Map Projection (SUR 314)
Lecture No: 4

MATHEMATICAL CYLINDRICAL PROJECTIONS

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Overview of Previous Lecture

- **Expected Learning Outcomes**
- **Cylindrical Map Projection**
- **Properties of Cylindrical Map Projections**
- **Cylindrical Map Projection - Shape of Graticules**
- **Cylindrical Map Projection – Distortion**
- **Conical Map Projection**
- **Properties of Conical Map Projections**
- **Conical Map Projection - Shape of Graticules**
- **Conical Map Projection – Distortion**

Overview of Today's Lecture

- Expected Learning Outcomes
- Mathematical Map Projection
- Global Properties of Mathematical Projections
- Mathematical Cylindrical Projection
- Types of Mathematical Cylindrical Projection
- Conical Map Projection
- Mercator Projection
- Loxodrome and Navigation
- Summary

Expected Learning Outcomes

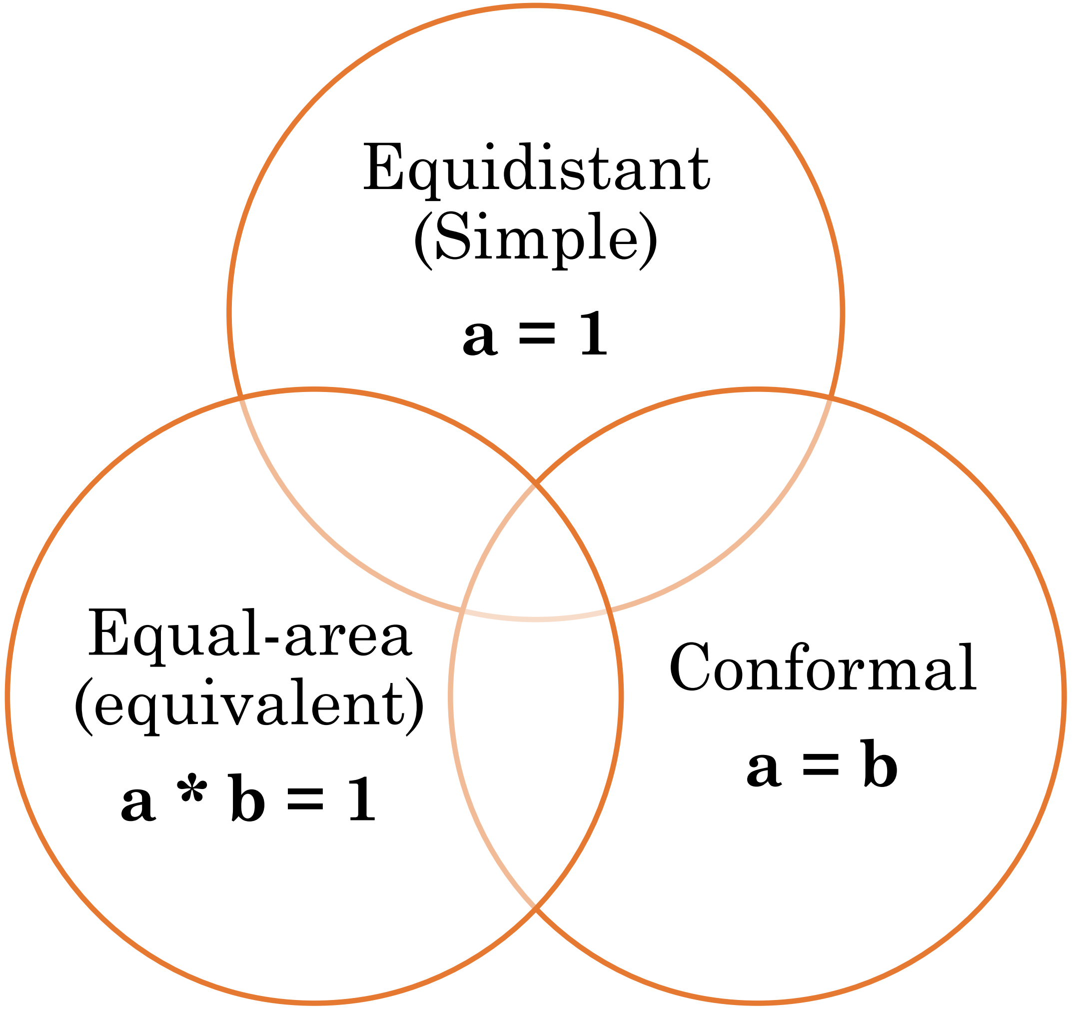
1. Understanding the concept of mathematical map projection.
2. Exploring the global properties of mathematical projections.
3. Explain the construction and characteristics of cylindrical projections.
4. Identify and describe specific examples of cylindrical projections commonly used in cartography.
5. Recognize and compare various types of cylindrical projections, such as the Mercator, Miller, Lambert, and Behrmann projections.
6. Define loxodromes (rhumb lines) and their properties.
7. Understand the use of loxodromes in navigation.
8. Explain the relationship between loxodromes and the Mercator projection.

Mathematical Map Projection

Mathematical Map Projection

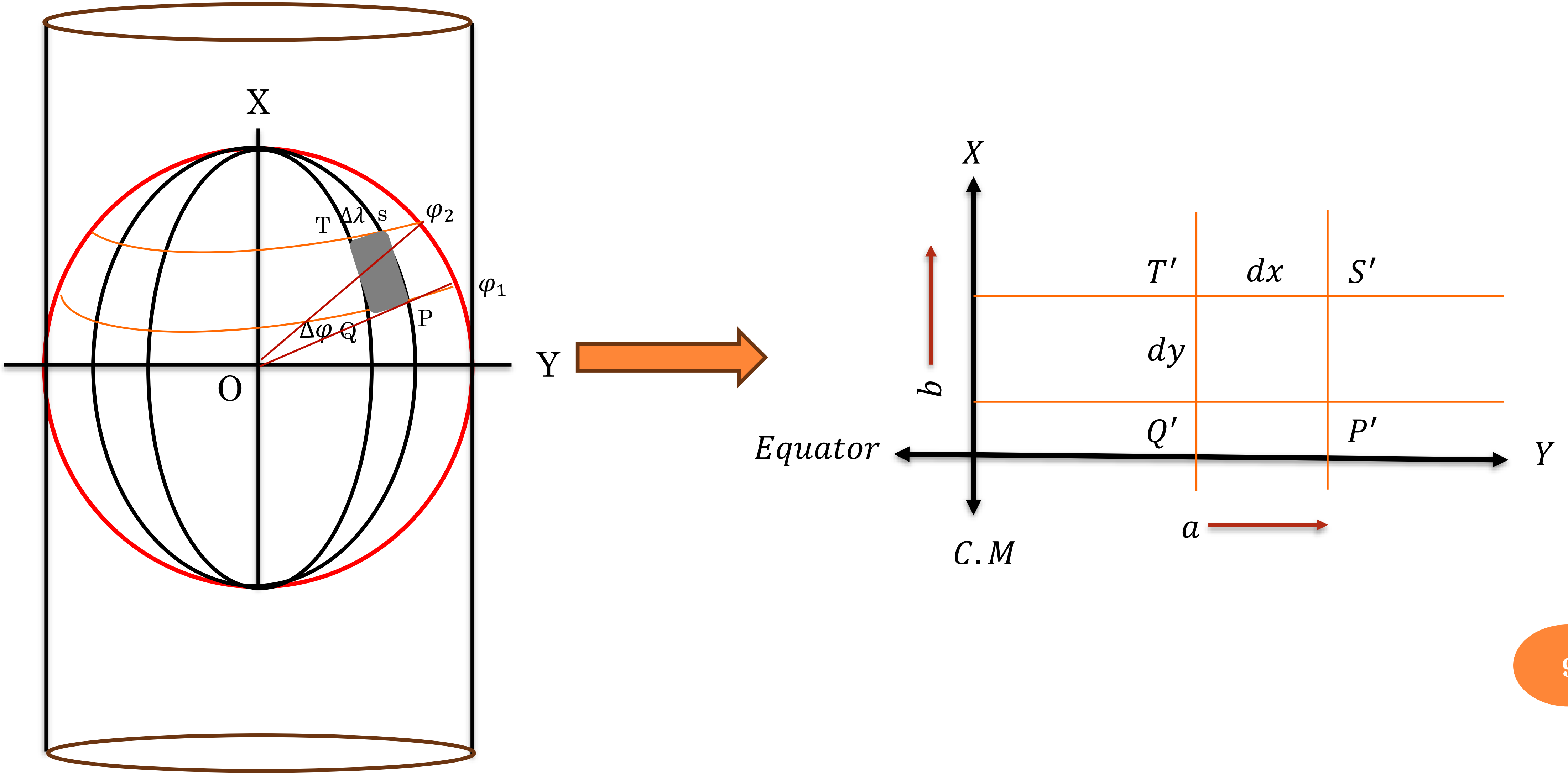
- Contrary to perspective map projections, mathematical map projection transfers the earth's surface to a map based on a pre-defined property such as conformality, equivalence, or equidistance.
- There are no radiations or perspective center (***non-perspective***).
- **Any mathematical projection is a modification of its counterpart perspective projection based on the developable surface.**
- Many mathematical projections are not related as easily to a plane, cylinder, or cone that may be placed under mathematical group of projections.
- These are altered versions of modified projections termed as pseudocylindrical, pseudoconic, and pseudoazimuthal.
- Pseudocylindrical projections are perhaps the most common type in the mathematical projections, with their meridians that curve toward the poles.
- To reduce the distortion, the necessary modifications are suggested for generating mathematical projections by including additional standard lines or changing the distortion pattern.

Mathematical Projection – Three Properties



Mathematical Cylindrical Map Projection

Mathematical Cylindrical Projection



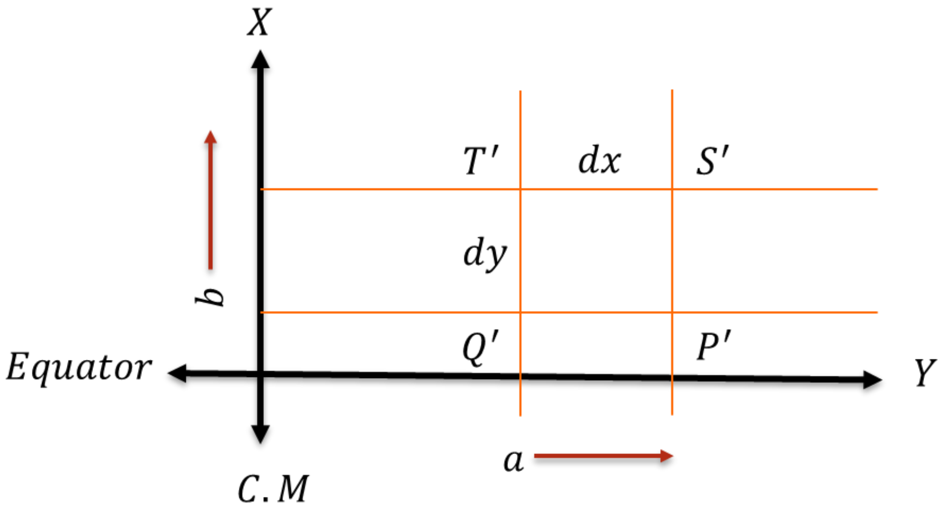
Mathematical Cylindrical Projection

- Assuming PQ is a parallel arc and its length on the earth's surface is given by: $R.\cos\varphi\,.(\Delta\lambda)^r$.
- Similarly, QT is a meridian arc with a length: $R.(\Delta\varphi)^r = R(\varphi_2 - \varphi_1)^r$.
- The distortion in parallel direction is given by:

$$a = \frac{P'Q'}{PQ} = \frac{dy}{R.\cos\varphi.(\Delta\lambda)^r} = \sec\varphi$$

- The distortion in meridian direction is given by:

$$b = \frac{Q'T'}{QT} = \frac{dx}{\partial(R\times\Delta\varphi)}.$$



MATHEMATICAL CYLINDRICAL PROJECTION

- **(1) Normal Simple Cylindrical Projection**

In this type, it is assumed that no distortion in meridian direction, ***b = 1***, therefore, the map coordinates are derived as follows:

$b = 1$, so

$$\frac{dx}{\partial(R\times \Delta\varphi)} = 1,$$

$dx = R\, d\varphi$

$X = R.\Delta\varphi$ ________________________________ (1)

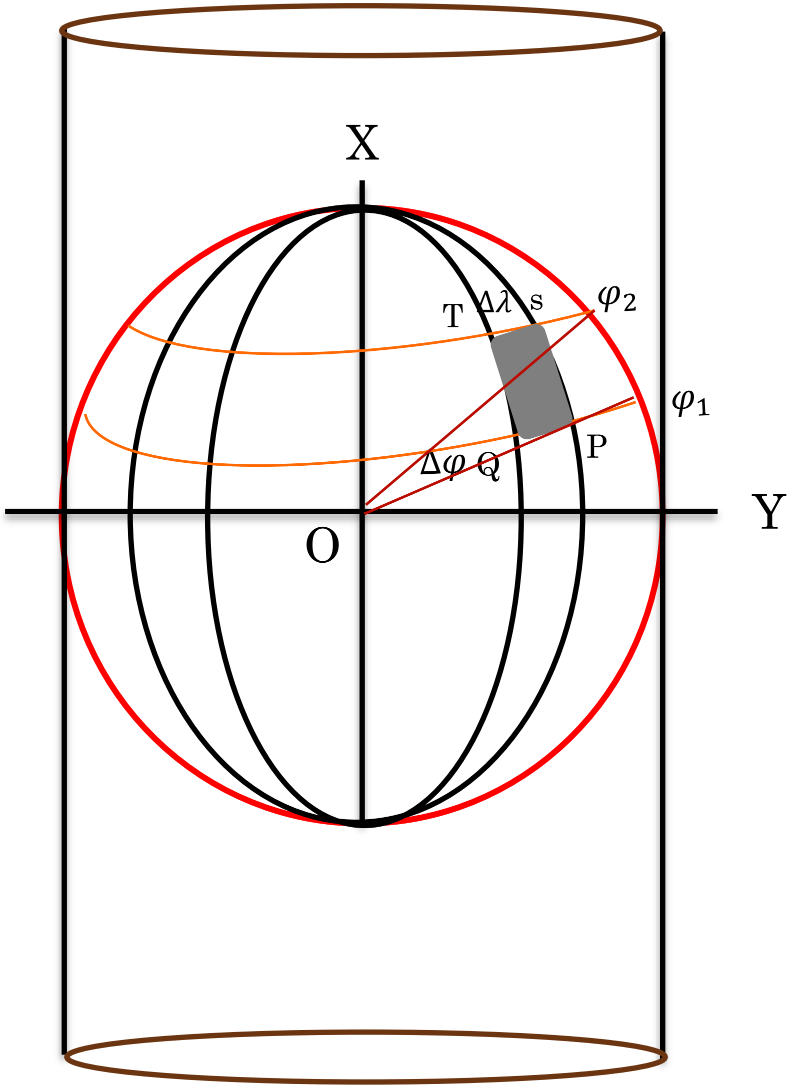
Similarly, $a = \sec\varphi$, so

$$\frac{dy}{R.\cos\varphi.(\Delta\lambda)^r} = \sec\varphi,$$

Then,

$\int dy = R \int d\lambda,$

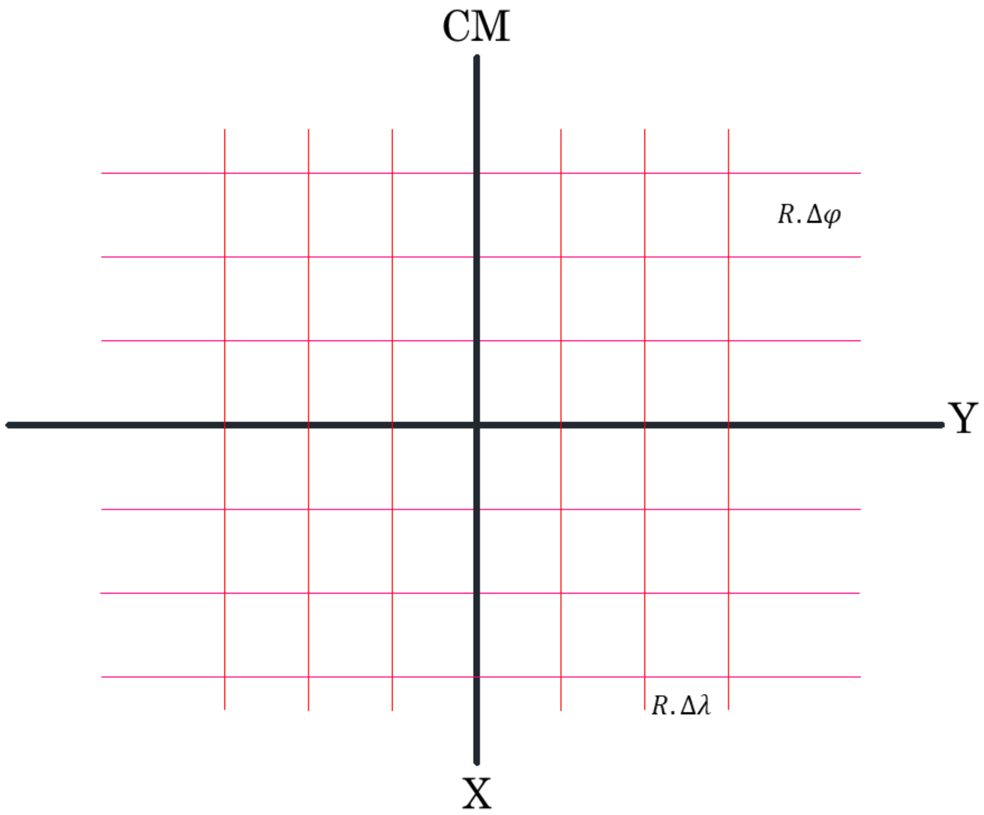
$Y = R.\Delta\lambda$ ________________________________ (2)



Mathematical Cylindrical Projection

- **(1) Normal Simple Cylindrical Projection**

- The graticules are equal squares in the X-Y directions with a separation of $R.\Delta\varphi = R.\Delta\lambda$.
- Consequently, this projection is also called "**chessboard**" or "**plate carree**"



Mathematical Cylindrical Projection

(2) Normal Equal-Area Cylindrical Projection

In this type, it is assumed that no distortion in areas, **a * *b* = *1***, therefore, the map coordinates are derived as follows:

$$a = \frac{dy}{R.\cos\varphi.(\Delta\lambda)^r} = \sec\varphi$$

$$b = \frac{dx}{\partial(R\times\Delta\varphi)}$$

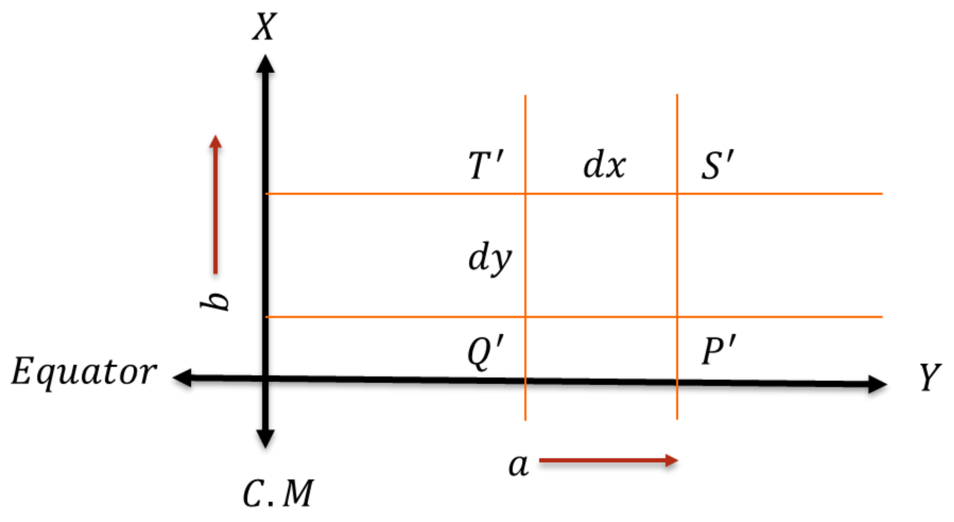
$$\frac{dx}{\partial(R\times\Delta\varphi)} \times \sec\varphi = 1$$

$$\int dx = R\int \cos\varphi \; d\varphi$$

$$X = R.\sin\varphi \qquad (3)$$

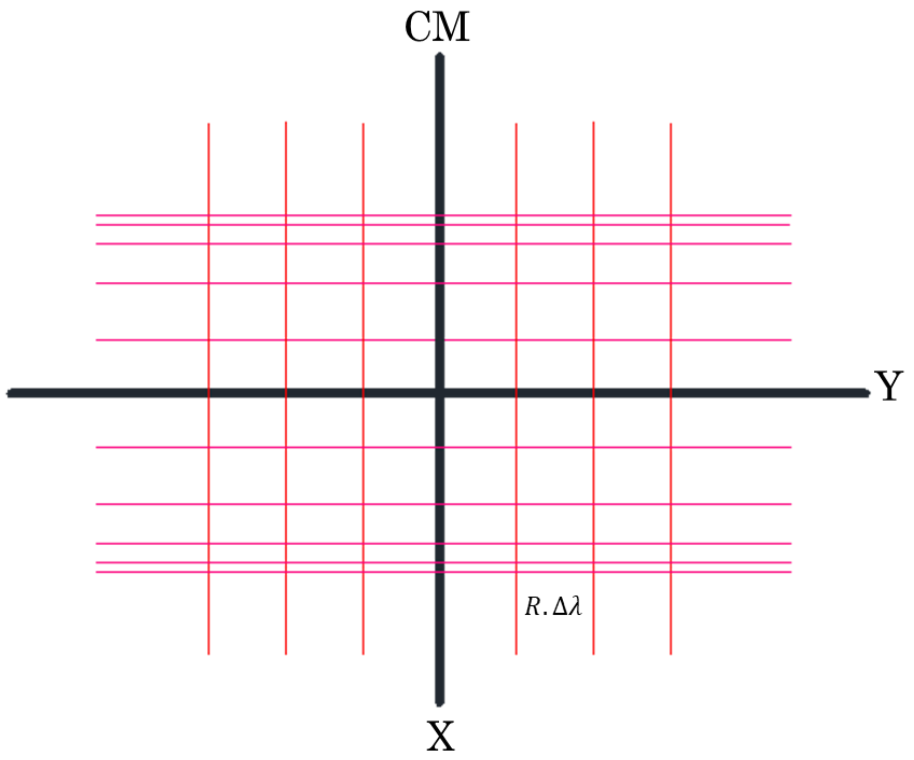
Also,

$$Y = R.\Delta\lambda^r \qquad (4)$$



MATHEMATICAL CYLINDRICAL PROJECTION

- **(2) Normal Equal-Area Cylindrical Projection**



- The graticules are rectangular in shape.
- Distances between parallels decrease apart from equator (to the pole) depending on φ.
- Meridians are straight lines which are equally-spaced and perpendicular to equator (no change).

Mathematical Cylindrical Projection

- **(3) Normal Conformal Cylindrical Projection**

- It is the most important projection because it preserves shapes therefore, it is commonly used for navigation purposes and surveying maps.

In this type, **a** = ***b*** , therefore, the map coordinates are derived as follows:

$$a = \frac{dy}{R.\cos\varphi.(\Delta\lambda)^r} = \sec\varphi$$

$$b = \frac{dx}{\partial(R\times\Delta\varphi)}$$

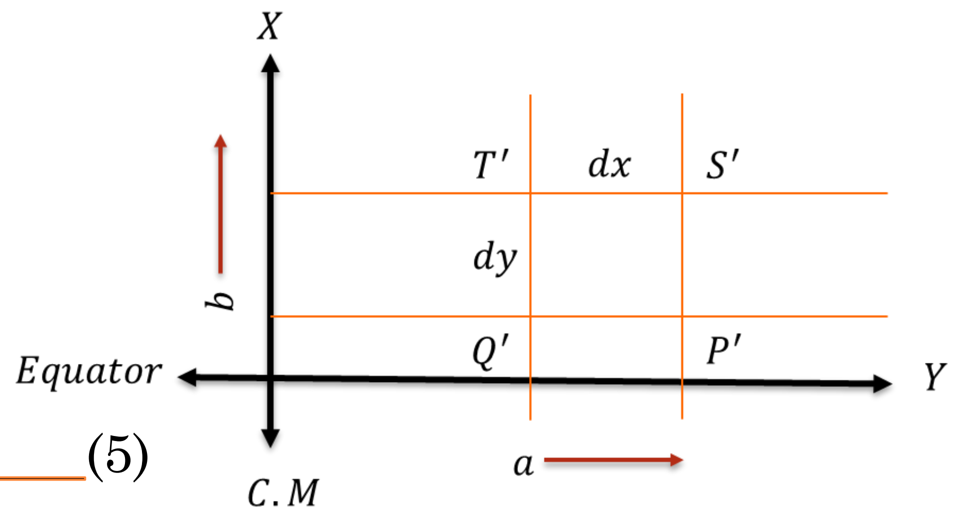
$$\frac{dx}{\partial(R\times\Delta\varphi)} = \sec\varphi$$

$$\int_0^x dx = R\int_0^\varphi \frac{1}{\cos\varphi}\, d\varphi$$

$$X = R.\,ln\left[\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right] \qquad (5)$$

Also,

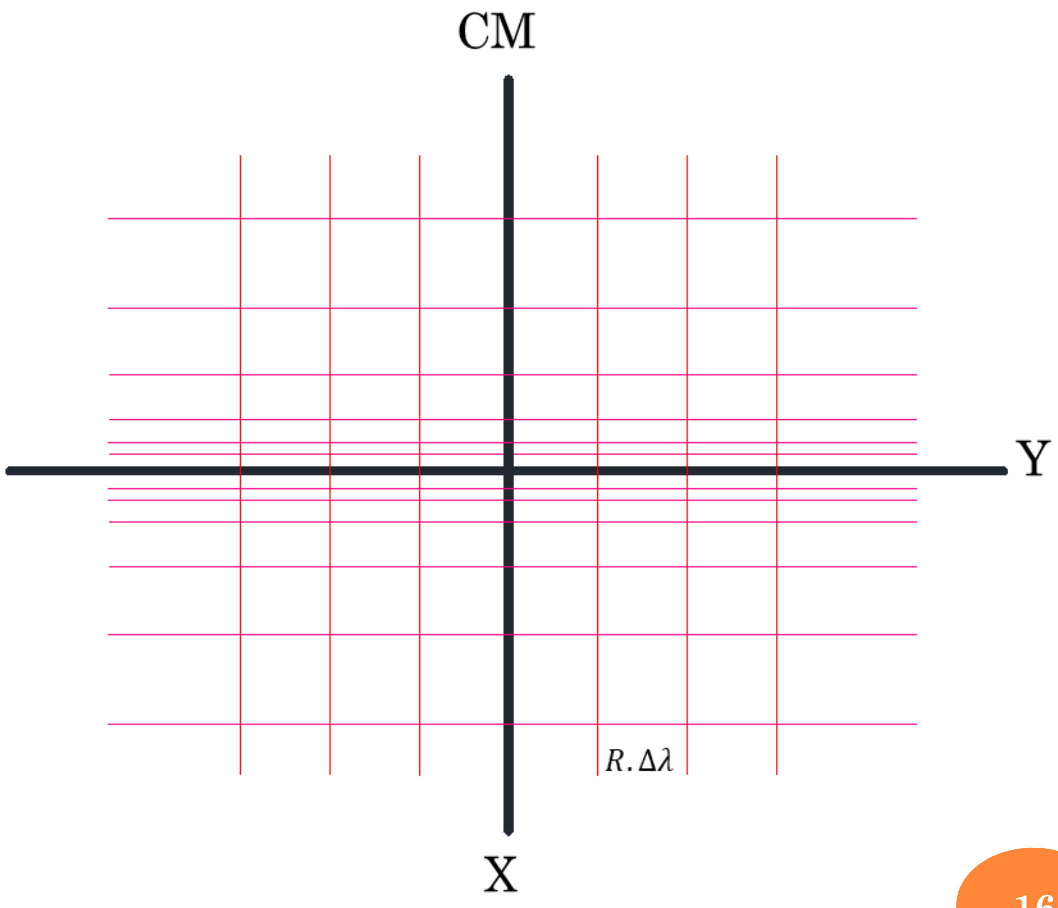
$$Y = R.\Delta\lambda^r \qquad (6)$$



Mathematical Cylindrical Projection

- **(3) Normal Conformal Cylindrical Projection**

- Graticules are rectangular in shape.
- Distances between parallels decrease near the equator and increase with high latitudes.
- Meridians are equally separated by $R.\Delta\lambda^r$.
- Also called "***Mercator***" projection.



Mathematical Cylindrical Projection – Mercator Projection

- A type of map projection introduced in 1569 by Gerardus Mercator.
- It is often described as a cylindrical projection, but it must be derived mathematically.
- The meridians are equally spaced parallel vertical lines.
- The parallels of latitude are parallel horizontal straight lines that are spaced farther and farther apart as their distance from the Equator increases.
- It is widely used for **navigation charts**, because any straight line on a Mercator projection map is a line of constant true bearing that enables a navigator to plot a straight-line course.
- It is less practical for world maps, however, because the scale is distorted; areas farther away from the Equator appear disproportionately large.

Mathematical Cylindrical Projection – Mercator Projection

- The projected coordinates are given by: -

$$x = Rk_o \frac{1}{2} ln \left[\frac{1+B}{1-B}\right] \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (7)$$

Such that: $B = \cos\varphi \sin(\lambda - \lambda_o)$

$$y = Rk_o \arctan\left[\frac{\tan\varphi}{\cos(\lambda - \lambda_o)}\right] \text{\_\_\_\_\_\_\_\_\_\_\_\_} (8)$$

Where,

k_o: Scale factor at central meridian, while the scale factor k at any point is: -

$$k = \frac{k_o}{\sqrt{1 - B^2}} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (9)$$

Mathematical Cylindrical Projection

- **Significance of Mercator Projection.**

- **A loxodrome, also known as a rhumb line** is a line of constant bearing on a map.
- In other words, it is a path on the Earth's surface that crosses all meridians at the same angle.
- Loxodromes maintain a constant compass direction throughout their length, which can be useful for navigation purposes, especially in the era of traditional nautical charts.
- Mercator map preserves loxodromes as straight lines however it is not the shortest path between two points on the earth's surface.

What is the shortest distance between any two points on the earth's surface?

Mathematical Cylindrical Projection

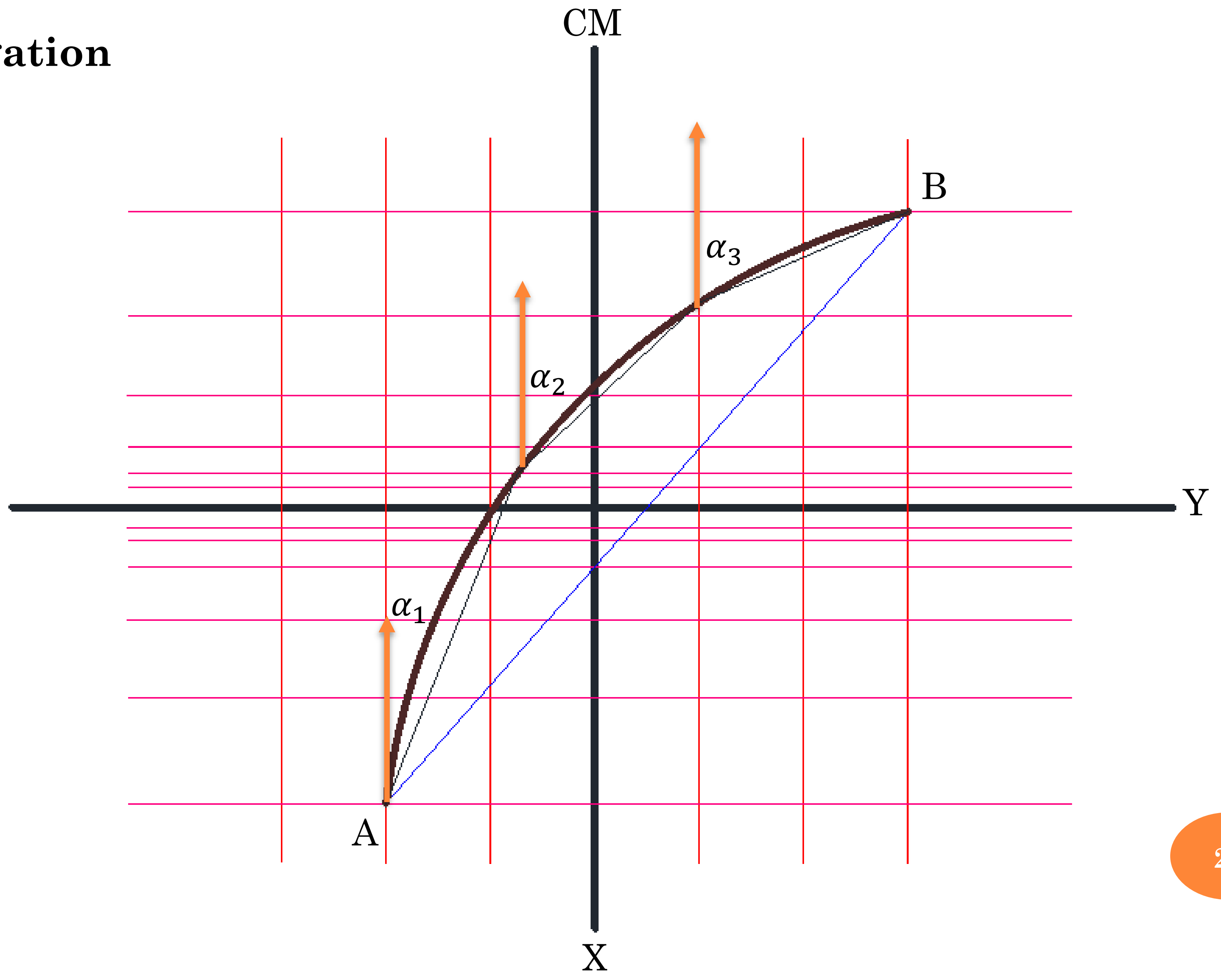
- **Notes on Mercator Projection.**

- **Loxodrome and Navigation**

1. Great circle (curved line) is projected on a Mercator map.
2. The projection of the great circle is divided into short intervals using Loxodrome with azimuths $\alpha_1, \alpha_2, \alpha_3$, etc., therefore, the route is as short as possible, and the sailing is most facilitated.

Notes on Mercator Projection.

- **Loxodrome and Navigation**



Quiz (1)

Lecture 1 to Lecture 3

To join, Please open "Quizzes" app on your Smart Phone and type in the PIN

If you have not installed the app, please use your browser by typing quizzes + join game

THANK YOU

End of Presentation