatic flame temperature which is calculated from the following equation:

\[ T_{ad} = \frac{hu}{c_{pg} (1 - \lambda L)} + T_a \]  
(19)

IV. SIMULATION PROCEDURE

In order to obtain the results for each case (A, B and C) the following steps were proceeding:
- The physical data and boundary conditions were inserted in the commercial code (pvb4c) program.
- The bvp4c solver requires two boundary conditions for the solution in which the solid inlet and outlet temperatures are known (inlet temperature of solid $T_{s,in}$, and outlet solid temperature $T_{s,f}$).
- For cases A the two equations 10 and 11 are solved to obtain the results.
- Regarding cases B and C the two equations 11 and 18 are implemented in the program to obtain results for those two cases.
- Furthermore, from the program results the temperature profiles for the gas and solid along the two zones (preheating and firing) are obtained. Some variables used in solving this system are listed in table 2.

V. RESULTS AND DISCUSSIONS

The solution of the system of ODEs is obtained for the three cases (A, B and C). The equations are solved by MATLAB program. A lot of results have been obtained from the model. Herein some of the results are shown for the three cases and the influence of fuel distribution along the firing zone. Table 2 represents some of the used data in the model calculations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{used}$</td>
<td>fuel type</td>
<td>natural gas H type</td>
<td></td>
</tr>
<tr>
<td>$L_k$</td>
<td>kiln length</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>$B_k$</td>
<td>kiln width</td>
<td>6</td>
<td>m</td>
</tr>
<tr>
<td>$H_k$</td>
<td>kiln height</td>
<td>3.5</td>
<td>m</td>
</tr>
<tr>
<td>$c_s$</td>
<td>specific heat of solid</td>
<td>0.85</td>
<td>kJ/kg.K</td>
</tr>
<tr>
<td>$M_k$</td>
<td>capacity of kiln</td>
<td>20000</td>
<td>t/year</td>
</tr>
<tr>
<td>$T_{s,in}$</td>
<td>solid inlet temperature</td>
<td>50</td>
<td>ºC</td>
</tr>
<tr>
<td>$T_{s,f}$</td>
<td>solid outlet temperature</td>
<td>1000</td>
<td>ºC</td>
</tr>
<tr>
<td>$L$</td>
<td>air demand</td>
<td>16.9</td>
<td>kg air/kg fuel</td>
</tr>
<tr>
<td>$h_u$</td>
<td>lower heating value</td>
<td>47300</td>
<td>kJ/kg fuel</td>
</tr>
</tbody>
</table>

4.1 Simple Case

The output results for the first case (A) are represented in Fig. 2. The figure shows the energy consumption as the function in a Stanton number for different excess air numbers. It is obvious that for constant excess air number, the energy consumption decreases as Stanton number increases. The figure also shows that the trend is the same for any excess air number. Moreover, the Stanton number increases if the heat transfer coefficient ($\alpha$) increases. The figure illustrates that for $E_s > 1.15$ a little increase in Stanton number results in a big decrease of energy consumption for same excess air number. Furthermore, the figure shows also that as excess air number increase the energy consumption increase. Moreover, the specific heat capacity ratio that is equal to one could be found.
The influence of excess air number on both temperature distribution and outlet gas temperature is shown in figure 3. The figure demonstrates that as the excess air number decreases the outlet gas temperature decreases, this is due to the decrease in energy consumption. The figure shows that at \( \lambda = 1.3 \) the flue gases have a lower temperature, which is required to prevent thermal pollution. Moreover, from these results the outlet gas temperature could be controlled and regulated according to the environmental requirements. In some cases the outlet flue gases are used in the drying process and its temperature should be a little bit higher.

4.2 Effect of firing zone length
The influence of the firing length ratio on the energy consumption for different excess air numbers and Stanton numbers is shown in figure 4. The figure illustrates that the energy consumption is always increasing with the firing zone length. Moreover, the figure demonstrates that by increasing the excess air number the energy consumption is increasing progressively with the firing zone length. Therefore, up to approximately 30%, the increase is relatively low. As a consequence, the firing zone length should be as short as possible for low energy consumption.

![Fig. 4: Influence of firing zone length on energy consumption.](image)

The influence of excess air number on the temperature profile for a constant Stanton number is shown in Fig. 5. The figure demonstrates a weak influence on the solid temperature. The increase of the solid temperature is approximately linear. The figure shows that for a higher excess air number the energy consumption has a higher value which resulting in a higher flue gas temperature.

![Fig. 5: Temperature profile, case C, with firing zone length of 25 m, Sts=3.](image)
A comparison of the temperature profiles for the three cases is represented in Fig. 6. The figure shows sample of the results for $St_s = 2$ and $\lambda = 1.7$ as a comparison between the three cases. From the figure it can be seen that, for the long firing zone (case B) the increase of the solid temperature is being stronger at the beginning. Furthermore, the long firing zone results in higher outlet temperature. Regarding case C, the slope of the temperature profile for gas is changing from firing zone to preheating zone. For case "A" a highly gas inlet temperature has occurred. This high gas temperature results in high NOx emissions and high quality and therewith expensive refractory materials. So that, an extreme short firing zone results in too high, gas temperatures and in a used proper profile of the solid temperature.

![Comparison between cases A, B and C with same outlet solid temperature for Sts=3.4.3 Effect of fuel distribution](image)

**Fig. 6:** Comparison between cases A, B and C with same outlet solid temperature for Sts=3.4.3 Effect of fuel distribution

Figure 7 shows different fuel distribution curves along the firing zone length. The figure shows four different fuel distribution curves; the first curve describes a constant fuel addition along the firing zone, the second curve divide the firing zone to two equal zones and with different constant amount along each division, the third curve describes another fuel addition curve with a linear descending fuel amount along the firing zone and finally the fourth curve describe a linear descending fuel addition along the firing zone with a zero fuel added at the end of the firing zone. The temperature profiles for the four different fuel distribution curves for $St_s = 2$ and $\lambda = 1.3$ as an example are shown in Fig. 8. From the figure it could be seen that, the fuel distribution along the firing zone length effects on the temperature profiles for both gas and solid. Furthermore, the energy consumption is also affected by the fuel distribution. Therefore, the thermal heating of products could be controlled by changing the fuel distribution along the firing zone. This thermal heating depends on the type of product and its final required specifications.
VI. CONCLUSION

A mathematical model for simulating the process in the tunnel kiln is developed in the present work. The model describes the way to obtain the ordinary differential equations for solids and gas in the preheating and firing zones. The developed mathematical model has been used to predict temperature profiles of different operating conditions (excess air number and Stanton number) and the energy consumption of the kiln. In addition, the model describes the effect of gas distribution along the firing zone on both energy consumption and temperature profiles.

Based on the model results, the following conclusions are drawn:

- The energy consumption is decreasing and more energy is saved during the production process by increasing the Stanton number. Moreover, the excess air number has a big influence on the heat transfer inside the kiln. But, the higher excess air number results in a higher energy
consumption and thereby higher flue gas temperature. Therefore, the desired application will control the optimum value of the excess air number.

- The firing zone length has a significant effect on the energy consumption and it should be as short as possible for low energy consumption. Up to approximately 30%, the increase is relatively low. As a consequence, the firing zone length should be as short as possible for low energy consumption.

- The gas distribution curve effects on the temperature profiles of both gas and solid. As a consequence, it is easy to control the thermal heating of the products inside the kiln. Therefore, it can be concluded that at lower excess air number with higher Stanton number, a lower flue gas temperature is obtained which, is important in chimney design.

VII. OUTLOOK FOR FUTURE WORK

In the present study basic simplified equations to simulate the process in tunnel kilns have been introduced. However, there are several areas where further analysis is required including:

- Experimental measurements should be carried out to verify the models (difficult in the present time).
- Including the radiation to the model especially in the firing zone.
- 3D simulation with the ANSYS program have to be done to part of the kiln.
- Studying the flow circulation in the cross section of the kiln.

REFERENCES


<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>A</td>
<td>total surface area of solid, m²</td>
</tr>
<tr>
<td>A_t</td>
<td>specific heat capacity of gas, kJ/(kg.K)</td>
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<td>c_pg</td>
<td>specific heat capacity of products, kJ/(kg.K)</td>
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<tr>
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<td>specific heat capacity of solid, kJ/(kg.K)</td>
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<tr>
<td>c_T</td>
<td>specific heat capacity of transportation, kJ/(kg.K)</td>
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<td>E_e</td>
<td>specific energy consumption, MJ/kg</td>
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<td>L_pre</td>
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<td>T_s,f</td>
<td>outlet solid temperature, K</td>
</tr>
<tr>
<td>T_s,in</td>
<td>inlet solid temperature, K</td>
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<tr>
<td>T_s,in</td>
<td>inlet solid temperature, K</td>
</tr>
</tbody>
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Subscript
- a: air
- conv: convection
- F: fuel
- g: gas
- prod: products
- s: inlet solid temperature, K
- T: transportation
- w: wall
- α: heat transfer coefficient, w/(m².K)
- λ: excess air number
- Ω: Specific heat capacity ratio
- bvp4c: Boundary value problem
- ODE: ordinary differential equation

**AUTHORS BIOGRAPHIES**

**Hassanein Refaey** was born in Kalyobia, Egypt, in 1978. He received the Bachelor degree in 2000 and the Master degree in 2006, both in Mechanical Power engineering from the University of Benha, Benha, Egypt. He is received the Ph.D. degree in Institute of Fluid Dynamics and Thermodynamics (ISUT) department from the University Otto-von Guericke, Magdeburg, Germany, in 2013. His research interests include heat transfer, heat exchangers, fluid dynamics, simulations, renewable energy and combustion.

**Mohamed Reda Salem** was born in Kalyobia, Egypt, in 1982. He received the Bachelor degree in Applied Science (B.A.Sc.), in 2004 and the Master degree in 2011. He is received the Ph.D. degree in 2014 all in mechanical engineering from University of Benha, Benha, Egypt. His research interests include heat transfer, heat exchangers, renewable energy and air conditioning.
Eckehard Specht studied Chemical Engineering and was then Scientific Research Fellow at the Institute for Energy Engineering at the TU Clausthal (1977-1993). After his Ph.D. in (1984) and Habilitation in (1993), he is professor from October 1, 1993 for thermodynamics and combustion at the Otto von Guericke University Magdeburg, Institute of Fluid Dynamics and Thermodynamics. His specializations are Combustion technology, heat and mass transfer, chemical process engineering, and ceramic materials.