Fetching the Most Appropriate Global Geopotential Model for Egypt

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Abstract

New additional Global Geopotential Models [GGMs] have now been released into the public domain, those including data from the CHAMP, GREACE and GOCE dedicated satellite gravimetric missions. Those satellite tracking data have resolved the long wave length component of the global gravity field with rather very high accuracy. Therefore, it is important to evaluate those new models over Egypt to determine which of them the most appropriate GGM there is. In this study, a comparison of the performance of three of the GGMs released between 1996 tell now (EGM96, EGM2008 and EIGEN-6C 2011) over Egypt is done. The gravity anomalies computed from the models are compared with point free air gravity anomalies on land. The results have indicated that the outstanding performance of EGM2008 to the other examined GGMs undoubtedly. EGM2008 has1.20 times better statistics than the EGM96 and 2.63 times than EIGEN-6C 2011, in terms of Root Mean Square (RMS).

Keywords: Global Geopotential Models (GGMs), Satellite Gravimetric Mission, Gravity Anomalies, Root Mean Square (RMS).

1. Introduction

Any element of the gravity field in local or regional areas are usually determined by combining the spherical expansion of the earth's potential (geopotential model) and a set of observed points or mean anomalies. The computation can be performed using e.g. Least Squares Collocation procedures, FFT, Stock's integral function,… etc. The determination of any element of gravity field is a repetitive task which, should be updated with time, as far as new gravity field data are collected and/or refined computational approaches are applied or new GGMs are released into the public domain **[3 and 4].** The higher accuracy of geoid computation required nowadays necessitates the need for an accurate GGM, which in turn necessitates the need for examining the performance of such newly released models in any local area to choose the best of them. Many of such studies have been done before in Egypt such as;**[1], [2], [5], [6], [9], [11] and [12]**.This study aims also to evaluate the behavior of those new models over Egypt to determine which; of them is the most appropriate GGM there.

2. Elements of the Earth's Gravity Field, Background and Relations

The spherical harmonic representation of the Earth’s gravitational potential **[14**], could be,

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n-2}^{\infty}\left[\frac{a}{r}\right]\sum_{m=-n}^{n}\bar{C}_{nm}^{s}\,\bar{Y}_{nm}(\theta,\lambda)\right] \quad [1]$$

Where:

r is the geocentric distance; θ is the geocentric co-latitude; and λ is the longitude; GM is the geocentric gravitational constant and "a" usually the equatorial radius of adopted mean earth ellipsoid is scaling factor associated with the fully normalized spherical "s" geopotential coefficients, $\bar{C}_{nm}$,

$$\bar{Y}_{nm}(\theta, r) = \bar{P}_{n|m|}(cos\theta).\begin{cases} cosm\lambda & if\ m < 0 \\ sin|m|\lambda & if\ m < 0 \end{cases} \qquad [2]$$

Where:

$\bar{P}_{nm}$ *[cosθ]* are the fully normalized associated Legendre functions of the first kind.

The disturbing potential T at a point $P\ [\ r\ ,\ \theta\ ,\ \lambda]$ is the differences between the actual gravity potential of the Earth and the normal potential of equipotential ellipsoid at P. Based on equation [1] the spherical harmonic representation of T is :

$$T(r, \theta, \lambda) = \frac{GM}{r}\left[1 + \sum_{n-2}^{\infty}\left[\frac{a}{r}\right]\sum_{m=-n}^{n}\bar{C}_{nm}^{s}\,\bar{Y}_{nm}(\theta, \lambda)\right] \qquad [3]$$

The above formula has been expanded in several processes to get any element of the earth's gravity field. The relationship between the coefficient of any spherical harmonic model and gravity anomalies $[\Delta g_{GM}]$ is given **[15]** as follows:

$$\Delta g_{GM} = \frac{GM}{r^2}\left[\sum_{n-2}^{n_{max}}(n-1)\left[\frac{a}{r}\right]^n\sum_{m=0}^{n}\bar{C}_{nm}^{s}\,cosm\lambda + \bar{C}_{nm}^{*}\,sin\,m\lambda\right]\bar{P}_{nm}(sin\emptyset) \qquad [4]$$

Where:

n_{max} is the maximum degree;

n, m is the degree and order respectively;

$\bar{C}^{*}_{nm}$ the relevant fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the WGS-84 reference ellipsoid,

$\bar{S}_{nm}$ the relevant fully normalized spherical harmonic S-coefficients of degree n and order m,

ϕ, λ is the geocentric latitude and longitude;

a is the scaling factor and r is the geocentric distance.

3. The Available Used Data

The local gravity data which are used in this study were all old available free-air gravity anomalies at 1440 points, where the sources of these data, their number and distribution are well documented in many previous works as e.g. **[5]**, **[6]**, **[7]**, **[8]** and **[13].** These old data were firstly, grouped in two sets as shown in figure (1).

The new free-air gravity anomaly values at 333 points were obtained from BGI [Bureau Gravimetric International], where their observational mean standard deviation is (0.24 mgal], while the standard deviation estimated for older gravity anomaly data distributed all over the whole territory of Egypt is 0.73 mgal in average. As can be seen from figure [1], free air gravity data distribution is not homogeneous over Egypt with significant gaps, particularly in the eastern and western deserts. The validation of the finally used data here were based on comparing the gravity value of each point to values at the nearest four surrounding stations, to identify any large discrepancies (more than 3segma), which were subsequently removed. After validation, the remaining used data were 1104 old data plus 333 new data obtained from BGI with observational standard deviation as shown in table [1].

Table (1): The raw and filtered data

Item	Data No. before filtration	Data No. after filtration	Average standard deviation (mgal)
Gravity anomalies [old]	1440	1104	0.73 -0.67
Old anomalies [first region]	-	333	0.45
Old anomalies [second region]	-	333	0.35
Gravity anomalies [new] [BGI]	333	333	0.24
Validated anomalies [old + new]	-	1437	0.63

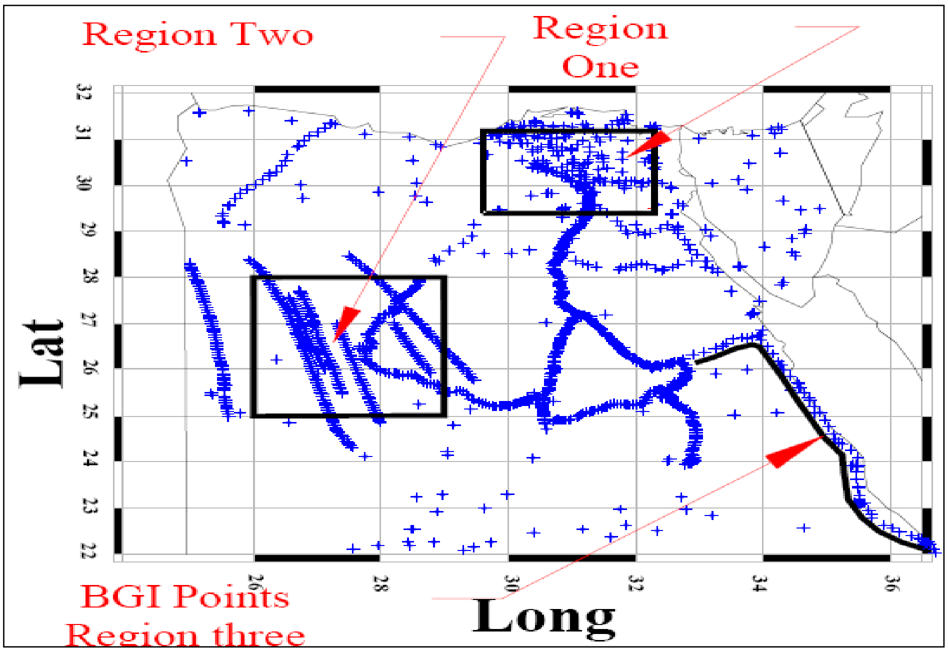


Figure (1): The Free Air Gravity Anomaly Data Three Highly Intensive Regions

4. Evaluation, methodology and used software

According to the available data and its distribution over Egypt, the evaluation process has been done once for three elected regions of highly intensive data points; 333 points, see figure (1). Two of these regions (region one lies between 29.5° to 31.5° N and 29.5° to 32.5° E and region two lies between 25° to 28° N and 26° to 29° E) contain old data only with different standard deviations, see table (1), while the third region contains the 333 new data points from BGI. The evaluation process was then repeated once more over the whole area of Egypt to check and confirm the results. The behavior of any model is judged here in this research through its precision and accuracy, where the model precision referred to different examined regions is determined in terms of standard deviation of the residual and computed as:

$$\sigma_{\Delta g_r} = \sqrt{\frac{\sum_{i=1}^{n}\left(\Delta g_{r_i} - \overline{\Delta g_r}\right)^2}{n-1}} \qquad [5]$$

Where:

$\Delta g_{r_i} = (\Delta g_i - \widehat{\Delta g_i})$,

$\overline{\Delta g_r} = \frac{1}{n} \sum_{i=1}^{n} \Delta g_{r_i}$ [6]

While the accuracy of the model is represented in term of root mean square (RMS) of the residual computed as follows:

$$\text{R.M.S.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (model_i - obseved_i)^2}$$

$$\text{R.M.S.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\Delta g_i - \widehat{\Delta g_i})^2} \quad [7]$$

4.1. Used software

The software used in computation is the Gravsoft 292 package [16] supplied by Prof. Tscherning, C.C. to whom we are much indebted. This package has been very valuable, since it contains many reliable programs used in calculations. We also feel very much indebted to personals of BGI, who agreed to supply us with some valuable gravity anomaly data of Egypt**.**

5. Results and Analysis

Tables (2 through 4) show the statistics of the comparison among the terrestrial gravity anomalies and those computed from the different harmonic models at scattering points of the three chosen regions.

Table (2): Statistics of the first region

Free air gravity anomaly[region 1]	**Min** mgal	**Max** Mgal	**Mean** mgal	**R.M.S**. mgal	**S. D. Of the residual** mgal
G.A$_{terrs}$-EGM96	-94.351	76.434	-2.451	24.812	24.524
G.A$_{terrs}$-EGM2008	-127.390	44.765	0.411	18.870	18.369
G.A$_{terrs}$-EGM2011	-29.034	203.773	105.619	57.499	57.787

Table (3): Statistics of the second region

Free air gravity anomaly[region2]	**Min** Mgal	**Max** Mgal	**Mean** mgal	**R.M.S.** mgal	**S. D. Of the residual** mgal
G.A$_{terrs}$-EGM96	-45.475	64.395	-9.766	16.030	15.857
G.A$_{terrs}$-EGM2008	-44.281	67.209	-11.933	14.953	14.358
G.A$_{terrs}$-EGM2011	125.059	302.591	184.821	29.240	31.309

Table (4): Statistics of the third region

Free air gravity anomalies[BGI]	**Min** Mgal	**Max** Mgal	**Mean** mgal	**R.M.S.** mgal	**S. D. Of the residual** mgal
$G.A_{terrs}$-EGM96	-44.969	43.809	4.833	12.422	12.443
$G.A_{terrs}$-EGM2008	-51.494	41.809	-1.030	11.383	11.399
$G.A_{terrs}$- EGM2011	125.059	302.591	184.821	27.810	28.144

Table (5) represents the results of the comparison among the terrestrial gravity anomaly data at scattering points over the whole territory of Egypt and those computed from the different harmonic models at the same scattering points.

Table (5): Statistics of the whole territory of Egypt

Free air gravity anomalies	**Min** mgal	**Max** mgal	**Mean** mgal	**R.M.S.** mgal	**SD of the residual** mgal
$G.A_{terrs}$-EGM96	-144.228	153.364	0.278	26.576	27.674
$G.A_{terrs}$-EGM2008	-98.474	131.428	1.901	19.943	19.710
$G.A_{terrs}$- EGM2011	67.535	390.737	214.911	61.200	60.039

The larger values of RMS and SD of EGM2011 shown in columns four and five in tables (2 through 5) is due to the source of data, satellite only tracking data, that mostly used to produce this model, which mainly recover the long wave length of the field.

Figures (3 through 5) and figures (6 through 8) represent the difference between terrestrial gravity anomalies and the corresponding gravity anomalies obtained from the three models, EGM96, EGM2008 and EGM2011-EIGEN6C respectively, relevant to the first and second elected areas of dense data, (first and second regions) respectively, where (latitude and longitude are in degree while gravity anomalies are in mgal).

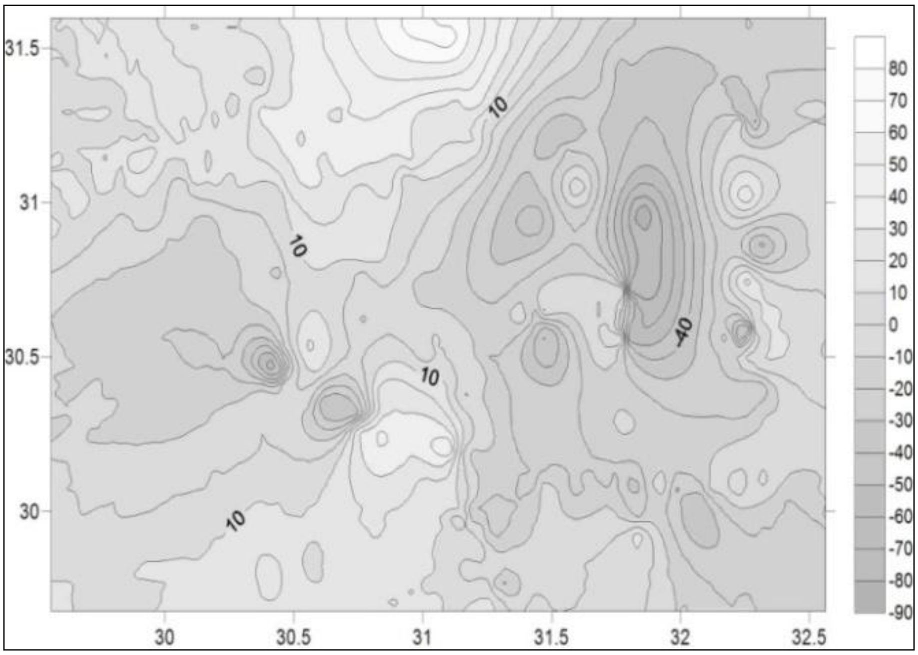


Figure (3): Δg_f difference in the first region referred to EGM96

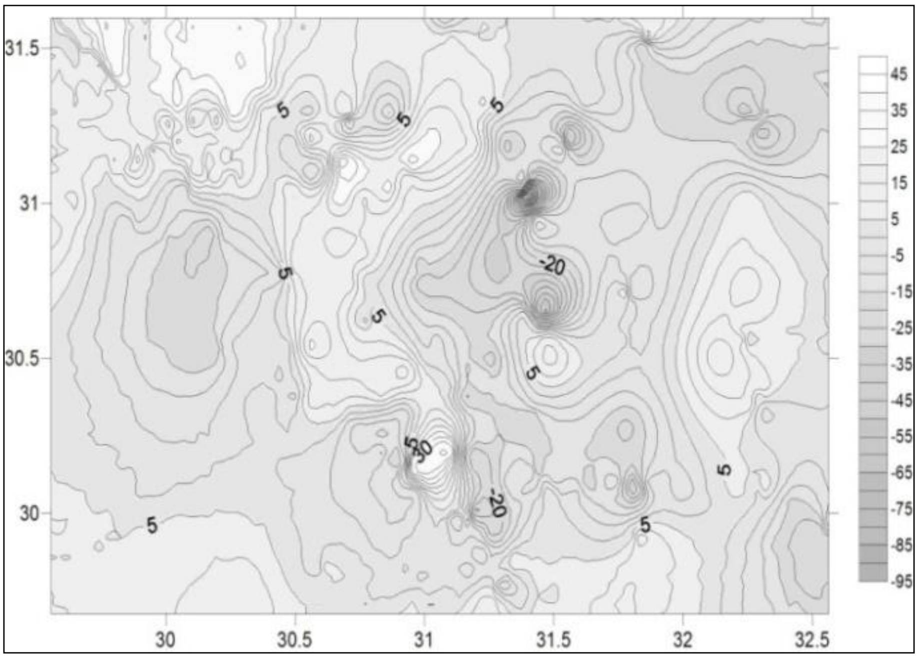


Figure (4): Δg_f difference in the first region referred to EGM2008

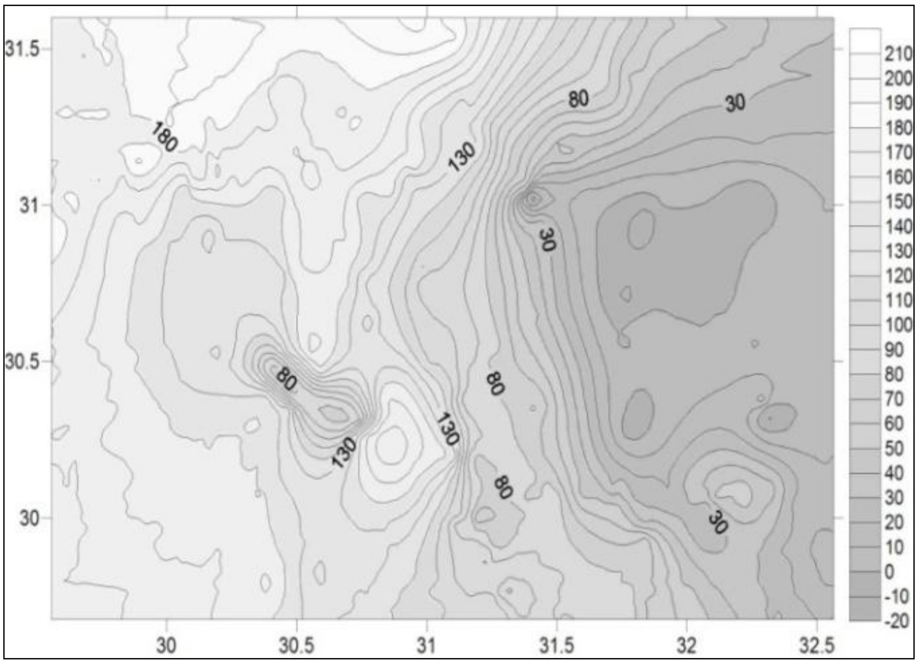


Figure (5): Δg_f difference in the first region referred to EGM2011

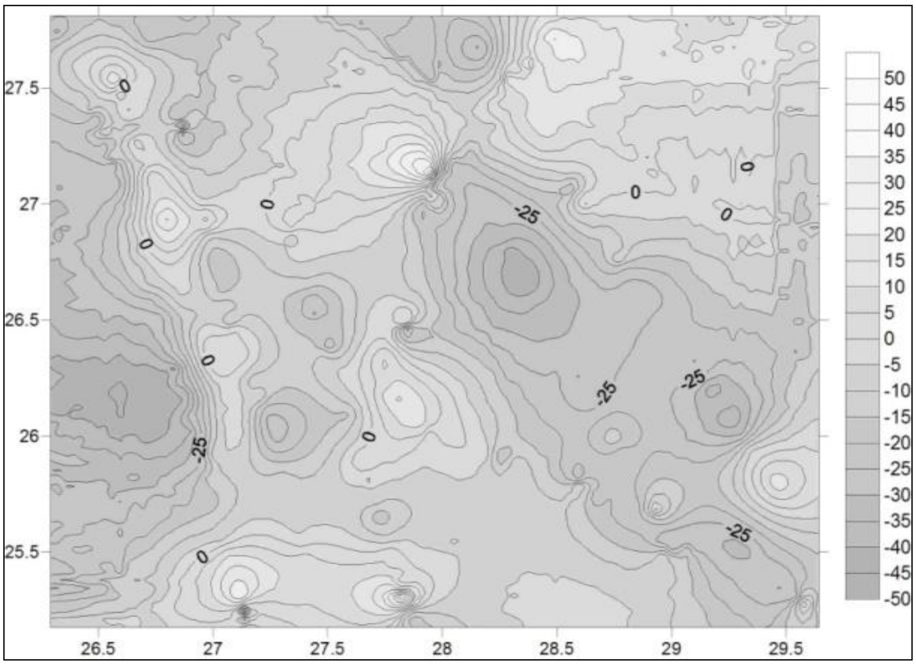


Figure (6): Δg_f difference in the second region referred to EGM96

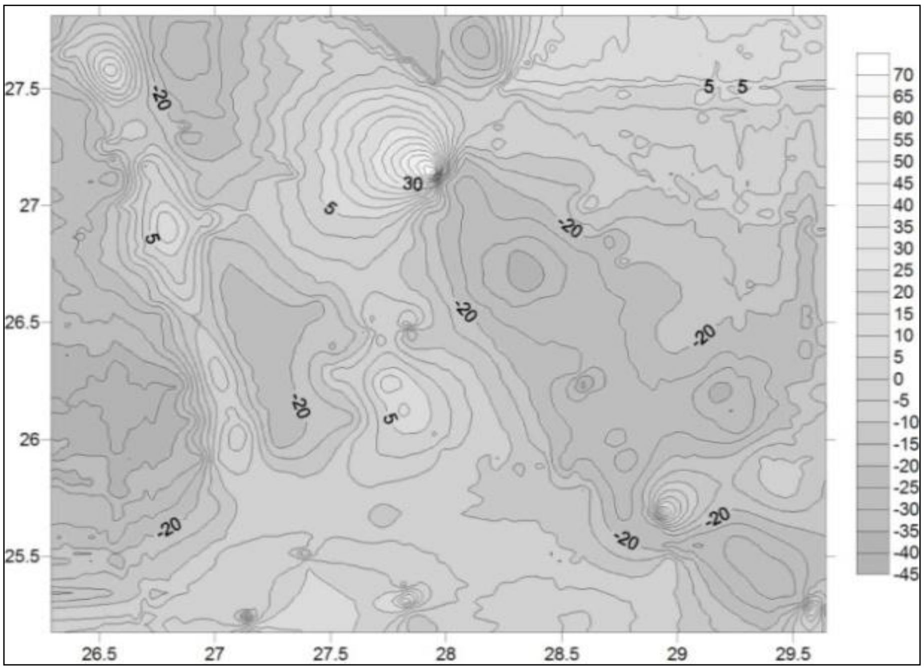


Figure (7): Δg_f difference in the second region referred to EGM2008

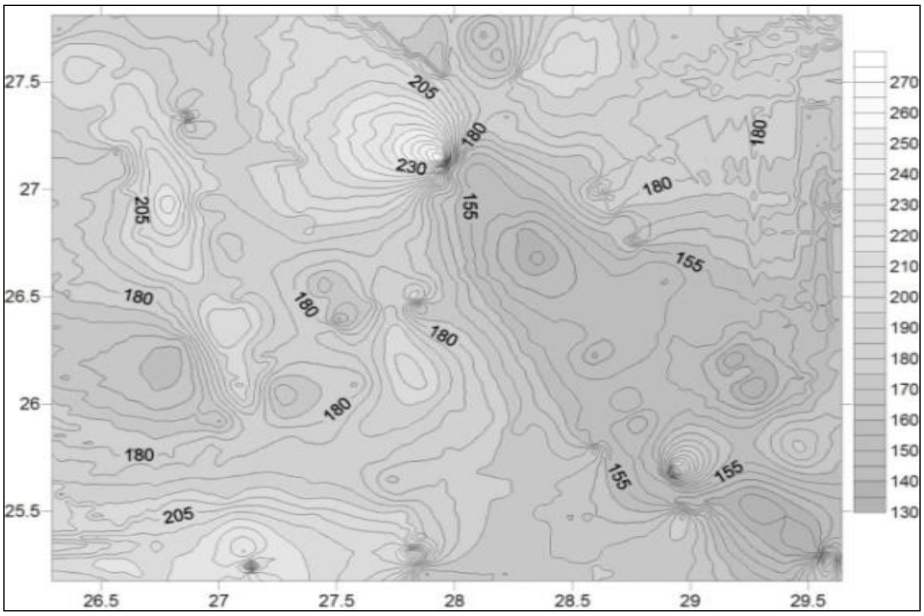


Figure (8): Δg_f difference in the second region referred to EGM2011

Figures (9), (10), and (11) represent Δg_f difference between terrestrial gravity anomaly data and those computed at the same observational points from the three models; EGM96, EGM2008 and EGM2011-EIGEN6C respectively, for the whole area of Egypt, where [latitude and longitude are in degree while gravity anomalies are i mgal.

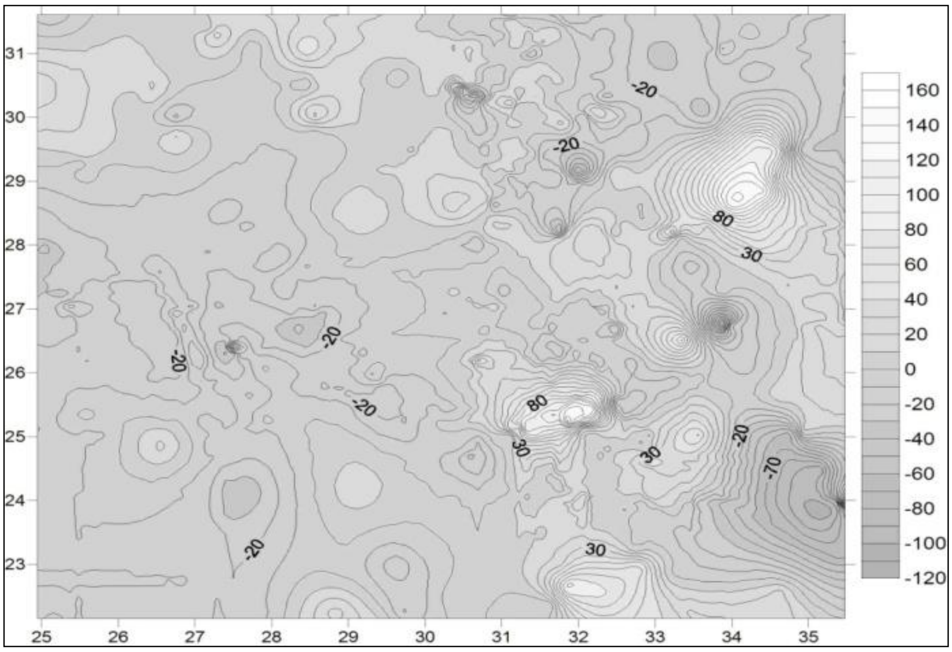


Figure [9], Δg_f difference referred to EGM96 for the whole area of Egypt

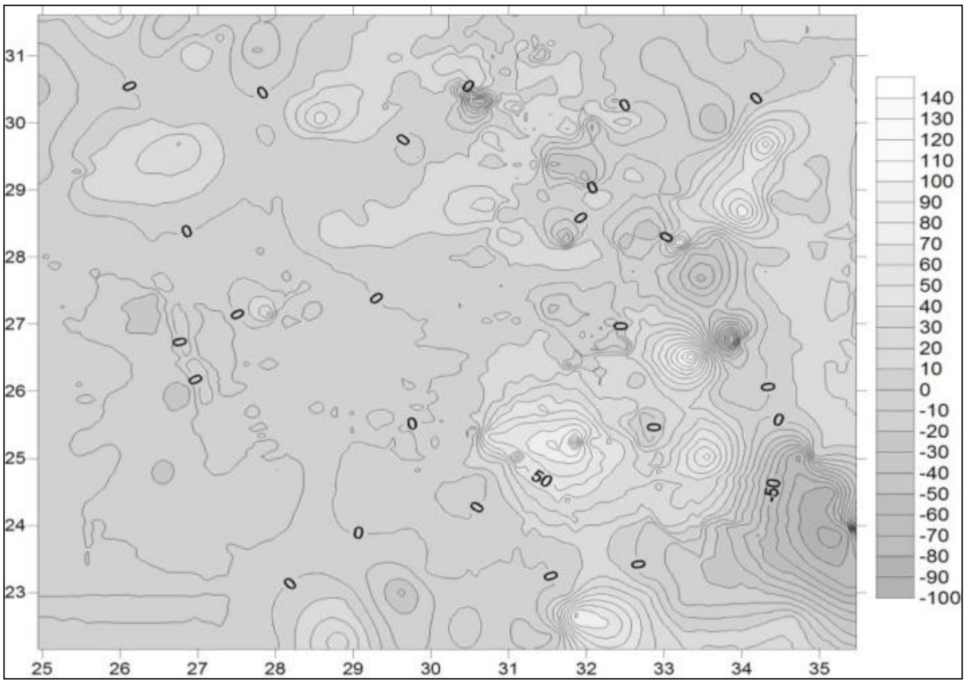
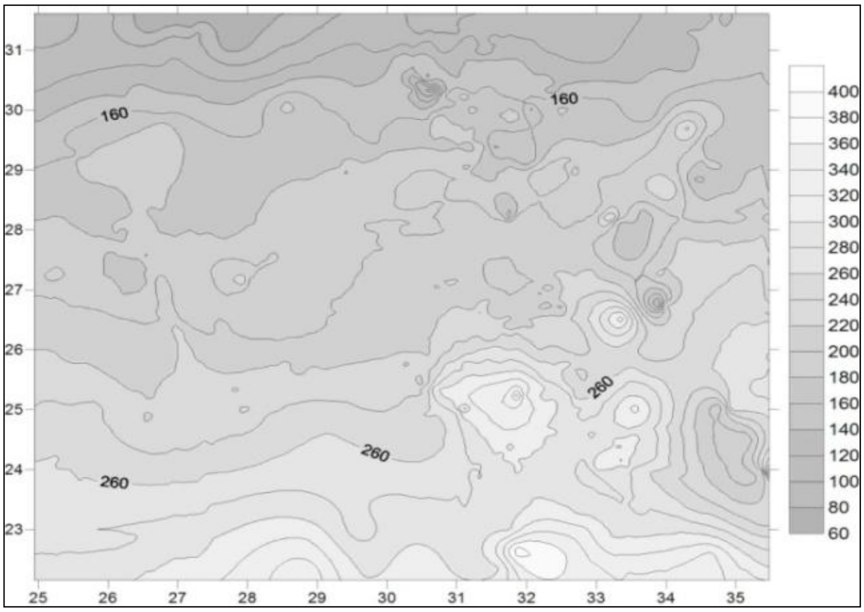


Figure [10], Δg_f difference referred to EGM2008 for the whole area of Egypt

Figure 

[11], Δg_f

difference referred to EGM2011for the whole area of Egypt

6. Conclusion and Recommendations

From the results shown in tables [2] through [4] we can notice that the best model of the three is EGM2008, since it gives the least [R.M.S.] compared with the other two models when applied over the three regions. The graduation in the values of the obtained [R.M.S.], related to EGM2008 model with respect to the three elected regions, that is [18.870 mgal], [14.953 mgal] and. [11.380 mgal] respectively, seems to be reasonable and was logically expected, since the observational mean standard deviations, referred to the three regions shown in table [1], of the terrestrial data used in their computation, were also graduated as; [0.45 mgal], [0.35 mgal] and [0.24 mgal] respectively. The results shown in tables [5] of the GGMs evaluation over the whole area of Egypt have indicated that the smallest [R.M.S.] is [19.94 mgal], referred also to EGM2008, which confirms the conclusion drawn on the previous paragraph i.e. this model is the best of the three. The reason that this value of [R.M.S.] is greater than those refereed to the same model but relevant to the three regains, given in tables
[2 through 4], is due to existence of several gap areas; empty of terrestrial data in the whole territory of Egypt.
The values of the standard deviations of residual shown in last column in tables
[2 through 5] of each model relevant to different regions, confirm also the previous conclusion, i.e. EGM2008 is the best of the three models.
Finally; from the values of R.M.S. shown in tables [2 through 5] we can conclude that EGM2008 has thus about 1.20 times better statistics than the EGM96 and 2.63 than EIGEN6C-EGM2011 in average , the jump over the last model is because data mostly used to produce the last model, are satellite only tracking data, which mainly recover the long wave length of the field, therefore; EGM2008 makes a significant improvement over all other models, and thus is advised to be used for geoid determination in Egypt.

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